Oscillations

P#1 15.25 - SHM vs. Uniform circular motion.

While riding behind a car travelling at 3.00 m/s you notice a small bump on the rim of the tire.

a) Explain why the bump appears to execute SHM

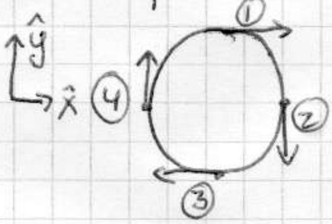
b) If the tire's radius is 0.300 m, what is the bump's period of oscillation.

(a) From behind the car you have a view like:

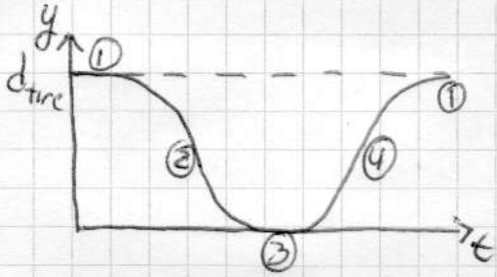
∴ You can only see the bump move in 1^D.

↳ recall: projection of uniform circular motion onto a single dimension → simple harmonic motion.

Explanation



side view of tire w/ velocities.



Since you can view the bump's motion only in $\hat{y}$, you need only consider the $\hat{y}$-component of the velocity.

at ① & ③ all velocity is $\hat{x}$ so no $\hat{y}$ motion
at ② & ④ the opposite happens
} reflected in plot.

Conclusion: 2^D Uniform Circular Motion ⇒ 1^D SHM

(b) Diameter: $d = 2\pi r$

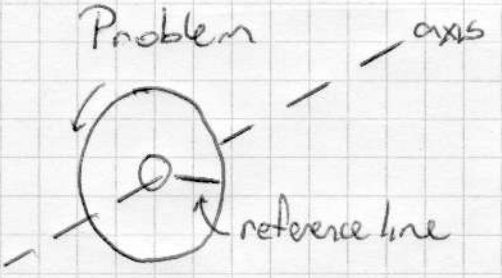
Period: $T = \frac{d}{v} = 0.6283$ s

∴ The period of oscillation is 0.628 s

P#5 - Angular Variables

Main point: Angular equations are analagous to the linear equations.

Problem



A grind stone (with a reference line) has constant angular acceleration ($\alpha = 0.35 \text{ rad/s}^2$). Assuming the wheel has initial angular velocity $\omega_0 = -4.6$ rad/s and starts at $\theta = 0$...

a) When will the wheel momentarily stop?

$$v = v_0 + at \rightarrow \omega = \omega_0 + \alpha t$$

$$\therefore t = \frac{\omega - \omega_0}{\alpha} = \frac{0 - (-4.6)}{0.35} = 13.1 s$$

$\therefore$ the wheel will momentarily stop at $t = 13$ s

b) At what time will the reference line have rotated such that its displacement is five revolutions in the positive direction?

$$x = x_0 + v_0 t + \tfrac{1}{2} a t^2 \rightarrow \theta = \theta_0 + \omega_0 t + \tfrac{1}{2} \alpha t^2$$

$$\therefore 5(2\pi) = 0 - 4.6t + \tfrac{1}{2}(0.35)t^2$$

Re arrange $t^2 - 26.3t - 180 = 0$

$$\therefore t = 31.9, -5.64$$

$\therefore$ the wheel will be at the desired position at

$t = 31.9$ s

P#6 - More challenging angular variables

Problem: A wheel rotates 40 times as it slows from an angular velocity of 1.5 rad/s to a stop.

$\theta_f = 40(2\pi) = 80\pi$ rad
$\omega_f = 0$ rad/s
$\omega_o = 1.5$ rad/s

(a) Assuming uniform acceleration, how long does it take it to stop?

