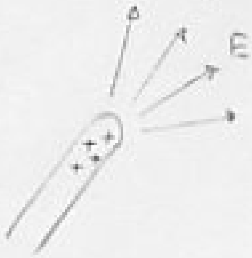
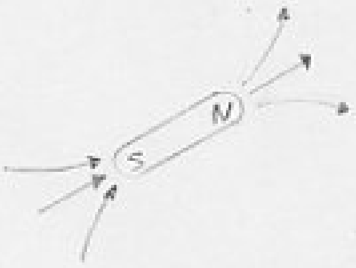
Chapter 30

The Magnetic Field

30-1 The magnetic field;



Similarly $\longrightarrow$



A charged Plastic produces $\bar{E}$

A magnet produces a vector field – the magnetic field $\bar{B}$

<u>Ex.</u>

Permanent magnets

Electromagnets

We have seen;

Electric charge $\longleftrightarrow$ $\bar{E}$

By the symmetry we <u>expect</u>

Magnetic charge $\longleftrightarrow$ $\bar{B}$

But, there seem to be <u>no</u> magnetic charges

$\longrightarrow$ there is <u>no</u> magnetic <u>monopoles</u>.

What is the source of the manetic field?

Experiment $\to$ moving charges

A charge $\xrightarrow{\text{sets up}}$ $\begin{cases} \bar{E}, \text{ at rest or in moving} \\ \bar{B}, \text{ only in moving} \end{cases}$

In the permanent magnet $\longrightarrow$ the electrons of the iron atom are responsible

" " electromagnet $\longrightarrow$ " " in the coil " "

Therefore in magnetism;

Moving charges $\longleftrightarrow \bar{B}$

or Electric currents $\longleftrightarrow \bar{B}$

Orsted (Danish physicist) in 1820 showed that the electric current in a wire could affect a magnetic compass needle.

30-2 The Def. of $\bar{B}$;

We defined the electric field by the force F_E acting on a charge q (at rest)

$$F_E = q\bar{E}$$

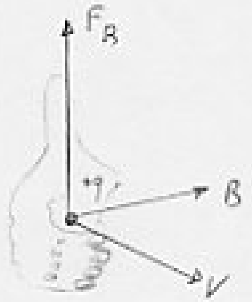
If a magnetic monopole were available, we could define $\bar{B}$ in a similar manner.

We define in the following way which originates from the experiment:

$$\bar{F}_B = q\bar{V}\times\bar{B}$$

This def. gives both the magnitude and the dir. of $\bar{B}$.

There are some points:



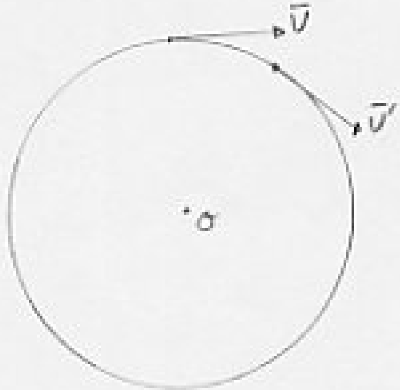
1- $\bar{F}_B \perp \bar{V}$

This means that if $\bar{B} = \begin{cases} \text{const} \\ \text{uniform} \end{cases}$

