Chapter 32

Faraday's law of Induction

32-1 Symmetries

A current carrying loop in an external mag. field $\overline{B}$ feels a torque $\overline{\tau}$.

Current $\longrightarrow$ torque (Principle of electromotor)

If a conducting loop is rotates by excerting a torque $\overline{\tau}$ in and external mag. field, a current appears in the loop.

torque $\longrightarrow$ current (Principle of electric generator)

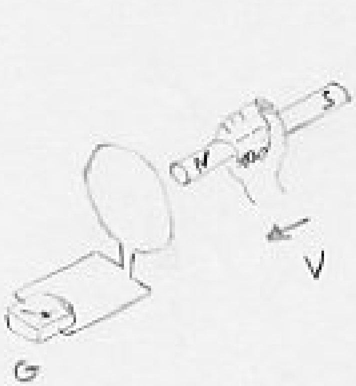
This phenomenon is on the basis of <u>Faraday's law</u> of induction.

32-2 Two Experiments:

Deflection of galvanometer $\sim |V|$

The dir. of the deflection $\xrightarrow{\text{depends on}}$ $\overline{V}$

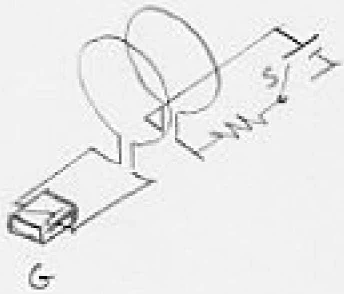
" " " $\xrightarrow{\text{"}}$ magnet pole



The appeared current in the loop is said to be an <u>induced</u> current.

The work done per unit charge in moving the charge of that current through the loop is said to be an induced emf.

Second Experiment



When we close or open the switch S an induced current appears in the second loop (momentarily) but in opposite dirs.

Conclusion:

An emf is induced only when something is changing. In a static situation, in which no physical objects are moving and the currents are steady, there is no induced emf. The key word is <u>change</u>.

32-3 Faraday's law of Induction

An emf is induced in the loop depends on the change of the field lines through the loop (the number of lines) and the rate of this change.

A Quantitative Treatment:

Consider a surface — which may or may not be a plane — bounded by a closed conducting loop.

$$\Phi_B = \int B \cdot dA \quad \text{magnetic flux} \sim \text{number of mag. lines}$$

For a planar (flat) surface with $B \perp A$;

$$\Phi_B = BA$$

unit: 1 weber = 1 Wb = 1 $T \cdot m^2$

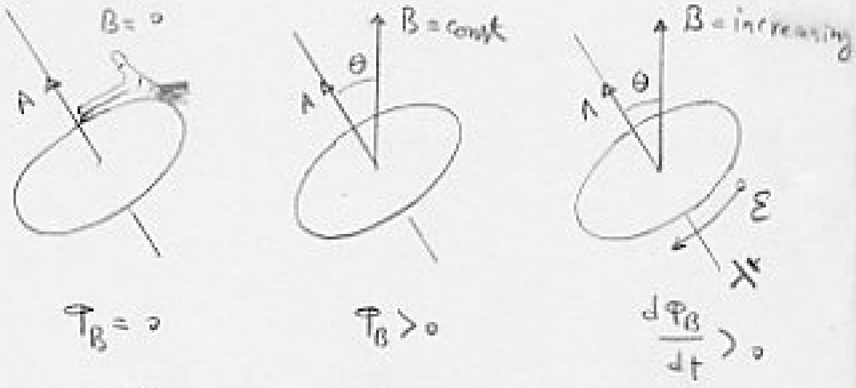
Faraday's law:

$$\mathcal{E} = -\frac{d\Phi_B}{dt} \quad \text{(volts)}$$

For N turns;

$$\mathcal{E} = -N\frac{d\Phi_B}{dt} \quad \text{(Faradays law)}$$

1- In Fig (c) if B increases in magnitude : $\Phi_B > 0$
also $\frac{d\Phi_B}{dt} > 0 \rightarrow \mathcal{E} < 0$



(a) (b) (c)

2- In Fig(c) if $\bar{B}$ decreases in magnitude : $\Phi_B > 0$
but $\frac{d\Phi_B}{dt} < 0 \rightarrow \mathcal{E} > 0$

3 - $\bar{B}$ points in the dir opposite that shown in Fig.(c) and is increasing in magnitude: $\Phi_B < 0$ and $\frac{d\Phi_B}{dt} < 0$
$\rightarrow \mathcal{E} > 0$

4 - $\bar{B}$ points in the dir. opposite that shown in Fig (c) and is decreasing in magnitude: $\Phi_B < 0$ and $\frac{d\Phi_B}{dt} > 0$
$\rightarrow \mathcal{E} < 0$

Ex.