Recall

$$\theta_f = \theta_o + \omega_o t + \tfrac{1}{2}\alpha t^2 \qquad (1)$$

$$\omega_f = \omega_o + \alpha t \Rightarrow \alpha t = \omega_f - \omega_o \qquad (2)$$

Sub (2) -> (1)

$$\theta_f = \theta_o + \omega_o t + \tfrac{1}{2}(\omega_f - \omega_o)t = \theta_o + \tfrac{1}{2}\omega_o t + \tfrac{1}{2}\cancelto{0}{\omega_f} t$$

$$\therefore t = \frac{2(\theta_f - \theta_o)}{\omega_o} = \frac{2(80\pi)}{1.5} = 335\ \text{s}$$

$\therefore$ it takes the wheel 340 s to stop.

(b) What is the wheel's angular acceleration?

from (2) $\alpha = \frac{\omega_f - \omega_o}{t} = \frac{-1.5}{335} = -4.48$ mrad/s^2

$\therefore$ the wheel's angular ~~vel~~ acceleration was -4.5 mrad/s^2

(c) How long does it take to complete its first 20 revolutions

$$\theta_f = \theta_o + \omega_o t + \tfrac{1}{2}\alpha t^2$$
$$\Rightarrow 20(2\pi) = 0 + 1.5t - 0.5(4.48\times10^{-3})t^2$$

$\therefore t = 98.2,\ 572$ s ← to come back to 20 once $\omega < 0$

$\therefore$ it takes 98 s to complete 20 revolutions

#12 from text (Cptr. 15)

P#7 - Simple Intro to springs and simple Harmonic Motion (SHM)

Goal: Use solution to SHM.

Problem: A 1.00 kg glider is attached to a 25.0 N/m spring and sits on a frictionless surface. At time $t=0$ it is at $x=-3.00$ cm.

1 a) Once released find the period of oscillation

3 b) Find maximum speed & acceleration

2 c) Find $x(t)$, $v(t)$, $a(t)$

↑ book order
my order

Solution:

a) Recall eq^n of motion $F = ma = m\ddot{x} = -kx$

$$\therefore \ddot{x} = -\frac{k}{m}x = -\omega^2 x \quad \text{where} \quad \omega = \sqrt{\frac{k}{m}}$$

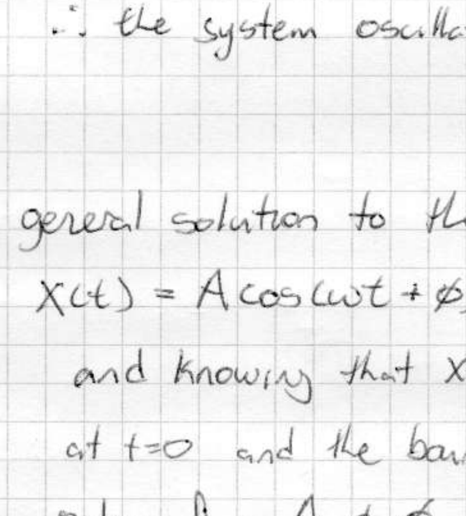
now $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{1}{25}} = \frac{2\pi}{5} = 1.257\ s$

$\therefore$ the system oscillates ~~at~~ with a period of 1.26 s

(c) A general solution to the equation of motion is:

$x(t) = A\cos(\omega t + \phi)$ where we know ω from above and knowing that x is at its maximum (negative) value at $t=0$ and the boundary condition [$x(0) = -3$ cm] lets us solve for A & ϕ.

Pictorally.



'see' $A = -3$ cm
$\phi = 0$
↑ start at max amplitude

alternate
$A = 3$ cm
$\phi = \pi$

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{25}{1}} \quad \left(\text{alternative } \omega = \frac{2\pi}{T}\right)$$

$$\therefore \; x(t) = -3\cos(5t) \text{ cm}$$

$$v(t) = \frac{dx}{dt} = 15\sin(5t) \text{ cm/s}$$

$$a(t) = \frac{dv}{dt} = 75\cos(5t) \text{ cm/s}^2$$

b) Since $-1 \leq \cos(\theta) \geq 1$ for all θ

$$\therefore \; v_{max} = 15 \text{ cm/s} = 0.15 \text{ m/s}$$

$$a_{max} = 75 \text{ cm/s}^2 = 0.75 \text{ m/s}^2$$