$\longrightarrow \bar{F}_B$ can change the dir. of $\bar{V}$ not its magnitude

$\longrightarrow T = \frac{1}{2} m v^2$ does not change

This does not violate $\bar{F} = m\bar{a}$



$|\bar{V}| = |\bar{V}'|$

2- $F_B = qvB\sin\varphi$

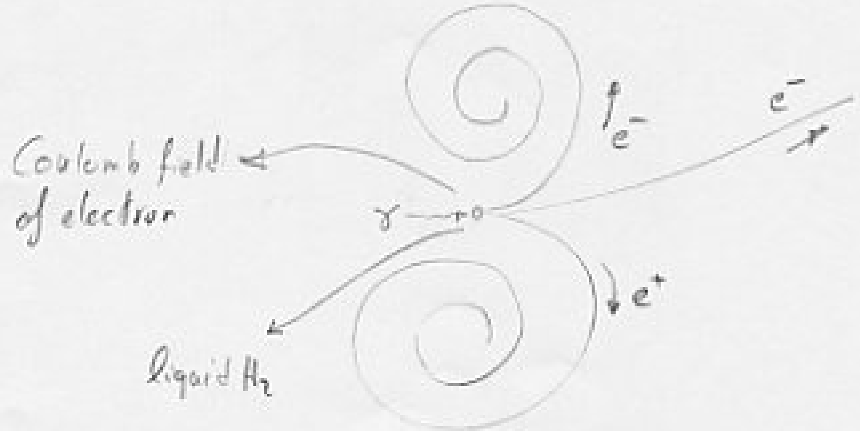
If $\bar{V} \parallel \bar{B} \rightarrow F_B = 0$

3- $F_{B_{max}} = qvB \qquad \varphi = 90^\circ$

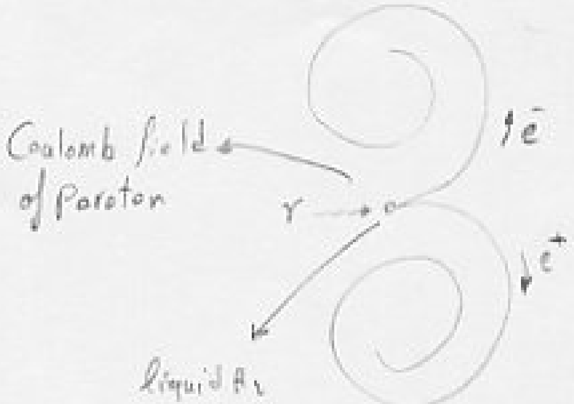
4- $F_B \sim q \quad , F_B \sim v$

5- Dir. of F_B depends on the sign of q.

Ex.



Units;



$$1\,\text{Tesla} = 1\,T = 1\,\frac{N}{C\left(\frac{m}{s}\right)}$$

Since $\frac{1C}{1S} = 1A$

$$1\,T = \frac{N}{A \cdot m} \qquad (SI)$$

$\vec{B}$ points out of the paper

An earlier unit;

$$1\,T = 10^4\,G \quad (gauss)$$

Some mag. fields

At the surface of neutron star	$10^8\,T$
An electromagnet	$1.5\,T$
Near a small bar magnet	$10^{-2}\,T$
At the surface of the Earth	$10^{-4}\,T$
Smallest value in a magnetically shielded room	$10^{-14}\,T$

Magnetic Field Lines:

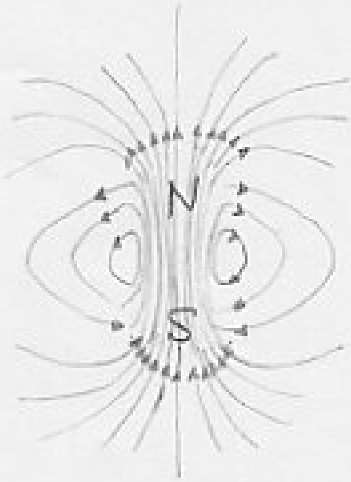
We can represent magnetic fields with field lines just as we did for electric fields.

1) The dir. of the tangent to a magnetic field line at any point gives the dir. of $\vec{B}$ at that point

2) The spacing of the lines is a measure of the magnitude of $\vec{B}$.

Experiment shows;

Opposite poles attract each other.



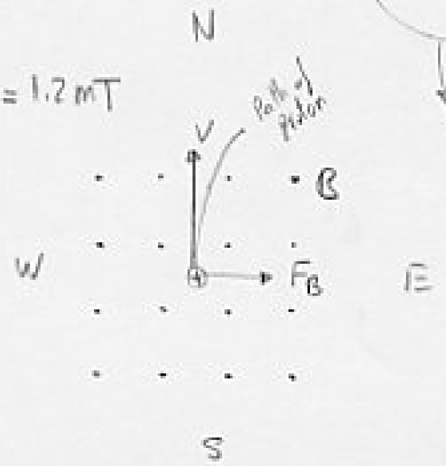
North pole of a compass needle points north of Earth.