A long solenoid has $n = 220$ turns/cm and carries $i = 1.5$ A, its diameter D is 3.2 cm. At its center we place a 130-turn close-packed coil C of diameter $d = 2.1$ cm. The current in the solenoid is reduced to zero and then increased to 1.5 A in the

other dir. at a steady rate over a period $T = 50$ ms.
What is the magnitude of the induced emf that appears in the central coil while the current in the solonoid is being changed?

Sol.

$$|\mathcal{E}| = \frac{N \Delta \Phi_B}{\Delta t}$$ N: number of turns in the inner coil C.

$$B = \mu i n = (4\pi \times 10^{-7}\ T \cdot m/A)(1.5\ A)(220\ \tfrac{turns}{cm} \times 100\ \tfrac{cm}{m}) = 4.15 \times 10^{-2}\ T$$

Max B due to i produced by solonoid at its center

$$A_c = \pi (\tfrac{d}{2})^2 = 3.46 \times 10^{-4}\ m^2$$

$$\Phi_B = B A_c$$

$$\Phi_B = B A_c = (4.15 \times 10^{-2}\ T)(3.46 \times 10^{-4}\ m^2) = 1.44 \times 10^{-5}\ Wb$$

Φ_B changes sign but not magnitude as the current is reversed

$$\rightarrow \Delta \Phi_B = 2 \times 1.44 \times 10^{-5} = 2.88 \times 10^{-5}\ Wb$$

$$|\mathcal{E}| = \frac{N \Delta \Phi_B}{\Delta t} = \frac{(130\ turns)(2.88 \times 10^{-5}\ Wb)}{50 \times 10^{-3}\ s} = 7.5 \times 10^{-2}\ V$$

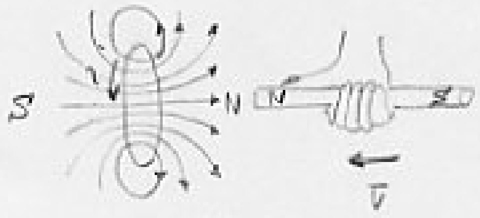
32-4 Lenz's law

An induced current in a closed conducting loop will appear in such a direction that it opposes the change that produced it.

The minus sign in $\mathcal{E} = -\frac{d\varphi}{dt}$ carries with it this symbolic notion of opposition.

By the dir. of current we can find the dir of $\mathcal{E}$.

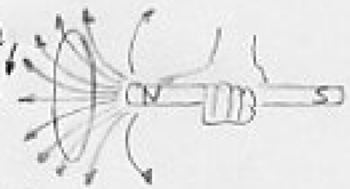
1- First Interpretation;

