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Quantum Mechanics-I

Useful Book : Quantum Mechanics
Stephen Gasiorowicz

Chapter 1

The Limits of Classical Physis

The end of 19th century
The begining of 20th " witnessed $\rightarrow$ a crisis in physics

A series of experimental results required concepts totally incompatible with classical physics.

$\longrightarrow$ { The particle properties of radiation
The wave properties of matter.

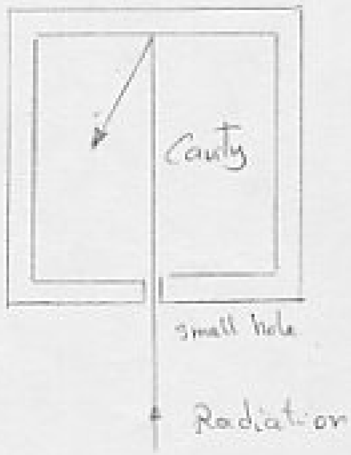
A Black Body Radiation:

Def. - A black body is an object that is perfect absorber of radiation of all wave length and reradiates the energy with universal wavelength spectrum.

The universal (black body) spectrum $= f$(T of the object)
$\neq f$(shape, size, chemical composition --)

To a reasonable approx., all matter in thermal equilibrium behaves like a black body.

- The radiation inside the cavity is in thermal equilibrium with the walls of the cavity and is continuously being absorbed and reemitted by the cavity walls.



Model of blackbody

The spectrum of the escaping radiation is identical to the spectrum inside the cavity.
If the temperature of the cavity is const., then as much energy escapes from the cavity as enters the cavity.

How much energy per volume in the form of radiation is inside the cavity?

Consider a cubical cavity;

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right) F = 0$$

$F(x,y,z,t)$: The components of electric and magnetic fields

$$\frac{\partial^2 F}{\partial x^2} + \frac{\partial^2 F}{\partial y^2} + \frac{\partial^2 F}{\partial z^2} = \frac{1}{c^2}\frac{\partial^2 F}{\partial t^2} \qquad (1)$$

One sol.:

$$F = C \sin(k_1 x) \sin(k_2 y) \sin(k_3 z) \sin(\omega t)$$

The boundary cond. : The electric field must vanish at the boundaries of the cube.

$(x=0, y=0, z=0, x=L, y=L, z=L)$

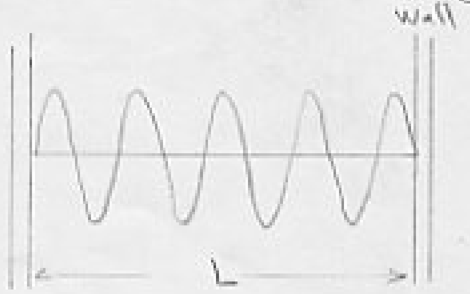
$\rightarrow K_i = \dfrac{n_i \pi}{L}$ $\quad i=1,2,3 \quad$ n_i : positive integers

also $\omega = \dfrac{2\pi c}{\lambda}$ $\quad \rightarrow F = C\,S\cdot\left(\dfrac{n_1\pi x}{L}\right) S\cdot\left(\dfrac{n_2 \pi y}{L}\right) S\cdot\left(\dfrac{n_3\pi z}{L}\right) S\cdot\left(\dfrac{2\pi c t}{\lambda}\right)$

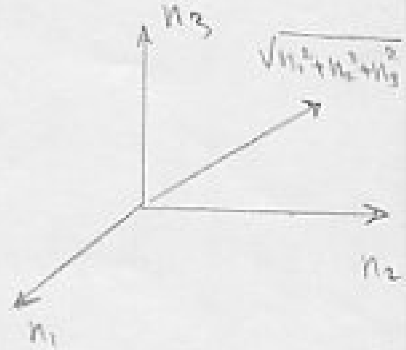
(2) standing wave

(2) in (1) $\longrightarrow \left(\dfrac{n_1\pi}{L}\right)^2 + \left(\dfrac{n_2\pi}{L}\right)^2 + \left(\dfrac{n_3\pi}{L}\right)^2 = \left(\dfrac{2\pi}{\lambda}\right)^2$

$\rightarrow n_1^2 + n_2^2 + n_3^2 = \dfrac{4L^2}{\lambda^2}$ (3)



Each (n_1, n_2, n_3) $\xrightarrow{\text{corresponds}}$ to one standing wave



The total number of modes $= \underbrace{\dfrac{1}{8}}\left(\dfrac{4}{3}\pi n^3\right)$

since n_i are positive

$N = (2)\left[\dfrac{1}{8}\left(\dfrac{4}{3}\pi n^3\right)\right] = 2\,\dfrac{1}{8}\,\dfrac{4}{3}\,\pi\,(n_1^2+n_2^2+n_3^2)^{3/2} = \dfrac{\pi}{3}(n_1^2+n_2^2+n_3^2)^{3/2}$ (4)

because of two possible relative orientation of E and B (two polarizations of the radiation)

