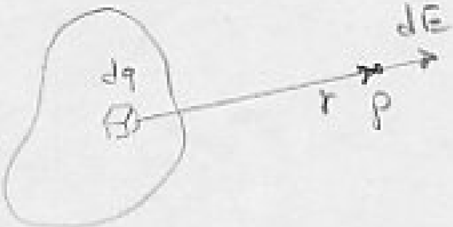
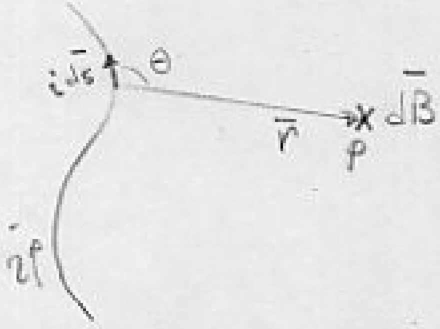
Chapter 31

Ampere's law

31-2 Calculating the Magnetic Field;

Comparison;





$$d\bar{E} = \left(\frac{1}{4\pi\epsilon_0}\right)\frac{dq}{r^3}\bar{r}$$

$$dB = \frac{\mu_0}{4\pi}\frac{i\,ds\,S\theta}{r^2}$$

$\mu_0 = 4\pi \times 10^{-7}$ T.m/A

$\simeq 1.26 \times 10^{-6}$ "

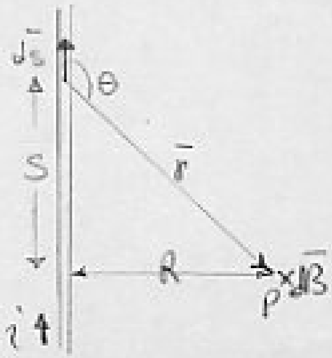
permeability const.

$$d\bar{B} = \left(\frac{\mu_0}{4\pi}\right)\frac{i\,d\bar{s}\times\bar{r}}{r^3}$$

Biot - Savart law

31-3 Magnetic Field due to a Long Straight Wire

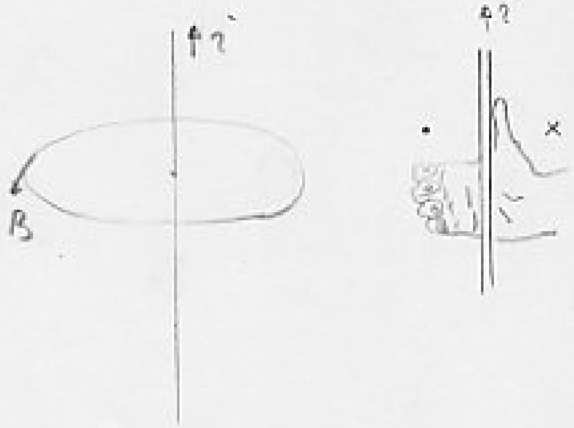
$$B = \int dB = 2\,\frac{\mu_0 i}{4\pi}\int_0^{\infty}\frac{S\theta\,ds}{r^2}$$



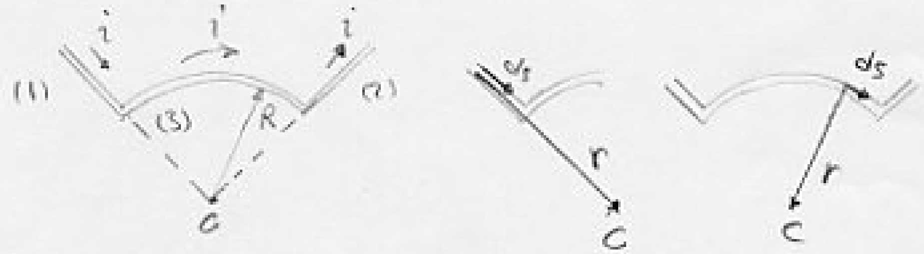
$$S\theta = S(\pi-\theta) = \frac{R}{\sqrt{s^2+R^2}} \qquad r = \sqrt{s^2+R^2}$$

$$B = \frac{\mu_0 i}{2\pi}\int_0^\infty \frac{R}{(s^2+R^2)^{3/2}}\,ds = \frac{\mu_0 i}{2\pi R}\left[\frac{s}{(s^2+R^2)^{1/2}}\right]_0^\infty$$

$$= \frac{\mu_0 i}{2\pi R}$$



Ex.



