The dissipation of transient gravity waves propagating in a shear flow

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Transient gravity waves that are propagating in a shear flow toward their critical levels are examined analytically. Three breaking regimes as a function of frequency spectrum width and the wave amplitude are found. When viscosity is considered, numerical experiments show that high transient waves never reach the convective instability threshold.

Theory: quasi-optic approximation

The horizontal velocity perturbation for a gravity wave superposition under WKB approximation and a non-rotating medium

$$u(x,z,t)=\frac{1}{2\pi}\iint \hat{u}(\omega)\left(\frac{m(\omega,z)}{m(\omega,0)}\right)^{1/2}\exp(i\psi)\,d\omega$$

with $\psi=\omega t-kx-\int_0^z m(\omega,z')\,dz'$ **and the dispersion relationship**

$$m=\frac{N_0 k}{\omega-ku_0(z)} \qquad (1)$$

ω : absolute frequency
k : horizontal wavenumber
m : vertical wavenumber
u_0 : basic zonal background wind
N_0 : basic buoyancy frequency
Ri_0 : basic Richardson number

Considering a Gaussian spectrum at $z=0$ **with central frecuency** ω_c **and spectral width** σ. ψ **is expanded in Taylor series up to second order in** $(\omega-\omega_c)$ **and the amplitude term is expanded only to first order, resulting**

$$u(x,z,t)=\frac{\hat{u}_0}{\sigma}\left(\frac{m_c}{m_{ic}}\right)^{1/2}\frac{\exp\left[i(\psi_c-\theta/2)-(\partial_\omega\psi)^2\left(2\left(\sigma^{-2}-i\partial^2_{\omega\omega}\psi_c\right)\right)^{-1}\right]}{\sqrt[4]{\sigma^{-4}+\left(\partial^2_{\omega\omega}\psi_c\right)^2}}$$

The maximum amplitude of the envelope occurs when $\partial_\omega\psi_c=0$,

$$a(t)=\hat{u}_0\left(\frac{m_c}{m_{ic}}\right)^{1/2}\left[1+\sigma^4\left(\partial^2_{\omega\omega}\psi_c\right)^2\right]^{-1/4} \qquad (2)$$

with a height width

$$\Delta z(t)=\Delta z_i\left(\frac{m_{ic}}{m_c}\right)^2\left[1+\sigma^4\partial^2_{\omega\omega}\psi_c\right]^{1/2} \qquad (3)$$

Assuming a linear background wind, it is obtained

$$a(t)=\frac{\hat{u}_0}{\sigma}\frac{\left(1+Ri^{-1/2}\omega_c t\right)^{1/2}}{\left[1/\sigma^4+(t/\omega_c)^2(2+Ri^{-1/2}\omega_c t)^2\right]^{1/4}} \qquad (4)$$

$$\Delta z(t)=\Delta z_i\frac{\left[1+(\sigma^2 t/\omega_c)^2(2+Ri^{-1/2}\omega_c t)^2\right]^{1/2}}{\left(1+Ri^{-1/2}\omega_c t\right)^2} \qquad (5)$$

Numerical Experiments

The numerical simulations were performed with a numerical model descript in Pulido (2005). This model solves numerically in the spectral space, the following equation,

$$d_z^6 w_1-\left[\frac{(\kappa+\nu)\Omega}{\kappa\nu}\right]d_z^4 w_1-\left[\frac{\Omega^2}{\kappa\nu}\right]d_z^2 w_1-\left[\frac{N_0^2k^2}{\kappa\nu}\right]w_1=0 \qquad (6)$$

w_1 **is the vertical velocity perturbation,** $\Omega=\omega-ku_0$, κ **is the thermal diffusivity coefficient and** ν **is the kinematic viscosity coefficient. Then, this solution is Fourier transformed.**
The background is characterized by $N_0=10^{-2}s^{-1}$, $Ri=100$ **and by a linear wind** $u_0=N_0Ri^{-1/2}z$ **. Typical values of the middle atmosphere for** κ **and** ν **are used.**
At $z=0$, **the wave packet modes conform a Gaussian spectrum in frequency centered at** ω_c**. The radiation condition is imposed in order to keep only upward waves. The vertical resolution was set at** 2m**. The disturbance is periodic in** x.