(3)(4) $\rightarrow N = \dfrac{8\pi L^3}{3\lambda^3}$ number of modes

$\rightarrow -\dfrac{dN}{d\lambda} = \dfrac{8\pi L^3}{\lambda^4}$ (5) number of modes / unit wave length

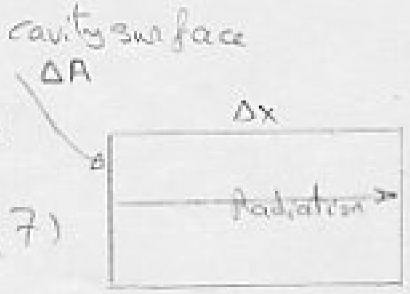
$$\rightarrow -\left(\frac{1}{L^3}\right)\frac{dN}{d\lambda} = \frac{8\pi}{\lambda^4} \quad \text{number of modes} / (\text{unit wave length} \times \text{cavity volume})$$

If $(\text{energy})_{\text{ave}} = KT$ for each mode

Then;
$$\frac{du}{d\lambda} = \underbrace{\left(\frac{1}{L^3}\right)\frac{dE}{d\lambda}}_{\text{The energy per unit wave length in the volume } L^3} = \left(\frac{1}{L^3}\right)\underbrace{(KT)\left(-\frac{dN}{d\lambda}\right)} = \frac{8\pi KT}{\lambda^4} \quad (6)$$

energy / (volume × wavelength)

How much is the power per area radiated from the cavity surface?



Now
$$\frac{dE'}{d\lambda} = 2\left(\frac{dR}{d\lambda}\right)_{\theta=0}(\Delta t)(\Delta A) = \left(\frac{dR}{d\lambda}\right)_{\theta=0}\frac{2\Delta x \Delta A}{c} \quad (7)$$

$\Delta t = \frac{\Delta x}{c}$

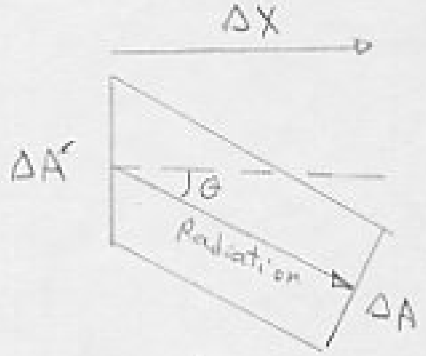
$\frac{dE'}{d\lambda}$: The energy per unit wave length in the volume $\Delta x \Delta A$

$\left(\frac{dR}{d\lambda}\right)_{\theta=0}$: Power per area per unit wave length (radiated by the wall)

2: because of two possible radiation dir. $(x, -x)$

$$\rightarrow \left(\frac{dR}{d\lambda}\right)_{\theta=0} = \left(\frac{dE}{d\lambda}\right)\left(\frac{c}{2\Delta x \Delta A}\right) = \left(\frac{du}{d\lambda}\right)\left(\frac{c}{2}\right) \quad (8)$$

For $\theta \neq 0$



$$\left(\frac{dR}{d\lambda}\right)_{\theta} = \left(\frac{dE}{d\lambda}\right)\left(\frac{1}{2\Delta t' \Delta A'}\right)$$
$$= \left(\frac{dE}{d\lambda}\right)\left(\frac{c\cos^2\theta}{2\Delta x \Delta A}\right) = \left(\frac{du}{d\lambda}\right)\left(\frac{c}{2}\right)\cos^2\theta \quad (9)$$

$\Delta t' = \frac{\Delta x}{c\cos\theta}$
the time taken to travel the distance Δx

Also $\Delta A' = \Delta A / \cos\theta$

$$\left\langle \left(\frac{dR}{d\lambda}\right)_\theta \right\rangle \equiv \frac{dR}{d\lambda} = \left(\frac{du}{d\lambda}\right)\left(\frac{c}{2}\right)\langle \cos^2\theta \rangle = \tag{10}$$

$$= \left(\frac{du}{d\lambda}\right)\left(\frac{c}{2}\right)\left(\frac{1}{2\pi}\right)\int_{-\pi}^{\pi} d\theta \, \cos^2\theta = \left(\frac{du}{d\lambda}\right)\left(\frac{c}{2}\right)\left(\frac{1}{2}\right) = \frac{c}{4}\frac{du}{d\lambda}$$