$\longrightarrow$ Earth's geomagnetic pole in the northern hemisphere is a south magnetic pole

Ex.

A proton enters in a region with $B = 1.2\,mT$ with $K = 5.3\,MeV$ (lab.), moving from S to N, $m_p = 1.67\times10^{-27}\,kg$

$F_B = ?$



Sol:

$$V = \sqrt{\frac{2k}{m}} = \sqrt{\frac{2(5.3\,MeV)(1.6\times10^{-13}\,J/MeV)}{1.67\times10^{-27}\,kg}} = 3.2\times10^{7}\,m/s$$

$$F_B = qvB\,S\varphi = (1.6\times10^{-19}\,C)(3.2\times10^{7}\,m/s)(1.2\times10^{-3}\,T)(S\,90°)$$
$$= 6.1\times10^{-15}\,N$$

This may seem small force, but it acts on small mass

$$a = \frac{F_B}{m} = \frac{6.1\times10^{-15}\,N}{1.67\times10^{-27}} = 3.7\times10^{12}\,m/s^2$$

Note that, $K << mc^2$ $(5.3 << 938)$

30-3 Discovering the Electron

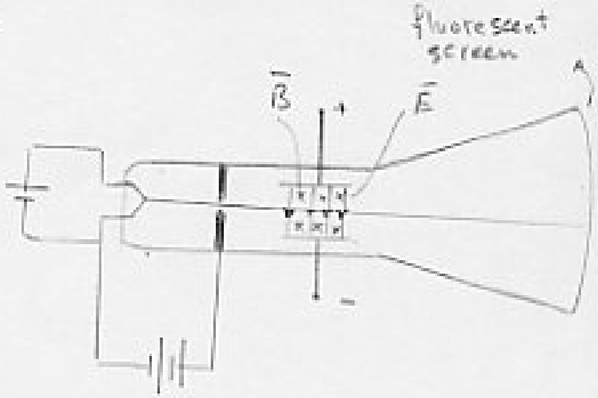
A beam of electron is deflected by a magnetic field.

Ex. TV screen, computer monitor.

Thomson measured the $\frac{m}{q}$ ratio.

Corpuscles (at that time) = electrons

No matter the sign of q, $\bar{E}$, and $\bar{B}$ will deflect q in opposite dir.



A modern version of Thomson device. (1897)

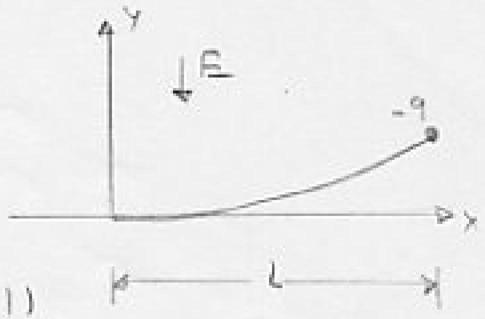
1) Set $E = 0$, $B = 0$ and note the position of the undeflected beam spot.

2) Apply the electric field $\bar{E}$, measure the deflection of the beam.

3) Leaving $\bar{E}$ in place, apply the mag. field $\bar{B}$ and adjust its value until the beam deflection is stored to Zero.

The deflection due to E;

$$\begin{cases} a_y = \frac{F}{m} = \frac{qE}{m} \\ Y = \frac{1}{2} a_y t^2 \quad L = V_x t \end{cases} \rightarrow y = \frac{qEL^2}{2mV_x^2} \quad (1)$$