$\bar{B}_c = ?$

$$dB = \frac{\mu_0}{4\pi}\frac{i\,ds\,S\theta}{r^2}$$

$$dB_1 = dB_2 = 0 \qquad (d\bar{s}\times\bar{r} = 0,\ \theta = 0)$$

$$dB_3 = \frac{\mu_0}{4\pi}\frac{i\,ds\,S90}{r^2} = \frac{\mu_0}{4\pi}\frac{i\,ds}{R^2}$$

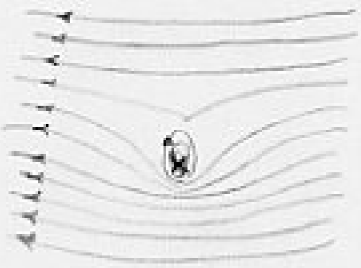
$$B_3 = \int dB_3 = \int_0^{\pi/2}\frac{\mu_0 i}{4\pi}\frac{R\,d\theta}{R^2} = \frac{\mu_0 i}{4\pi R}\left(\frac{\pi}{2}-0\right) = \frac{\mu_0 i}{8R}$$

$B_c = B_1 + B_2 + B_3 = \dfrac{\mu_0 i}{8R}$ pointing <u>into</u> the plane of the page

31-3 The Magnetic Force on a Current-Carrying Wire;

From chapter 30;

$$\bar{F} = i\,\bar{L} \times \bar{B}_{ext}$$



A long straight wire carrying a current i into the page is immersed in a uniform external field B_{ext} that points to the left.

$$B = B_{ext} + B_{intr}$$

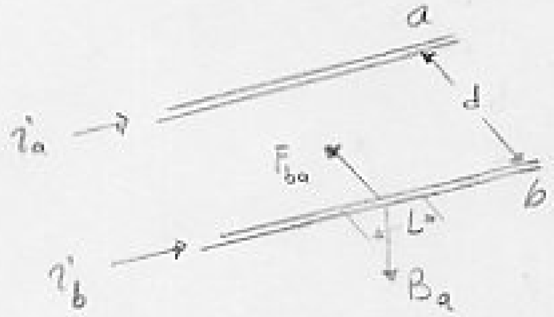
31-4 Two Parallel Conductors;

Two long parallel wires carrying currents exert forces on each other.

$$B_a = \frac{\mu_0 i_a}{2\pi d} \quad \text{(at the sight of wire b)}$$

$$F = i\,\bar{L} \times \bar{B}_{ext} \qquad \bar{F}_{ba} = i_b\,\bar{L} \times B_a$$

$$\rightarrow F_{ba} = i_b L B_a \cos 0^\circ = \frac{\mu_0 L\, i_a i_b}{2\pi d}$$

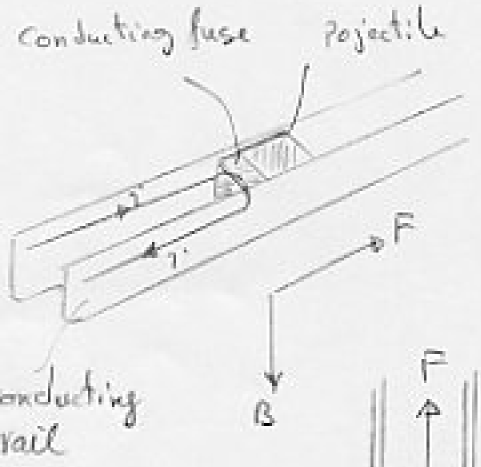


$$\text{Also } F_{ab} = \frac{\mu_0 L\, i_a i_b}{2\pi d}$$

Result: Parallel currents attract each other and antiparallel currents repel.

Def. — The ampere is that const current which, if maintained in two straight parallel conductors of intrinsic length, of negligible circular cross section, and placed 1 meter apart in vacuum would produce on each of these conductors a force equal to 2×10^{-7} N per meter of length.

Rail Gun:

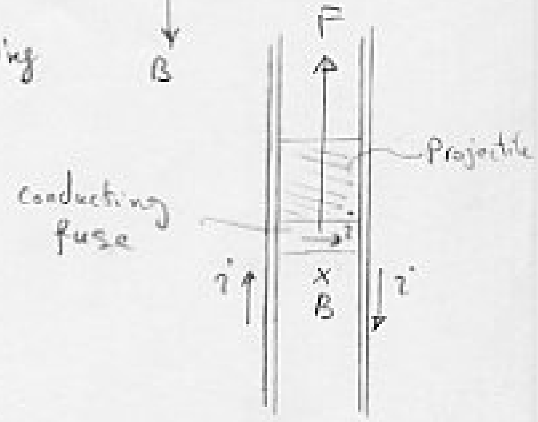


B is produced by i (between two rails)

Conducting fuse melts due to passing i across it.

F is excerted on vaporized fuse (gas) due to B and i.

This force is transfered to the projectile.



<u>Ex.</u>

Show that the equ. $F_{ba} = \frac{\mu_0 L i_a i_b}{2\pi d}$ is consistant with the def. of the ampere.