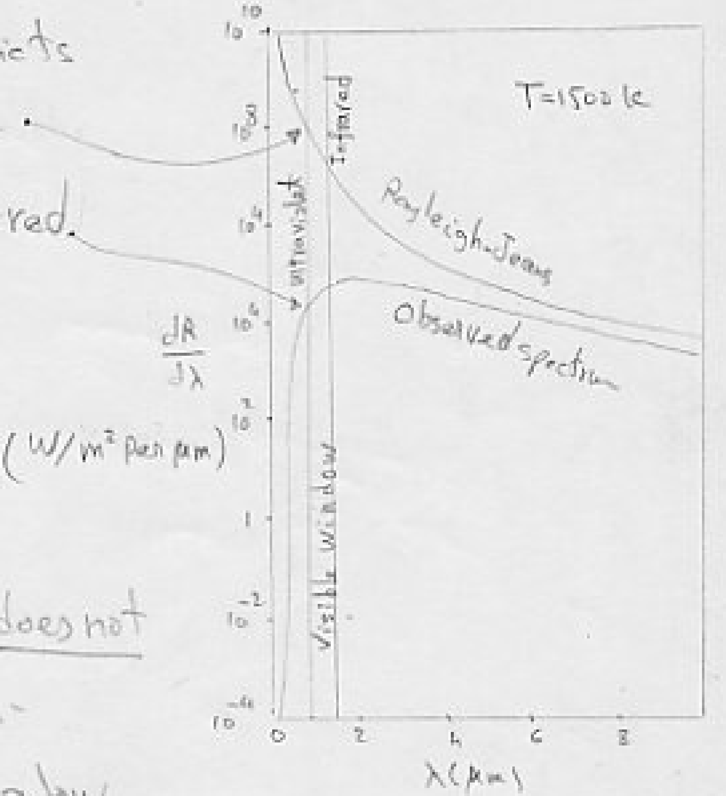
Power/(area x unit wave length)

$$(6)(10) \rightarrow \frac{dR}{d\lambda} = \frac{du}{d\lambda}\left(\frac{c}{4}\right) = \frac{2\pi c k T}{\lambda^4} \tag{11}$$

Rayleigh-Jeans formula

The Rayleigh-Jeans distribution predicts that the object is glowing blue.

Our experience tells us it is glowing red.



As $\lambda \to 0 \quad \frac{dR}{d\lambda} \to \infty$

(ultraviolet catastrophe)

We know that the power radiated <u>does not</u> become large at small wavelength. Objects in a dark room do not glow <u>blue</u> from their thermal radiation.

Something is wrong:

$$\langle \text{energy per oscillator} \rangle_{\text{ave.}} \neq kT \quad \text{for small } \lambda$$

The Empirical Formula of Wien

$$\frac{dR}{d\lambda} = \frac{a\, e^{-\frac{b}{\lambda T}}}{\lambda^5} \qquad (12) \quad \text{Wien hypothesis}$$

As $\lambda \to 0$, $\frac{dR}{d\lambda} \to 0$ (exponentially)

As $\lambda \to \infty$ $\frac{dR}{d\lambda} \to 0$ (as $\frac{1}{\lambda^5}$)

. Wien distribution predicts <u>two</u> important results:

I - The total power per area $\sim T^4$

Proof: Change of variables: $x = T\lambda \to dx = T d\lambda$

$$(12) \longrightarrow \frac{dR}{d\lambda} = \frac{a\, e^{-\frac{b}{\lambda T}}}{\lambda^5} = \frac{a T^5 e^{-b/x}}{x^5}$$

$$R = \int_0^\infty d\lambda \frac{dR}{d\lambda} = a T^5 \int_0^\infty \frac{dx}{T} \frac{e^{-b/x}}{x^5} = a T^4 \int_0^\infty dx \frac{e^{-b/x}}{x^5} \qquad (13)$$

consistent with Boltzman deduction from thermodynamics.

II - $\lambda_{max} \sim \frac{1}{T}$

Proof: $$\frac{d}{d\lambda}\left(\frac{dR}{d\lambda}\right) = \left[\frac{-5a e^{-b/\lambda T}}{\lambda^6} + \left(\frac{a}{\lambda^5}\right)\left(\frac{b e^{-b/\lambda T}}{T\lambda^2}\right)\right]_{\lambda=\lambda_m} = 0$$

$$\longrightarrow \lambda_m = \frac{b}{5T} \qquad (14)$$

Quantization of the Energy Levels:

Planck's hypothesis: The energy distribution of atomic oscillators is descrete.

$$f_n = C e^{-E_n/kT}$$

and $E_n = nhf = \frac{nhc}{\lambda}$ Energy quantization $n = 0, 1, 2, \ldots$ (15)