y ~ displacement of the spot on screen

The dir. of deflection ⟶ gives information about the sign of charge

when $\bar{B}$ is adjusted ⟶ $F_E = F_B$

$$qE = qVB \qquad (\varphi = 90^\circ)$$

$$\rightarrow V = \frac{E}{B} \quad (2) \qquad \text{we can measure } V$$

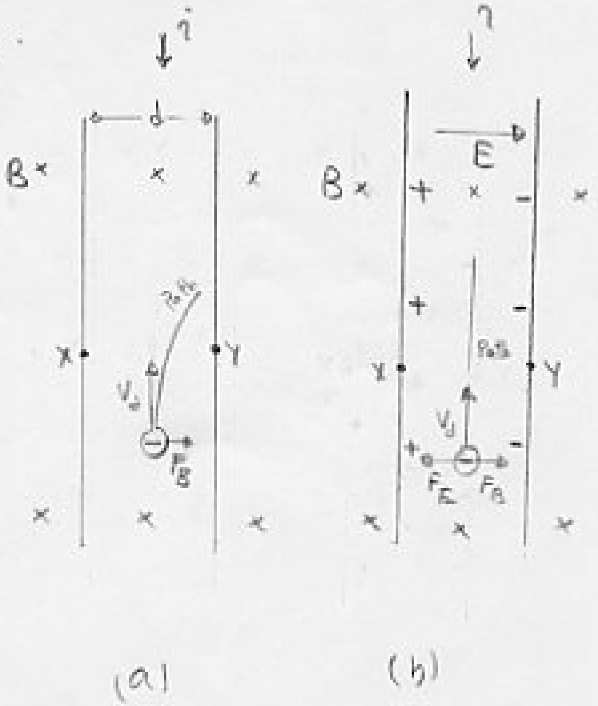
$$(1)(2) \rightarrow \frac{m}{q} = \frac{B^2 L^2}{2YE}$$

30-4 The Hall Effect:

Edwin H. Hall 1879;

The Hall effect allows us to find out whether the charge carriers in a conductor carry a <u>positive</u> or <u>negative</u> charge.

We can also measure the <u>number</u> of such carriers per unit volume of the conductor.



(a) The situation immediately after the mag. field $\bar{B}$ is turned on

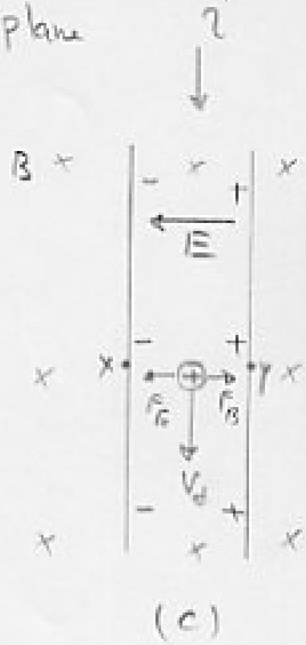
(b) The situation at equilibrium

At equilibrium $F_E = F_B$

$\bar{B}$: pointing into the plane

The electric field $\bar{E}$ that builds up is associated with a Hall potential difference V,

$$E = \frac{V}{d}$$



For positive charge

V can be measured by the terminals x and y.

The <u>polarity</u> of V determines the sign of <u>charge</u>.

The dir of the current i remains the same for both negative and positive carriers.

But Hall effect distingushes these two (the polarity at x and y)

At equilibrium: $(-e)E = (-e)V_d B$ (1)

But $V_d = \frac{J}{ne} = \frac{i}{neA}$ (2) A: cross section of the strip

$E = \frac{V}{d}$ (3)

(1), (2), (3) $\rightarrow n = \frac{Bi}{Vle}$ $l = \frac{A}{d}$: thickness of the strip

V_d can also be measured;

We move the strip mechanically in a dir. opposite to V_d. The speed of the moving strip is then adjusted until the Hall potential difference vanishes.

At this time $|V_d| = |V_{strip}|$

$\rightarrow$ The velocity of charge carrier with respect to $\bar{B}$ is zero $\rightarrow$ There is no Hall effect.

Ex.