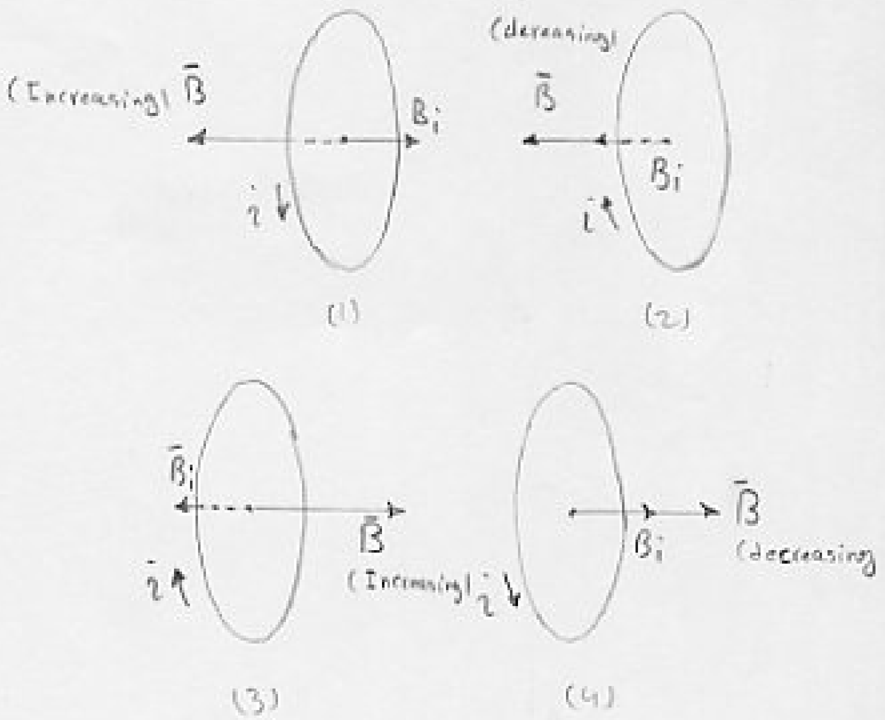


The induced current set up a mag. field that opposes the motion of magnet.

2- Second Interpretation;

The induced current i set up a mag. field that opposes the increases in flux produced by the external agent.





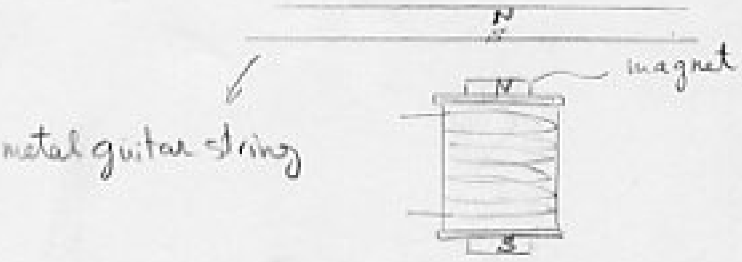
Lenz's Law and Energy Conservation:

What would happen if Lenz's law were turned the other way around, that is, if the induced current acted, to aid the change that produced it.

This would be in contradiction with energy conservation and can not happen.

Work done on the magnet (to compensate resisting force) = thermal energy in the coil + electromagnetic radiation

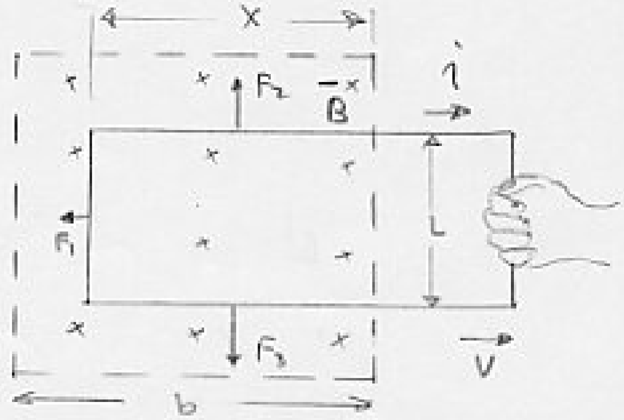
Ex



Pickup

32-5 Induction: A quantitative study;

Change in Φ_B may occur due to the motion of the loop while $\bar{B}$ = const.



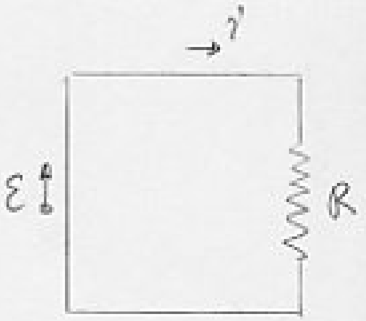
Rate of Doing Work:

To move the loop a force F must be applied to the loop by hand (which is in opposite dir. of the force acted to the loop but the same magnitude to give a const. velocity v).

$$P = FV$$

$$\Phi = BLx$$

$$\mathcal{E} = \frac{d\Phi}{dt} = \frac{d}{dt}(BLx) = BL\frac{dx}{dt} = BLV$$



To find i we cannot apply the loop rule, because we can not define a potential for an induced emf.