Now

$$\langle E_n \rangle = \frac{\sum_{n=0}^{\infty} E_n f_n}{\sum_{n=0}^{\infty} f_n} = \frac{\sum_{n=0}^{\infty} \left(\frac{nhc}{\lambda}\right) e^{-nhc/\lambda kT}}{\sum_{n=0}^{\infty} e^{-nhc/\lambda kT}} \quad (16)$$

change of variables: $x \equiv \frac{hc}{\lambda kT}$ $\quad Y \equiv e^{-x}$

$$\sum_{n=0}^{\infty} e^{-nhc/\lambda kT} = \sum_{n=0}^{\infty} e^{-nx} = 1 + Y + Y^2 + Y^3 + \cdots = \frac{1}{1-Y} \quad (17)$$

because $(1-Y)(1+Y+Y^2+Y^3+\cdots) = 1$

And since $\frac{d}{dx}\left(e^{-nx}\right) = -ne^{-nx}$

then; $\sum_{n=0}^{\infty} \left(\frac{nhc}{\lambda}\right) e^{-nhc/\lambda kT} = -\left(\frac{hc}{\lambda}\right) \frac{d}{dx} \sum_{n=0}^{\infty} e^{-nx}$

$$= -\left(\frac{hc}{\lambda}\right) \frac{d}{dx}\left(\frac{1}{1-Y}\right) = -\left(\frac{hc}{\lambda}\right)\left(\frac{-Y}{(1-Y)^2}\right) \quad (18)$$

$$(3)(4) \rightarrow \langle E \rangle = \left(\frac{hc}{\lambda}\right)\left(\frac{Y}{1-Y}\right) = \frac{hc}{\lambda\left(e^{hc/\lambda kT} - 1\right)} \quad (19)$$

Replacing kT with $\langle E \rangle$ in (11)

$$\frac{dR}{d\lambda} = \frac{2\pi hc^2}{\lambda^5\left(e^{hc/\lambda kT} - 1\right)} \quad (20)$$

Thermal Radiation formula discovered by Planck

This formula is consistent with the measured radiation spectrum emitted from objects in thermal equilibrium at a temperature T.

At large wavelength; $\frac{hc}{\lambda} << KT$

$$\rightarrow e^{hc/\lambda KT} \approx 1 + \frac{hc}{\lambda KT}$$

$$(20) \rightarrow \frac{dR}{d\lambda} = \frac{2\pi c KT}{\lambda^4} \quad \text{Rayleigh-Jeans formula}$$

At 300T the radiation of the object is not visible.

At 1500T the object glows red.

" 5900T " " " white.

$\frac{dR}{d\lambda}$

(W/m² per μm)

The area of each curve $\sim T^4$

Using Eq. (20)

Planck din not realize, why the energy levels are quantized.

We now know this is because of the wave nature of the particle (electors in atoms).

For the same reason the atom does not collapse.

Planck argued that for some unknown reason the atoms in the walls of the cavity emitted radiation in quanta with energy nhf.

The Photoelectric Effect:

As successful as the Planck formula was, The conclusion from it of the quantum nature of the radiation is hardly compelling. An important contribution to its acceptance came from the work of Albert Einstein, who, in 1905 used the concept of the quantum nature of light to explain some peculiar properties of metals, when these are irradiated with visible and ultraviolet light.

Experimental facts:

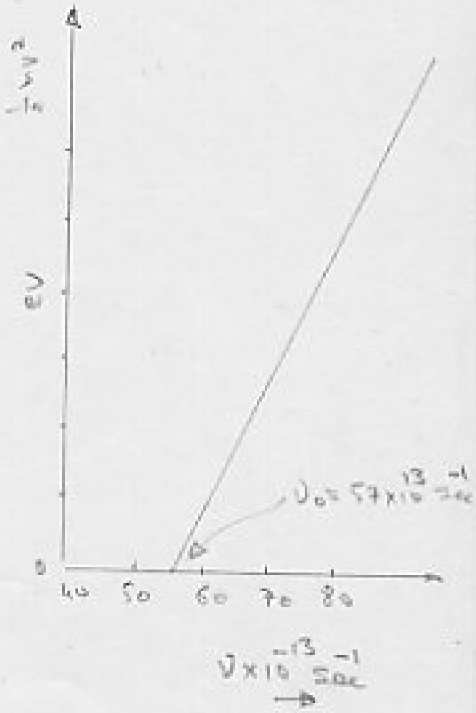
1 – Polished metal plates emit electrons under irradiation.
2 – There is a threshold frequency
3 – The magnitude of the current $\sim$ the intensity of the light
4 – The energy of the photoelectrons is indep. of the intensity of the light source but $\sim$ frequency of the light.

Acc. to classical theory: The energy of the electromagnetic wave $\sim$ the intensity of the source (not frequency)

There is no time delay between arrival of the radiation and the departure of the electron (at least no longer than 10^{-9} sec)

Einstein considered the radiation to consist of a collection of quanta of energy $h\nu$ (Photon) (Particle nature of light)

$$\frac{1}{2}mV^2 = h\nu - W$$

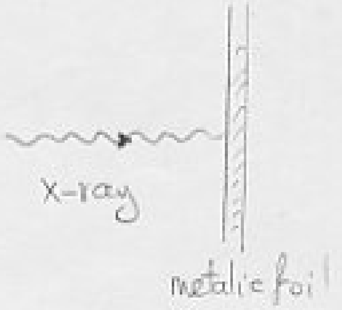


The Compton Effect:
(Evidence for particle nature of radiation)