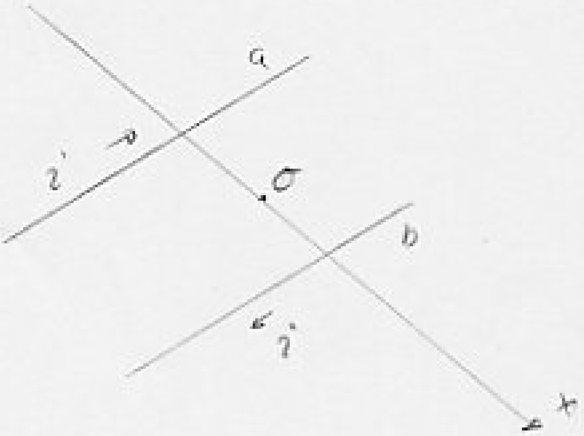
Let $i_a = i_b = 1\,A$, $d = 1\,m$

$$\frac{F}{L} = \frac{\mu_0 i_a i_b}{2\pi d} = \frac{(4\pi\times10^{-7}\,T.m/A)\,(1A)(1A)}{2\pi\,(1m)} = 2\times10^{-7}\,T.A$$
or N/m

$$F = \frac{\mu_0 L i_a i_b}{2\pi d} \longrightarrow 1\,N = 1\,T.A.m$$

or $1\,T.A = 1\,N/m$

Ex.

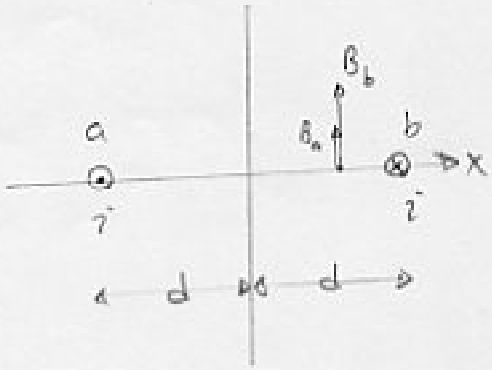


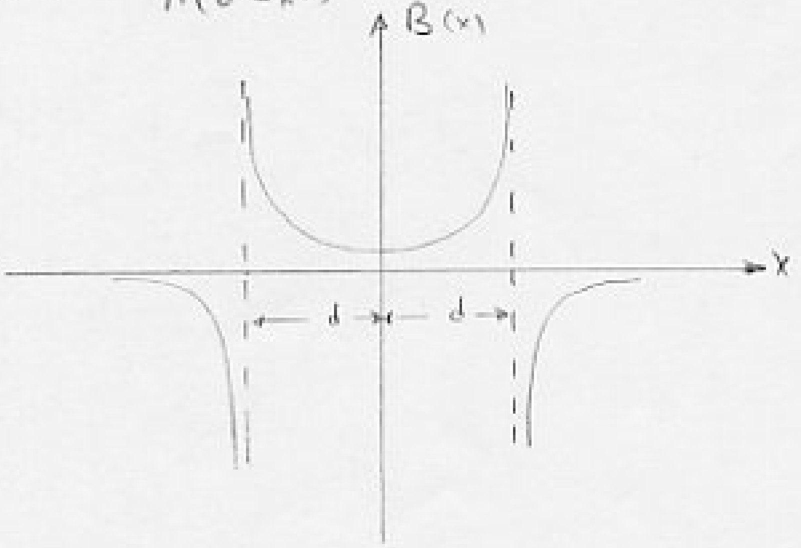
$B(x) = ?$

$$B(x) = B_a(x) + B_b(x)$$

$$B(x) = \frac{\mu_0 i}{2\pi(d+x)} + \frac{\mu_0 i}{2\pi(d-x)}$$

$$B(x) = \frac{\mu_0 i d}{\pi(d^2-x^2)}$$



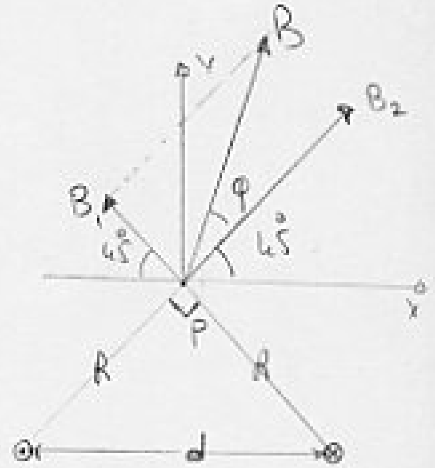


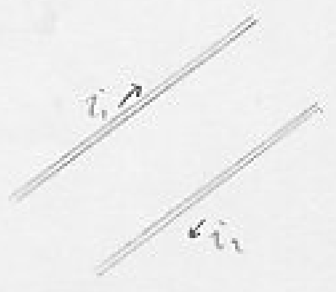
Ex.

Two long Parallel wires carrying currents i_1 and i_2 in opposite dirs.

$i_1 = 15\,A$, $i_2 = 32\,A$ $d = 5.3\,cm$

$\overline{B}_p = ?$





$$B_1 = \frac{\mu_0 i_1}{2\pi R} = \frac{\mu_0 i_1}{2\pi (d/\sqrt{2})} = \frac{\sqrt{2}\mu_0}{2\pi d} i_1 \qquad R = \frac{d}{\sqrt{2}}$$

$$B_2 = \frac{\mu_0 i_2}{2\pi R} = \frac{\mu_0 i_2}{2\pi (d/\sqrt{2})} = \frac{\sqrt{2}\mu_0}{2\pi d} i_2$$

$$B = \sqrt{B_1^2 + B_2^2} = \frac{\sqrt{2}\mu_0}{2\pi d}\sqrt{i_1^2 + i_2^2}$$

$$B = \frac{\sqrt{2}(4\pi \times 10^{-7}\ T.m/A)\sqrt{(15A)^2 + (32A)^2}}{2\pi (5.3 \times 10^{-2} m)} = 189\ \mu T$$

$$\varphi = \tan^{-1}\frac{B_1}{B_2} = \tan^{-1}\frac{i_1}{i_2} = \tan^{-1}\frac{15}{32} = 25^\circ$$