But we can use energy method;

$$dW = \mathcal{E}\,dq = \mathcal{E}i\,dt$$

$$\mathcal{E}i\,dt = i^2R\,dt \rightarrow \mathcal{E} = iR$$

$$i = \frac{\mathcal{E}}{R} = \frac{BLv}{R}$$

Also

$$\bar{F} = i\bar{L}\times\bar{B}$$

The contributions from $\bar{F}_2$ and $\bar{F}_3$ cancel each other.

B is perpendicular to $\bar{L}$

$$\bar{F} = -\bar{F}_1$$

$$F = F_1 = iLB\,S\,90 = iLB$$

$$F = \frac{B^2L^2V}{R}$$

B, L, R are consts.

$\longrightarrow$ If $F = \text{const} \longrightarrow V = \text{const}.$

$$P = FV = \frac{B^2L^2V^2}{R}$$

The Thermal Energy,

$$P = Ri^2 = R\left(\frac{BLV}{R}\right)^2 = \frac{B^2L^2V^2}{R}$$ thermal energy rate

which is exactly equal to the rate of work done by $\bar{F}$ $(P = FV)$.

Ex.

Suppose the loop in Fig (P196) has N=185 Turns Copper wire.

L= 13Cm, B.15 T, R=6.2 Ω, V = 18 Cm/s.

a) $\mathcal{E}=?$ $\mathcal{E} = NBLV =$ (85 turns)(1.5T)(0.13m)(0.18 m/s)

$\mathcal{E} = 2.98V$

b) $i=?$ (induced current) $i = \frac{\mathcal{E}}{R} = \frac{2.98V}{6.2Ω} = 0.48$ A

c) F=? (the applied force)

F = NiLB = (85 turns)(0.48A)(0.13m)(1.5T) = 8.0N

d) P=? P = FV = (8.N)(0.18 m/s) = 1.4W

Ex.

Fig. shows a rectangular conducting loop of resistance R, width L, and length b being pulled at const. speed V through a region of width d in which uniform $\vec{B}$.