Acc. to the classical theory prediction;

Reradiation of light by electrons are set into forced oscillations by the incident radiation

Prediction $\longrightarrow$ The intensity $\sim (1+\cos^2\theta)$
$\neq f(\lambda)$

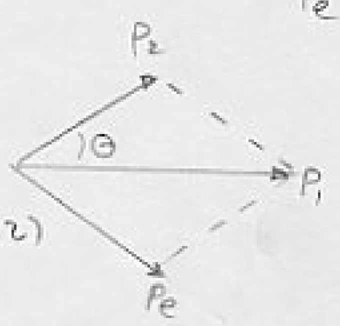


$$E = \sqrt{(m_0c^2)^2 + (pc)^2} \qquad (21)$$

$\longrightarrow E = PC$ for photon with $m_0 = 0$

$$\rightarrow P = \frac{E}{c} = \frac{h\nu}{C}$$

$$\bar{P}_1 = \bar{P}_2 + \bar{P}_e \rightarrow \bar{P}_e^2 = (\bar{P}_1 - \bar{P}_2')^2 = P_1^2 + P_2^2 - 2\bar{P}_1\cdot\bar{P}_2' \qquad (22)$$



$$P_1c + m_ec^2 = P_2'c + \sqrt{m_e^2c^4 + P_e^2c^2} \qquad (23)$$

(23) → $h\nu + m_e c^2 = h\nu' + \sqrt{m_e^2 c^4 + P_e^2 c^2}$

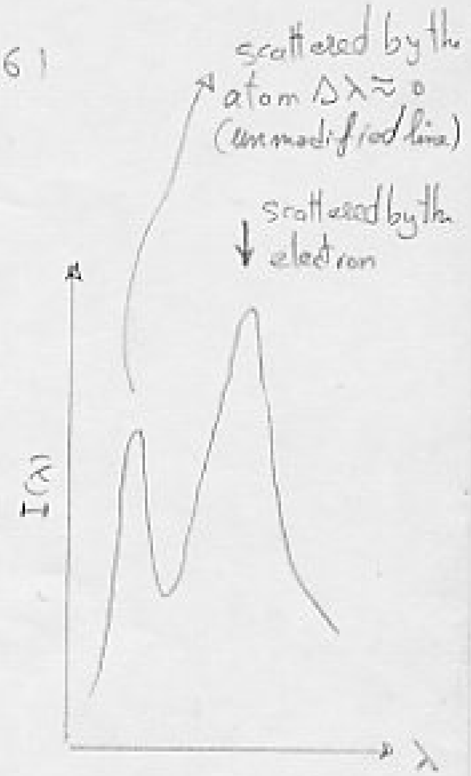
$$m^2 c^4 + \underbrace{P_e^2 c^2} = (h\nu - h\nu' + mc^2)^2 = \underbrace{(h\nu - h\nu')^2 + 2 m_e c^2 (h\nu - h\nu') + m^2 c^4}_{(24)}$$

(22) → $P_e^2 = \left(\frac{h\nu}{c}\right)^2 + \left(\frac{h\nu'}{c}\right)^2 - 2\frac{h\nu}{c}\frac{h\nu'}{c}\cos\theta$ (25)

→ $P_e^2 c^2 = (h\nu - h\nu')^2 + 2(h\nu)(h\nu')(1-\cos\theta)$ (26)

(24)(26) → $h\nu\nu'(1-\cos\theta) = m_e c^2(\nu - \nu')$

→ $\lambda' - \lambda = \frac{h}{m_e c}(1-\cos\theta)$ (27) Consistant with the experimental results



where $\frac{h}{m_e c} = \frac{hc}{m_e c^2} = \frac{1240 \text{ eV.nm}}{0.511 \text{ MeV}} = 2.43 \times 10^{-6}$ m

Compton wavelength of electron

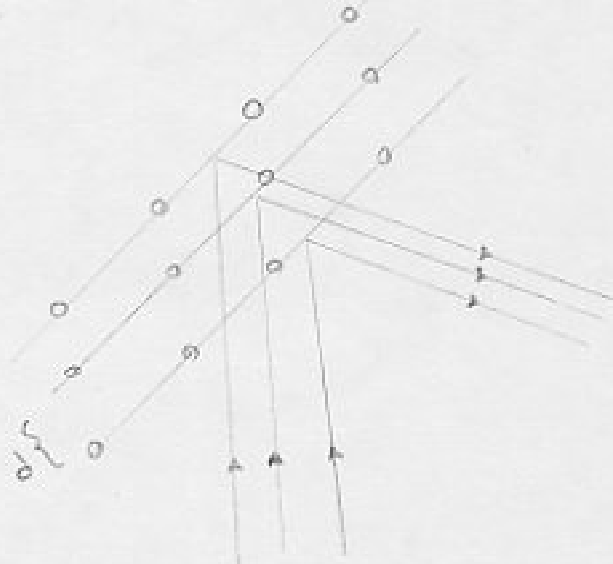
(But it is not a true wavelength rather is a change of wavelength)