The angle between $\bar{B}$ and x-axis;

$$\varphi + 45^\circ = 25^\circ + 45^\circ = 70^\circ$$

31-5 Ampere's Law

$$dE = \frac{1}{4\pi\epsilon_0}\frac{dq}{r^2} \longrightarrow \oint \bar{E}.d\bar{A} = \frac{q}{\epsilon_0}$$

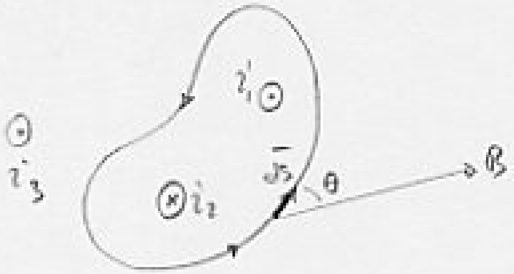
$$d\bar{B} = \frac{\mu_0}{4\pi}\frac{i d\bar{s} \times \bar{r}}{r^3} \longrightarrow \oint \bar{B}.d\bar{s} = \mu_0 i \qquad \text{Ampere's law}$$

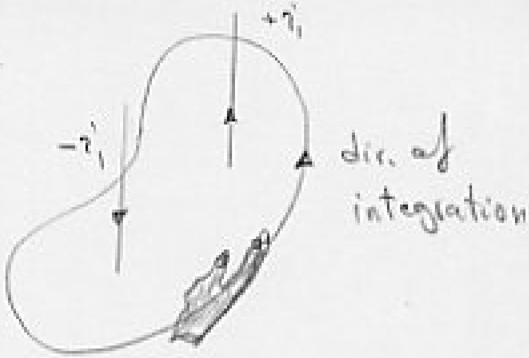
closed loop

(amperian loop)

i: the net current encircled by the loop.

Ex.





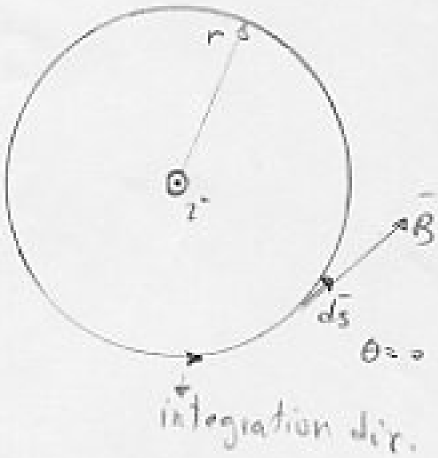
$$\oint \bar{B}\cdot\bar{ds} = \oint B\cos\theta\, ds$$

$$i = i_1 - i_2$$

$$\oint B\, ds\cos\theta = \mu_0(i_1 - i_2)$$

Ex.

A long straight wire carrying a current i.



$$\oint \bar{B}\cdot\bar{ds} = \mu_0 i$$

$$\oint B\cos\theta\, ds = B\oint ds = B(2\pi r)$$

$$B(2\pi r) = +\mu_0 i \quad \Rightarrow B = \frac{\mu_0 i}{2\pi r}$$

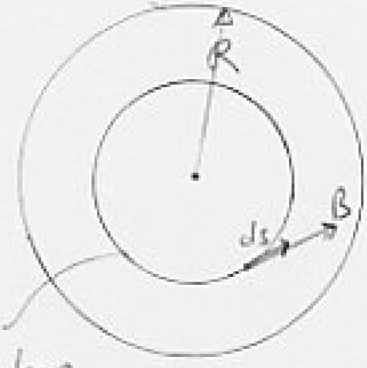
(easier than using Biot-Savart law)

In general we choose the dir. of $\bar{B}$ arbitrary, if we get (+) sign for B in the final result, the chosen dir. is correct, otherwise we get a (−) sign.

Ex.

A long straight wire with a cross section of radius R carrying i_0 (uniformly distributed)

$B(r) = ?$ for $\begin{cases} r > R \\ r < R \end{cases}$



i_0: imerging from the page

$$\oint B \cdot ds = B \oint ds = B(2\pi r)$$

$$i = i_0\left(\frac{\pi r^2}{\pi R^2}\right)$$

$$B(2\pi r) = \mu_0 i_0 \frac{\pi r^2}{\pi R^2} \qquad B = \left(\frac{\mu_0 i_0}{2\pi R^2}\right) r \qquad r < R$$

linear in r

And:

$$B' = \frac{\mu_0 i_0}{2\pi r} \qquad r > R$$

At $r = R$ $\quad B = B' = \dfrac{\mu_0 i_0}{2\pi R}$

Ex.