a) plot $\Phi_B(x)$,

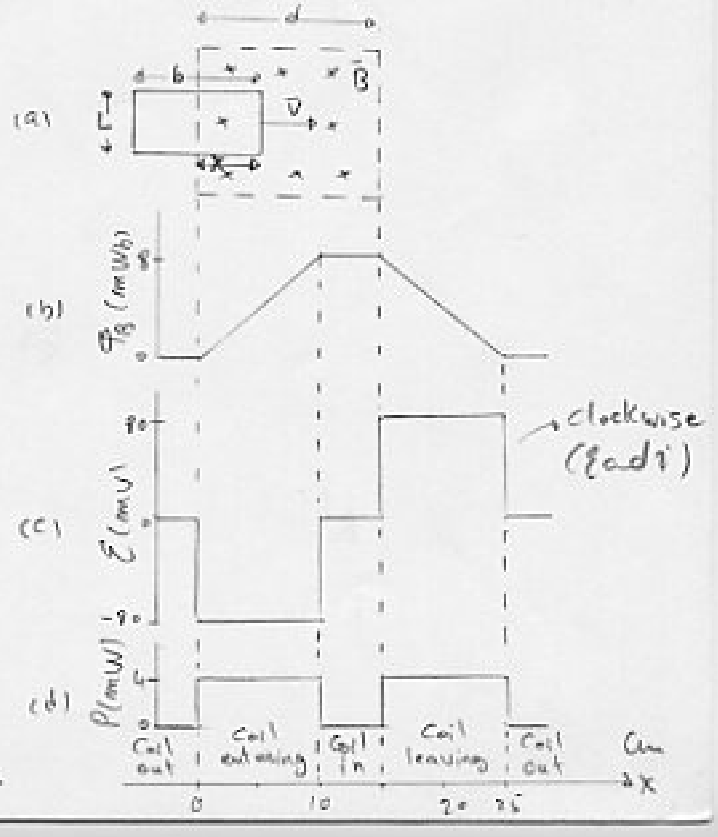
L = 40 mm, b = 10cm, d = 15cm

R = 1.6 Ω, B = 2.0T, V = 1.0 m/s

$\Phi_{min} = 0$ $\Phi_{Max} = BLb = 8$ mWb

$\Phi = BLx$ entering

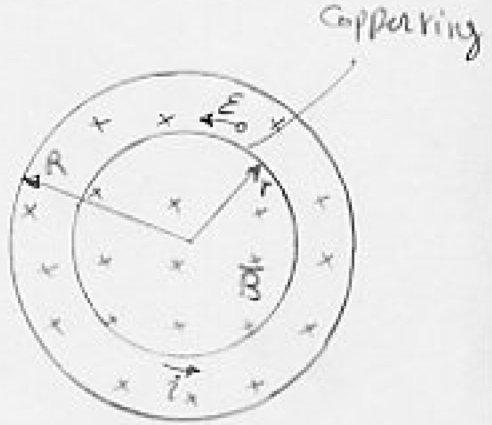
$\Phi = BL[b-(x-d)]$ leaving



b) Plot $\mathcal{E}$, $\mathcal{E} = -\frac{d\Phi_B}{dt} = -\frac{d\Phi_B}{dx}\frac{dx}{dt} = -\frac{d\Phi_B}{dx} V$

c) Plot P, $P = i^2 R = \frac{\mathcal{E}^2}{R}$ also $P = \frac{B^2 L^2 V^2}{R}$

32-6 Induced $\bar{E}$lectric Fields



(a)

$\bar{B} = \alpha t \hat{k}$ $\alpha = \text{const} > 0$

(increasing

Fig(a): $\bar{B}$ is increasing at a steady rate.

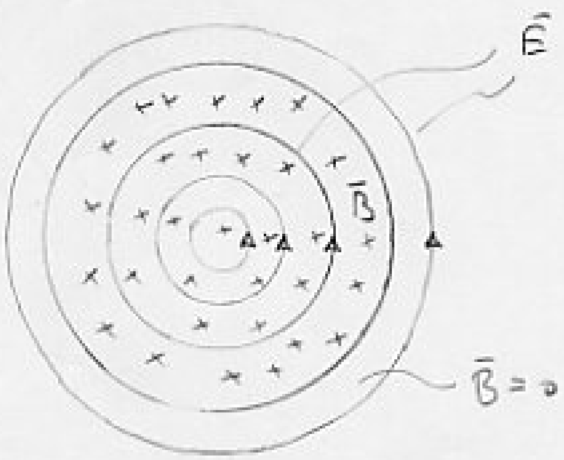
$\longrightarrow$ $\mathcal{E}$ and i are induced.

if there is induced i $\longrightarrow$ there should be induced $\bar{E}$ within the loop

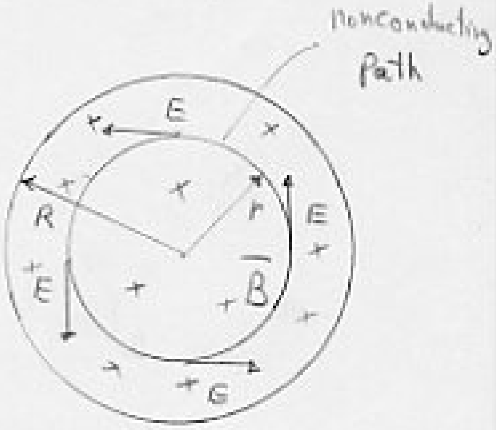
$\xrightarrow{\text{leading}}$ a force $q_0 E$ on a test charge

Changing $\bar{B}$ $\longrightarrow$ produces $\bar{E}$

$\bar{E}$ is produced even if there is <u>no</u> conducting ring.



$\bar{B} = \alpha t \hat{k}$ $\alpha = \text{const} > 0$



(b)

$\bar{B} = \alpha t \hat{k}$ $\alpha = \text{const.} > 0$

A Reformulating of Faraday's law:

$W = \mathcal{E} q_0$ the work done on q_0 in <u>one</u> revolution

on the other hand $W = \int F \cdot ds = (q_0 E)(2\pi r)$

$\rightarrow \mathcal{E} = E 2\pi r$

In general case $\mathcal{E} = \oint E \cdot ds$

New def. for <u>induced</u> $\mathcal{E}$:

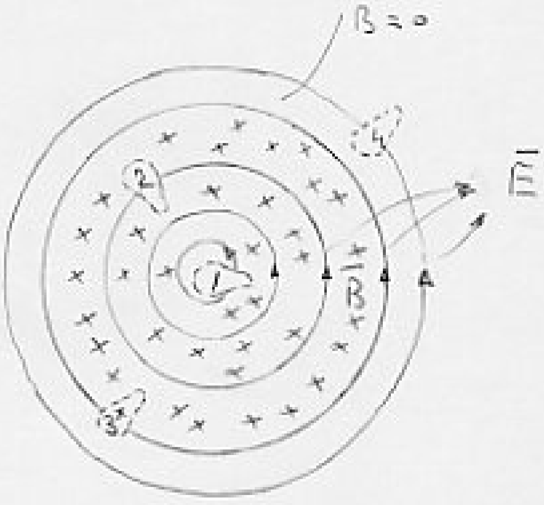
Old def: The work done per unit charge in maintaining current in a circuit owing to a changing Φ_B.

or: The work done per unit charge on a test charge that moves around a <u>closed</u> path in a changing Φ_B.

New def : An induced $\mathcal{E}$ is the sum – via integration – of quantities $E \cdot ds$ around a closed path, where $\bar{E}$ is the electric field induced by a changing Φ_B and $d\bar{s}$ is a differential length vector along the path.

$$\mathcal{E} = -\frac{d\varphi_B}{dt} \longrightarrow \oint E\cdot ds = -\frac{d\varphi_B}{dt} \quad \text{(Faraday's law)}$$

$$\mathcal{E} = \oint E\cdot ds$$



(d)

$\bar{B} = \alpha t \hat{k} \quad \alpha = \text{const} > 0$

$$\mathcal{E} = \oint E\cdot ds = -\frac{d\varphi_B}{dt}$$

$$\oint_1 E\cdot ds = \oint_2 E\cdot ds$$

$$\left(\text{because } \frac{d\varphi_1}{dt} = \frac{d\varphi_2}{dt}\right)$$

$$\oint_3 E\cdot ds < \oint_1 E\cdot ds$$

$$\oint_4 E\cdot ds = 0 \quad \left(\frac{d\varphi_B}{dt} = 0\right)$$

A New look at Electric Potential:

Induced $\bar{E}$ <u>is not</u> produced by static charges, <u>but</u> by changing φ_B.

The important difference;

$\bar{E}$ produced by static charge <u>start</u> from positive charge and <u>end</u> on negative charges.

Induced $\bar{E}$ form a closed loop.

In a more formal sence;

Electric potential has meaning only for electric fields that are produced by static charges; it has no meaning for electric fields that are produced by induction.

Explanation:

consider a test charge that makes a single journey around a single path (Fig 2b, P 200).

During this single round it has experienced an emf $\mathcal{E}$ of say $\mathcal{E} = C$ volt

$\longrightarrow$ Its potential should have increased by this amount

This is impossible, however, because otherwise a single point in space would have two different values of V.

$\longrightarrow$ V has no meaning for $\bar{E}$ induced by changing Φ_B.

More formal look;

$$V_f - V_i = \frac{W_{if}}{q_0} = -\int_i^f E \cdot ds$$

if $i = f \longrightarrow \oint E \cdot ds = 0$

But there is a changing $\Phi_B \longrightarrow \oint E \cdot ds = -\frac{d\Phi_B}{dt} \longrightarrow$ V has no meaning

Ex. In (Fig b, P200), $R = 8.5\,cm$, $\frac{dB}{dt} = 0.13\,T/s$

a) Induced $\bar{E} = ?$ when $r = 5.2\,cm$

$$\oint E \cdot ds = (E)(2\pi r) = -\frac{d\Phi_B}{dt}$$

$r < R$, $\Phi_B = B(\pi r^2)$

$$(E)(2\pi r) = -(\pi r^2)\frac{dB}{dt} \qquad E = \frac{1}{2}\left(\frac{dB}{dt}\right) r$$