Electron Diffraction;

In 1923 De Broglie, guided by the analogy of Fermat's principle in optics, and the least action principle in mechanics, was led to suggest that the dual wave-particle nature of radiation should have its counterpart in the dual particle-wave nature of matter. De Broglie suggested;

$$\lambda = \frac{h}{p}$$ wavelength for particle with momentum p

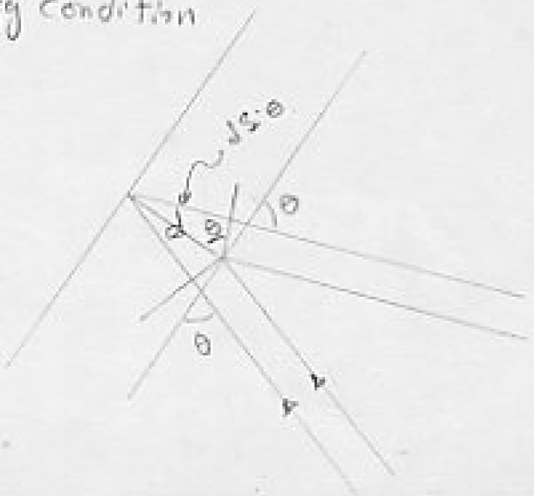
The experiment of Davinson and Germer is an evidence for the wave nature of the particle.



$$\Delta\varphi = K\Delta x = \frac{2\pi}{\lambda}(2d\sin\theta)$$

$$\Delta\varphi = 2\pi n$$ for constructive interference

$$2d\sin\theta = n\lambda \Rightarrow \lambda = \frac{2d\sin\theta}{n} \quad (28)$$ Bragg condition

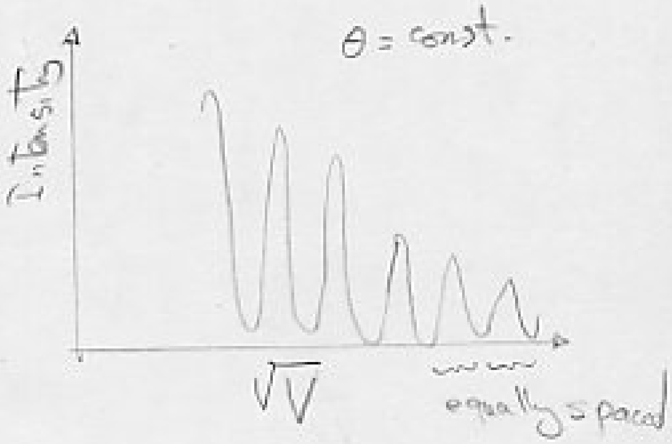


$$E = eV = \frac{1}{2}mv^2$$

$$\sqrt{V} = v\sqrt{\frac{m}{2e}} \quad (29)$$

$v \sim \sqrt{V}$

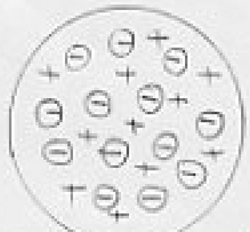
Fig $\rightarrow \frac{1}{\lambda} \sim V$



$\lambda = \frac{h}{p} = \frac{h}{mv}$ together with Bragg condition explain the experimental data

$\longrightarrow$ Are Electrons Waves?

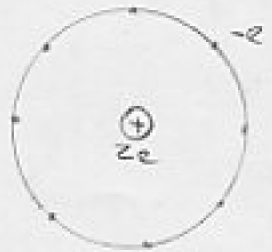
The Bohr Atom;



Thomson Model

Large scale angle scatt. of α particles by thin foils observed by Geiger and Marsden. (inconsistent with Thomson model)

$\longrightarrow$ Rutherford model



Rutherford Model

Rutherford model could explain the large scale angle scatt. of α particles, but it faced two main difficulties;

1- It could not account for the spectra of radiation from atoms, which did not have the expected harmonic structure (i.e. a vibrating string).

But instead it had the structure

$$\frac{1}{\lambda} = \text{const.}\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) \qquad n_1, n_2 : \text{integers}$$

2- It also lacked a mechanism for stabilizing atoms.

Bohr Model; The postulates;

1- The electrons move in orbits restricted by the requirement;

$$L = mvr = n\frac{h}{2\pi} \qquad (30)$$

Furthermore the electrons in these orbits do not radiate in spite of their acceleration. They were said to be in stationary states.