A long straight wire of radius $R = 1.5$ mm carries a steady current i of 32 A

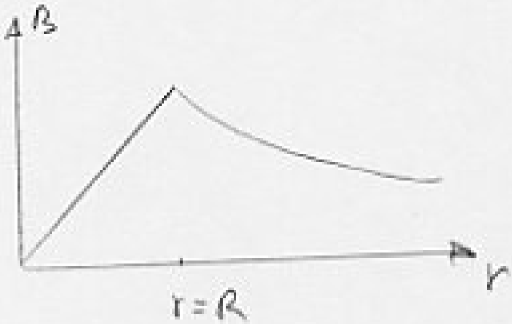
a) $B = ?$ at $r = R$

$$B = \frac{\mu_0 i_0}{2\pi r} = \frac{(4\pi \times 10^{-7}\ T.m/A)(32\ A)}{2\pi(1.5 \times 10^{-3}\ m)} = 4.27 \times 10^{-3}\ T$$

b) $B = ?$ at $r = 1.2$ mm

$$B = \frac{\mu_0 i_0}{2\pi R^2} r = \frac{(4\pi \times 10^{-7}\ \mathrm{T \cdot m/A})(32\,\mathrm{A})}{2\pi\,(1.5\times 10^{-3}\,\mathrm{m})^2}(1.2\times 10^{-3}\,\mathrm{m})$$

$$= 3.41\times 10^{-3}\ \mathrm{T}$$



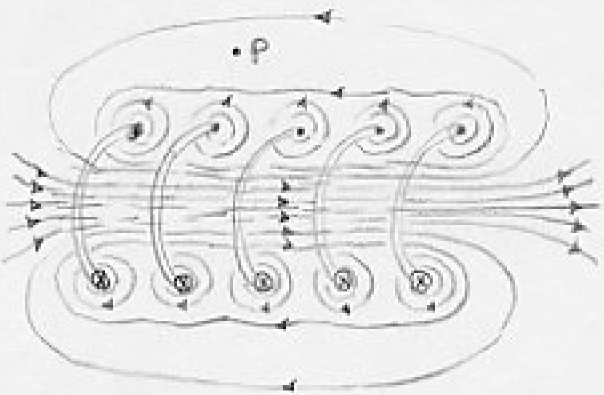
31-6 Solonoids and Toroids

L>>2R

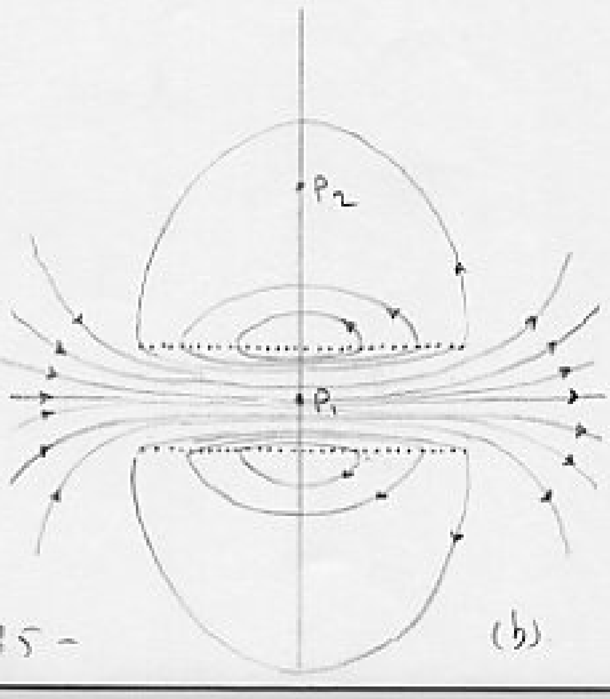
The Solonoid;

Ideal solonoid: $L \to \infty$, tightly packed turns of square wire.

$\longrightarrow$ The field inside the coil is <u>uniform</u> and Parallel to the solonoid axis.



(a)



(b)

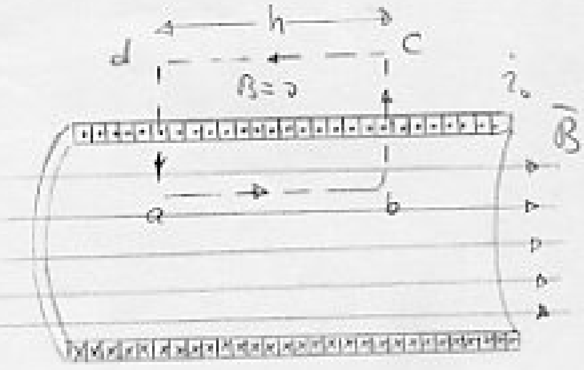
(a) and (b) are <u>real</u> solenoids.

Note that the field is $\begin{cases} \text{strong and uniform at the interior points } (P_1) \\ \text{weak at external point } (P_2) \end{cases}$

At points like P (Fig a) the contributions from up and down sides cancel somehow each other.