Results

The waves become unstable when $N^2=N_0^2-N_0\overline{\partial_z u}\le 0.$
Three breaking regimes are found under the quasi-optic approximation as a function of the $\beta=(\sigma/\omega)Ri^{1/4}$ **and of the wave amplitude (fig.3). Waves with large initial amplitude and small** β **attain the convective instability (**$N^2=0$**) before they reach the maximum wave amplitude. Waves with small initial amplitude and large** β, **break near the critical level for long times.**
The viscous effects were included in the numerical model. In this case, the transient waves attain their convective instability at longer times and higher altitudes compared to the inviscid case. Moreover, high transient gravity waves never attain the convective instability threshold.

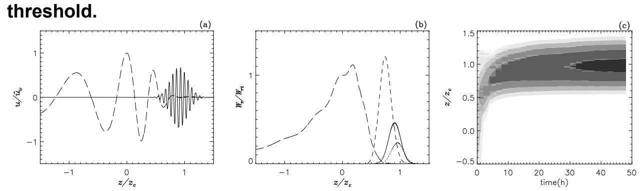

Figure 1: (a) Profile of u_1 **obtained with the inviscid numerical model at t=0h (long dashed) and t=12h (continuous) with** $\sigma=0.15\ \omega$ **and** $Ri=100$**. (b) Wave energy density at t=0h (long dashed), 4h (dashed), 12h (continuous) and 24h (dotted). (c) Path of the envelope of** $\partial_z u_1$ **(lighter contours show lower values).**

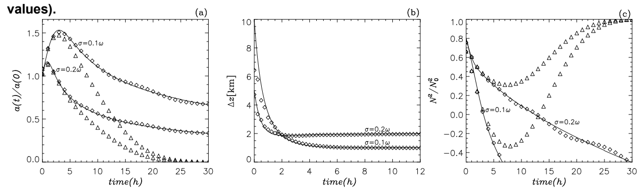

Figure 2: (a) Evolution of the wave amplitude. (b) Evolution of the height width. (c) Evolution of the minimum buoyancy frequency for $\tilde{u}=0.2$**. Numerical Results for** $\nu=\kappa=10^{-1}m^2s^{-1}$**(Δ) and for the inviscid case (◊). Results of quasi-optic approximation (continuous line).**

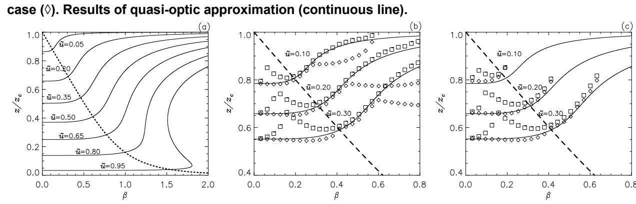

Figure 3: Breaking height as a function of β **for different initial amplitudes** $\tilde{u}=uk/\omega$**. (a) Theoretical results. (b) Numerical results for the inviscid case. (c) Numerical results for** $\nu=\kappa=10^{-1}m^2s^{-1}$ **(height ~ 70km). Minimum breaking height for all time (Δ). First breaking height (□). Quasi-optics results (solid lines). Height of the maximum wave amplitude for quasi-optics (dashed curve).**

Conclusions

- **The quasi-optic approximation enables to relate the initial wave and the medium characteristics with the evolution of wave field. It enables to obtain the dependencies of the convective instability height (a quantity that may be useful for gravity parameterizations) on the Richardson number, spectral with and initial amplitude.**
- **If viscous effects are considered, transient waves with small amplitude and broad spectral width never reach their convective instability threshold. In these cases the momentum deposition is governed by viscosity.**

References

Pulido M, 2005. *Q. J. R. Meteorol. Soc.*, 131, 1215-1232.

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