2- Discontinuous transitions by the electrons from one orbit to another:

$$\nu = \frac{E - E'}{h} \qquad (31)$$

Assuming $M_{nucleus} \simeq \infty$

$$F = m\frac{v^2}{r} \longrightarrow \frac{Ze^2}{r^2} = \frac{mv^2}{r} \qquad (32)$$

$$(30)(32) \longrightarrow v = \frac{2\pi e^2 Z}{hn} \qquad (33)$$

$$\text{also} \quad r = \frac{1}{4\pi^2}\frac{n^2h^2}{Ze^2m} \qquad (34)$$

$$E = \frac{1}{2}mv^2 - \frac{Ze^2}{r} = -\frac{2\pi^2 e^4 Z^2 m}{h^2 n^2} \qquad (35)$$

$$(31)(35) \longrightarrow \quad \frac{1}{\lambda} \sim \left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right) \qquad (36)$$

Define:

$$\hbar = \frac{h}{2\pi} = 1.0545 \times 10^{-27} \text{ erg sec}$$

$$\longrightarrow \quad \omega = \frac{E - E'}{\hbar} \qquad (\omega = 2\pi\nu)$$

$$E = h\nu = \hbar\omega$$

$$\lambda\!\!\!{}^{-} = \frac{\lambda}{2\pi} = \frac{c}{\omega} \qquad \text{also } p = \frac{h}{\lambda} = \frac{\hbar}{\lambda\!\!\!{}^{-}}$$

Also $mvr = n\hbar \qquad n = 1, 2, 3, \ldots$

$$\alpha = \frac{e^2}{\hbar c} = \frac{1}{137.0388} \quad \text{fine structure const.}$$

Remark: Indeed $\alpha = \dfrac{e^2}{4\pi\epsilon_0 \hbar c} = \dfrac{Ke^2}{\hbar c}$, $K = 8.99 \times 10^{9}$ $J\cdot m/C^2$

$Ke^2 = 2.3 \times 10^{-28}$ $J\cdot m$

K is absorbed in e

$\rightarrow e = 1.52 \times 10^{-14}$ $(J\cdot m)^{1/2}$

In terms of these quantities:

$$\frac{v}{c} = \frac{Z\alpha}{n}, \quad r = \frac{n^2}{Ze}\frac{\hbar}{mc} \qquad E = -\frac{1}{2}mc^2 \frac{(Z\alpha)^2}{n^2}$$

In atomic calculations we shall express our results in terms of mc^2 (energy), $\frac{\hbar}{mc}$ (length), $\frac{\hbar}{mc^2}$ (time) and mc (momentum)

$$mc^2 \simeq 0.51 \text{ MeV}, \quad \frac{\hbar}{mc} \simeq 3.9 \times 10^{-11} \text{ cm}, \quad \frac{\hbar}{mc^2} \simeq 1.3 \times 10^{-21} \text{ sec}$$

Also;

a) Bohr radius; (34) $\rightarrow$ $r_1 \equiv a_0 = \frac{137}{Z} \frac{\hbar}{mc} = \frac{0.53}{Z}$ Å (n=1)

b) Binding energy of the electron;

(35) $\rightarrow$ $E_b = \frac{1}{2} mc^2 (Z\alpha)^2 = 13.6\,ev$ (transition from n=1 to n=∞)

$$E_{photon} = \frac{mc^2\alpha^2}{2} \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right) = (1.36\,ev) \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right) \quad (37)$$

Difficulties;

1- In this model it is said nothing about when the electrons would make their jumps.

2- The quantization rule was restricted to periodic systems; a more general statement by Sommerfeld and Wilson

$$\oint_{\text{closed path}} p\,dq = nh \qquad (38)$$

was of no help in treating problems other than those associated with atomic levels of hydrogen.

From the Bohr theory emerged:

1- The correspondence principle, which, in essence states that classical Physics results should be contained as limiting cases of quantum mechanical results.
This principle was very helpful in guiding theoretical guesses, and led Heisenberg to the point from which he could make his giant leap to Q.M.

Example: A transition from quantum number $(n+1)$ to n when $n \rightarrow$ large we may ask for the classical limit, since the angular momentum $n\hbar$ is $\gg \hbar$.

Classically; an electron moving in a circular orbit with velocity v would be expected to radiate with the frequency of its motion;

$$\nu_{cl} = \frac{v}{2\pi r} = \frac{Z\alpha c}{n}\,\frac{Z\alpha mc}{2\pi n^2\hbar^2} = \frac{(Z\alpha)^2 mc^2}{2\pi\hbar}\,\frac{1}{n^3} \qquad (39)$$

on the other hand

$$(31) \rightarrow \nu = \frac{\omega}{2\pi} = \frac{1}{2\pi\hbar}\,\frac{mc^2}{2}\,(Z\alpha)^2\left[\frac{1}{n^2} - \frac{1}{(n+1)^2}\right] \qquad (40)$$

for $n \gg 1 \quad \rightarrow \nu_{cl} \simeq \nu$

2- The quantization of the angular momentum held in other situations as well (in Sommerfeld model for the elliptic orbits there are two deg. of freedom r and φ, each one should be quantized))

$$\int P_\varphi \, d\varphi = k h \quad , \quad \int P_r \, dr = m h$$

The Wave-Particle Problem

The fact that radiation exhibits both wave and particle properties raises a deep conceptual difficulty, as can be seen from the following considerations:

1- Gedanken-experiment;

Assume we stretch out the Time, such that in two-slit diffraction experiment the photons that come to the plate are an hour apart (so they are not correlated).