In an ideal solenoid $\begin{cases} R \ll L \\ \text{external field} = \text{zero} \end{cases}$

Let us apply Ampere's law:



$$\oint \mathbf{B}\cdot d\mathbf{s} = \mu_0 i$$

$$\oint \mathbf{B}\cdot d\mathbf{s} = \int_a^b \mathbf{B}\cdot d\mathbf{s} + \int_b^c \mathbf{B}\cdot d\mathbf{s} + \int_c^d \mathbf{B}\cdot d\mathbf{s} + \int_d^a \mathbf{B}\cdot d\mathbf{s}$$

$$\oint \mathbf{B}\cdot d\mathbf{s} = Bh + 0 + 0 + 0$$

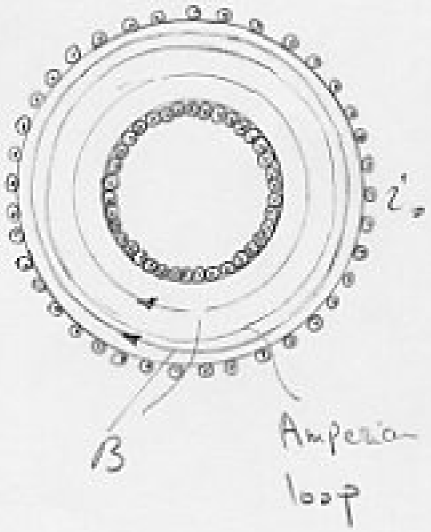
$$i = i_0 (nh) \qquad (n: \text{number of turns per unit length})$$

$$Bh = \mu_0 i_0 nh \rightarrow B = \mu_0 i_0 n \quad (\text{ideal solenoid})$$

It holds quite well for actual solenoids (if we choose the integration path properly).

The Toroid:

Toroid may be described by a solenoid bent into the shape of a doughnut.



$\bar{B}_{in} = ?$

$\oint B \cdot ds = \mu_0 i$

$B(2\pi r) = \mu_0 i_0 N$ N: total number of turns

$B = \frac{\mu_0 i_0 N}{2\pi} \frac{1}{r}$ (toroid)

In contrast to the situation for a solenoid, B is not const over the cross section of a toroid.

Ampere's law $\xrightarrow{\text{shows}}$ $B_{out} = 0$

Since $\frac{N}{2\pi r}$ is the number of turns per unit length

$B = \mu_0 i_0 n$ which is similar to the solenoid's field.

Ex. A solenoid with $L = 1.23\,m$ and inner diameter $d = 3.55\,cm$ has five layers of windings of 850 turns each and carries a current $i_0 = 5.57\,A$. $B = ?$

$B = \mu_0 i_0 n = (4\pi \times 10^{-7}\ T\cdot m/A)(5.57A)\left(\frac{5 \times 850\ \text{Turns}}{1.23\ m}\right)$

$= 2.42 \times 10^{-2}\ T$

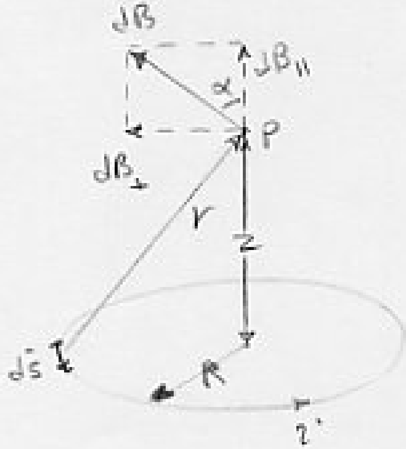
31-7 A Current Loop as a Magnetic Dipole;

Consider a single current loop; We have seen;

$$\bar{\tau} = \bar{\mu} \times \bar{B}_{ext}$$

such a loop behaves like a magnetic dipole;

Magnetic Field of a Current Loop:



The vector sum of $dB_{\perp} = 0$

$B = \int dB_{\parallel}$

Using Biot-Savart law;

$$d\bar{B} = \left(\frac{\mu_0}{4\pi}\right)\frac{i\,d\bar{s} \times \bar{r}}{r^3}$$

$$dB = \left(\frac{\mu_0}{4\pi}\right)\frac{i\,ds\,\sin 90}{r^2} \qquad dB_{\parallel} = dB\cos\alpha$$

$$dB_{\parallel} = \frac{\mu_0 i \cos\alpha\, ds}{4\pi r^2}$$

$$\cos\alpha = \frac{R}{r} = \frac{R}{\sqrt{R^2 + z^2}} \qquad r = \sqrt{R^2 + z^2}$$

$$dB_{\parallel} = \frac{\mu_0 i R}{4\pi (R^2 + z^2)^{3/2}} ds$$

$$B = \int dB_{\parallel} = \frac{\mu_0 i R}{4\pi (R^2 + z^2)^{3/2}} \int ds$$

$$B(z) = \frac{\mu_0 i R^2}{2 (R^2 + z^2)^{3/2}}$$

If $z >> R \longrightarrow B(z) \approx \dfrac{\mu_0 i R^2}{2 z^3}$

Since $\pi R^2 = A \longrightarrow B(z) = \dfrac{\mu_0}{2\pi} \dfrac{N i A}{z^3}$

And since $\bar{\mu}$ and $\bar{B}$ have the same dir.;

$$\bar{B}(z) = \frac{\mu_0}{2\pi} \frac{\bar{\mu}}{z^3}$$ for current loop.