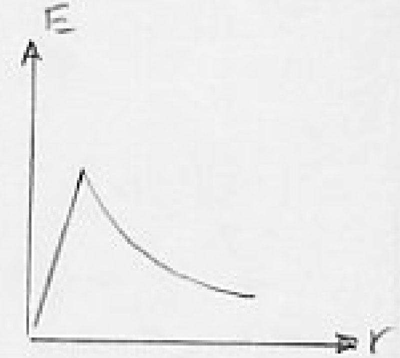
$$E = \left(\tfrac{1}{2}\right)(0.13\,T/s)(5.2\times 10^{-2}\,m) = 0.0034\,V/m$$

b) Induced $\bar{E} = ?$ at $r = 12.5\,cm$

$r > R$ $\quad \Phi_B = B(\pi R^2)$

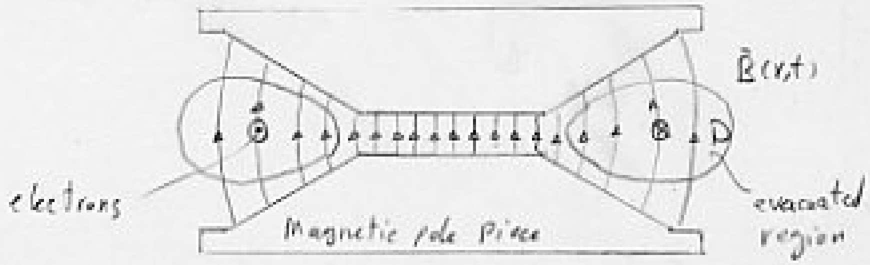
$$(E)(2\pi r) = -\frac{d\Phi_B}{dt} = -(\pi R^2)\frac{dB}{dt} \qquad E = \frac{1}{2}\left(\frac{dB}{dt}\right)\frac{R^2}{r}$$

$$E = \frac{\left(\frac{1}{2}\right)(0.13\,T/s)(8.5\times 10^{-2}\,m)^2}{12.5\times 10^{-2}\,m} = 3.8\times 10^{-3}\,V/m$$



32-7 The Betatron

The device accelerates electrons by allowing them to be acted on by induced $\bar{E}$.



Time varying $\bar{B}(r,t)$ has several funcs.:

1- It guides the electrons in a circular path

2- The changing Φ_B generates an induced $\bar{E}$ accelerating the electrons in this path.

3- It keeps the radius of the electron orbit essentially const. during the acceleration process.

4- In injects the electrons into the orbit initially and extracts them from the orbit after they have reached their full energy.

5- It provides a resorting force that resist any tendency for the electrons to stray away from their orbit, either vertically or radially.

These are done by proper shaping and control of the $\bar{B}(r,t)$

B_{orb} at orbit position serves to guide the electrons in their orbit.

The field in the area enclosed by the orbit, with the average value B_{av} ($= 2B_{orb}$) contributes to the central flux Φ_B.

The time variation of this central flux induce $\bar{E}$, accelerating the electrons

Ex.

In a 100 MeV Betatron, R = 84 cm.

B in the region enclosed by the orbit rises and falls with the frequency $f = 60\ Hz$ from zero to a maximum average of $B_{av_{max}} = 0.80\ T$.

An electron is fully accelerated in $\frac{1}{4}$ of a magnetic-field period (4.2 ms) $\left(4.2\times10^{-3} \simeq \frac{1}{4}\frac{1}{60}\right)$

a) How much energy does the electron gain in one average trip around its orbit in this changing Φ_B?

$$\Phi_{max} = (B_{av_{max}})(2\pi R) = (0.80\,T)(\pi(0.84\,m)^2) = 1.8\,Wb$$

$$\left(\frac{d\Phi_B}{dt}\right)_{av} = \frac{1.8\,Wb}{4.2\times10^{-3}\,s} = 430\,Wb/s$$

Since $\mathcal{E} = -\frac{d\Phi_B}{dt} \rightarrow \mathcal{E}_{av} = 430\,Wb/s$ (Volts) $(430\times1.6\times10^{-19}\,J)$

→ The energy of the electron increase by an average of 430 eV/revolution

Full final energy = 100 MeV

$$\frac{100\times10^{6}\,eV}{430} = 230{,}000 \text{ revolutions} \longrightarrow 230{,}000(2\pi R) = 1200\,km$$

b) $\overline{V}_{av} = ?$ average speed during its acceleration

$$\bar{V} = \frac{1200\times10^{3}\,m}{4.2\times10^{-3}\,s} = 2.86\times10^{8}\,m/s \quad (95\%)$$

(the actual value $\bar{V} = \%99.9987\,C$ considering relativity)