Add a small monitor telling us whether the photon went through slit 1 or 2.

For the first class, we could have closed down slit 2 and vice versa.

We might thus expect that the pattern on the potoelectric place should be the <u>same</u> if we repeated the experiment with one slit closed for half of the time, and the other slit closed for the other half of the time.

This, however cannot be, since the second experiment <u>does not</u> give an interference pattern.

If we accept that the presence of the monitor <u>does not</u> affect the experiment, then we have an <u>inconsistency</u>.

But when we discuss the Heisenberg uncertainty principle, we shall see that the action of the monitor destroys the interference pattern, so there is <u>no</u> inconsistency.

But we can still speak of an average intensity of radiation at each slit, (i.e. for individual photons we can <u>only</u> speak of a probability of going through one slit or another).

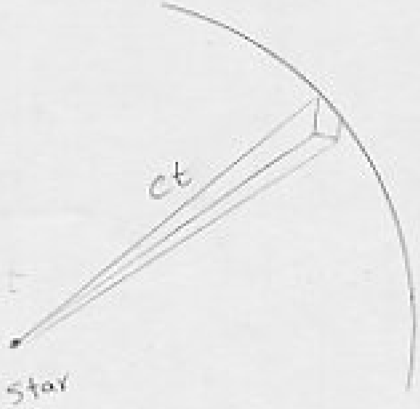
2— The notion of probability must again be invoked in understanding the passage of polarized radiation through an analyzer.

I_0 (initial intensity of radiation) $\longrightarrow$ $I_0 \cos^2\alpha$ attenuated radiation

α: angle between polarizer and analyzer

In terms of single photons that are individuals, such an attennuation is only explainable if we state that a given photon will either go through or be locked by the system, with a probability of transmission governed by the angle α.

3 - In terms of individual photons it is not sensible to think of the photon as spread thinly over a sphere of radius ct. We may however interpret the spherical distribution as given us the probability of finding photon at given solid angle.

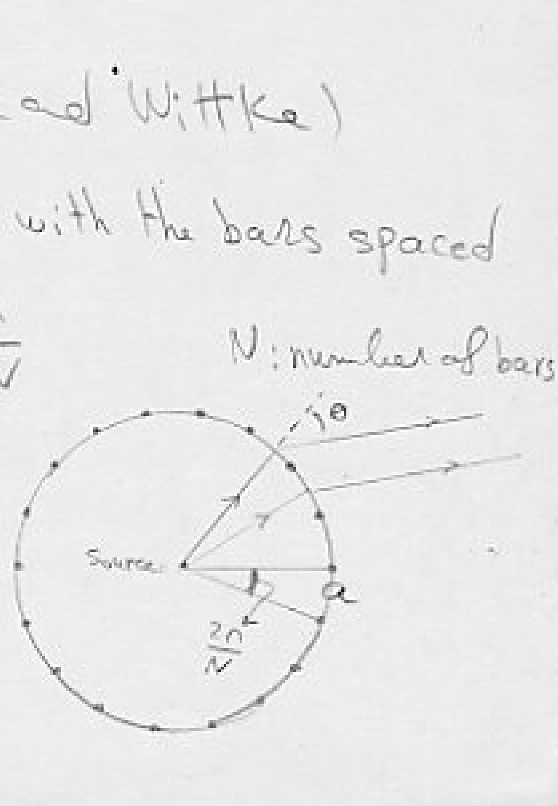


4 — Sometimes it is possible to interpret experiment both in particle and in wave language, but then nonclassical acpect creeps in elsewhere.

Gedanken-experiment (by Dicke and Wittke)

Consider a cylindrical bird cage with the bars spaced regulary, and spacing $a = 2\pi \frac{R}{N}$ N: number of bars

Consider radiation emitted from a source placed on the axis of the cylinder.



The bars act as diffraction grating

$$a\, S\theta = n\lambda \qquad (n=1,2,\ldots) \quad \text{for maxima}$$

$$\Rightarrow \lambda = \frac{a\, S\theta}{n} = \frac{2\pi R\, S\theta}{Nn}$$

We could also interpret the intensity peak by assuming that the particles scattered through an angle θ off the bars of the bird cage.

$pS\theta$: the mom. transferred to the cage

$L = RpS\theta$ " ang. mom. " " "

$\lambda = \frac{2\pi\hbar}{p}$ DeBroglie wave length

$$L = \underbrace{\frac{2\pi\hbar Nn}{2\pi R S\theta}}_{p} \cdot R S\theta = Nn\hbar \qquad \text{quantized!}$$