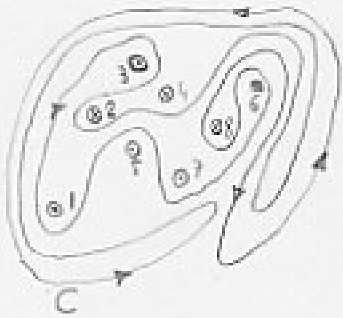
We remind that for the electric dipole we had;

$$\bar{E}(z) = \frac{1}{2\pi\epsilon_0} \frac{\bar{P}}{z^3}$$

41E- Eight wires carying the current $I_k = k i_0$

$(k = 1, 2, \dots 8)$

$I_{k(odd)} \uparrow$, $I_{k(even)} \downarrow$ (antiparallel)

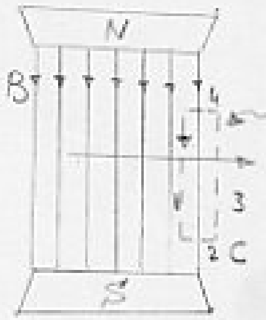


$$\oint_C B \cdot dl = ?$$

Sol.

$$\oint B \cdot dl = \mu_0 I = \mu_0 (I_1 + I_3 + I_6 + I_7) = \mu_0 (i_0 + 3i_0 - 6i_0 + 7i_0)$$
$$= 5 \mu_0 i_0$$

45P- Show that a uniform mag. field $\bar{B}$ can not drop abruptly to zero.



Sol.

$$\oint_C B \cdot dl = \mu_0 I = \mu_0 (0) = 0 \qquad (1)$$

$$\oint_C B \cdot dl = \int_1 B \cdot dl + \int_2 B \cdot dl + \int_3 B \cdot dl + \int_4 B \cdot dl$$

$$\int_1 B \cdot dl = Ba \qquad \int_2 B \cdot dl = \int_3 B \cdot dl = 0 \quad (B \perp dl)$$

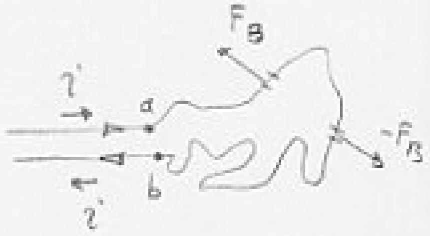
$$\int_3 B \cdot dl = 0 \quad (B = 0 \text{ if } \bar{B} \text{ abruptly drop to zero})$$

$$\rightarrow \int B \cdot dl = Ba \qquad (2)$$

(1) and (2) are in contradiction.

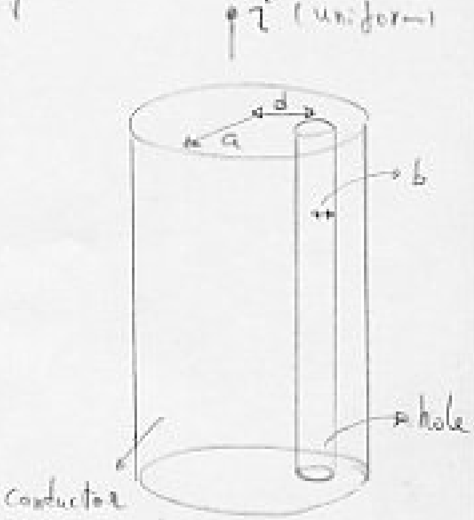
12- A messy loop of limp wire.

Will the current i try to form a circular loop or will it try to bunch up further



<u>Sol.</u>

Since each two element of the wire exert repulsion force on each other, it will change to a <u>circular</u> loop.

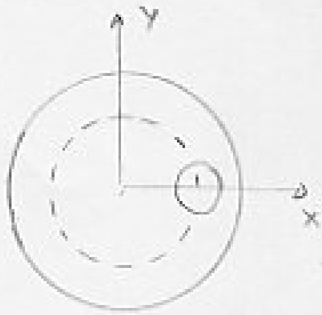


50P-

a) $B = \dfrac{\mu_0 i d}{2\pi(a^2-b^2)}$ (at the center of hole)

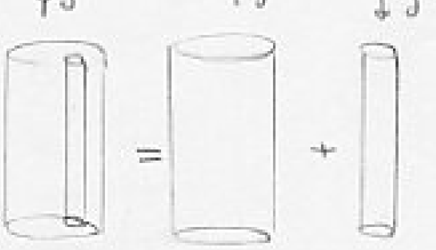
b) Discuss the cases $b=0$ and $d=0$

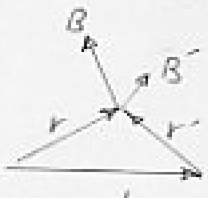
c) Use the Ampere's law to show that the mag. field in the hole is uniform.



<u>Sol.</u>
a) $J = \dfrac{i}{\pi(a^2-b^2)}$

We apply superposition





$\oint B \cdot dl = \mu_0 I = \mu_0 \int J \cdot dA = \mu_0 J A$

$B(2\pi r) = \mu_0 J \pi r^2 \rightarrow B = \dfrac{\mu_0 J}{2} r$

$B(2\pi r') = \mu_0 J \pi r'^2 \rightarrow B' = \dfrac{\mu_0 J}{2} r'$

$$\bar{B} = \frac{\mu_0 J}{2}(\hat{k}\times\bar{r}) \qquad B' = \frac{\mu_0 J}{2}(-\hat{k}\times\bar{r}')$$

$$\bar{r}' = \bar{r} - \bar{d} \quad \rightarrow \quad B' = \frac{\mu_0 J}{2}\left[-\hat{k}(\bar{r}-\bar{d})\right]$$

$$\bar{B}_{TOT} = \bar{B} + \bar{B}' = \frac{\mu_0 J}{2}(\hat{k}\times\bar{d})$$

Since $\bar{d} = d\hat{i} \quad \rightarrow B_{TOT} = \frac{\mu_0 J d}{2}\hat{j}$

$$B_{TOT} = \frac{\mu_0 i d}{2\pi(a^2-b^2)}\hat{j}$$ (everywhere inside the hole) uniform

b) $b=0 \rightarrow B_{TOT} = \frac{\mu_0 i d}{2\pi a^2}$ consistent with the previous results.

$d=0 \rightarrow B_{TOT} = 0$ (inside the hole) consistent with the Ampere's law

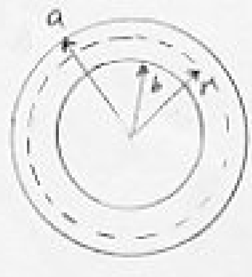
c) we found it in (a).

46P – A hollow cylindrical conductor of radii a and b carrying a uniformly current i.

a) $B = \frac{\mu_0 i}{2\pi(a^2-b^2)}\left(\frac{r^2-b^2}{r}\right)$ for $b<r<a$

b) Discuss the cases $r=a$, $r=b$ and $b=0$

c) Assume $a=2$ cm, $b=1.8$ cm, $i=100$ A and plot $B(r)$ for the range $0<r<6$ cm.



Sol.

a) $\oint B \cdot dl = \mu_0 i \qquad B(2\pi r) = \mu_0 J A = \mu_0 J \pi (r^2 - b^2)$

$J = \dfrac{i}{\pi(a^2-b^2)} \quad \rightarrow \quad B = \dfrac{\mu_0 i}{2\pi(a^2-b^2)} \left(\dfrac{r^2-b^2}{r} \right)$

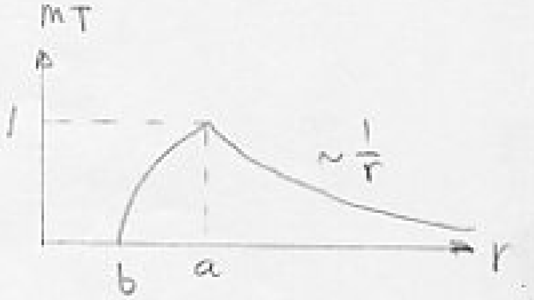
b) $r = a \quad B = \dfrac{\mu_0 i}{2\pi r}$ (the mag. field of a long straight wire)

$r = b \quad B = 0$

$b = 0 \quad B = \dfrac{\mu_0 i}{2\pi a^2} r$ (" " " inside a solid conductor)

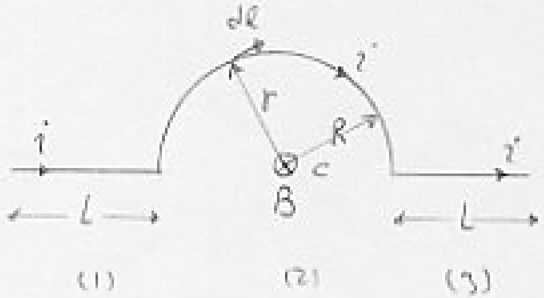
$r > a \quad B = \dfrac{\mu_0 i}{2\pi r}$

c)



11P— $B_c = ?$

Sol.



For the L-pieces $d\bar{s} \times \bar{r} = 0$

$\rightarrow B_1 = B_3 = 0$

$d\bar{B}_2 = \dfrac{\mu_0}{4\pi} \dfrac{i\, d\bar{l} \times \bar{r}}{r^3} \qquad d\bar{l} \perp \bar{r} \quad \rightarrow |dB_2| = \dfrac{\mu_0}{4\pi} \dfrac{i\, dl}{r^2}$

$B_2 = \dfrac{\mu_0 i}{4\pi} \displaystyle\int_0^{\pi R} \dfrac{dl}{R^2} = \dfrac{\mu_0 i}{4\pi R^2} \pi R = \dfrac{\mu_0 i}{4R}$

$B = B_1 + B_2 + B_3 = \dfrac{\mu_0 i}{4R}$