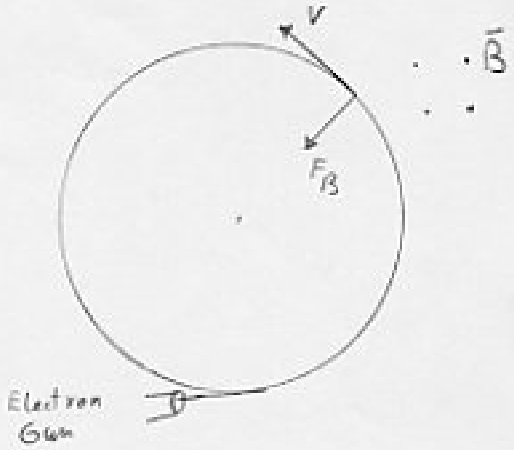
A copper strip $l = 150\,\mu m$, in a mag. field $\bar{B}$ of $|B| = 0.65T$, and $i = 23A$. $V_{Hall} = ?$

Sol.

$$n = \frac{N_A \rho}{M} = \frac{(6.02\times10^{23}\ mol^{-1})(9.0\times10^{3}\ kg/m^3)}{64\times10^{-3}\ kg/mol} = 8.47\times10^{28}\ elec/m^3$$

$$V = \frac{Bi}{nel} = \frac{(0.65\ T)(23\ A)}{(8.47\times10^{28}\ m^{-3})(1.6\times10^{-19}\ C)(150\times10^{-6}\ m)} = 7.4\times10^{-6}\ V$$

30-5 A Circulating charge



$F_B = q\bar{v}\times\bar{B}$

The circular path become visible if the electrons pass through a gass.

B: out of the plane uniform

$$\begin{cases} |F_B| = qvB \\ F = m\dfrac{v^2}{r} \end{cases} \rightarrow \quad qvB = m\frac{v^2}{r}$$

$r = \dfrac{mv}{qB}$, radius

$T = \dfrac{2\pi r}{v} = \dfrac{2\pi}{v}\dfrac{mv}{qB} = \dfrac{2\pi m}{qB}$ Period

$f = \dfrac{1}{T} = \dfrac{qB}{2\pi m}$ frequency

$\omega = 2\pi f = \dfrac{qB}{m}$ angular frequency

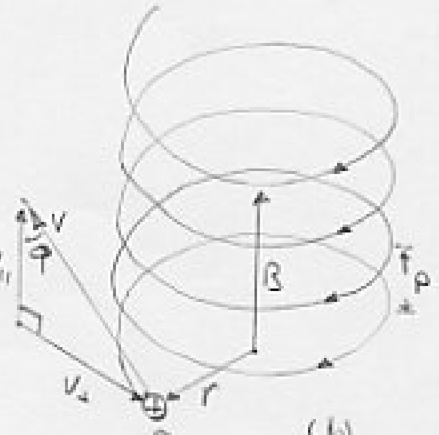
Note that $f \neq f(V)$ (if $v \ll c$)

$r = r(V)$, But $T = T(q/m)$ and $T \neq T(v)$

(a)

If V has a component along $\bar{B}$:

$V_{\parallel} = V \cos\varphi$ $\quad V_{\perp} = V \sin\varphi$

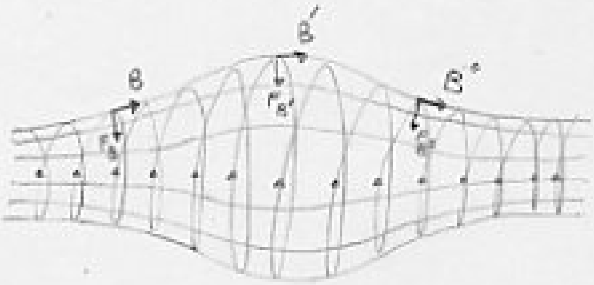


(b)

$V_{\parallel}$, determines the pithch of the helix,
$V_{\perp}$, is responsible for circular motion.

If $\bar{B}$ is strong enough at one end, the particle reflects from that end.

If the particle reflects from both ends, it is said to be trapped in a magnetic bottle. (spiraling back and forth between the ends)



(C)

$B > B'$

A charged particle spiraling in an inhomogeneous $\bar{B}$.

Note that $\bar{F}_B, \bar{F}_{B''}$ have components pointing toward the center of bottle.

Protons and electrons are trapped in this way by the Earth's mag. field, forming the Van Allen radiation belts

When a large solar flare shoots additional energic electrons and protons into the radiation belts, an $\bar{E}$ is produced in the region where electrons normally reflect

→ This $\bar{E}$ eliminates the reflection,

→ and drives electrons down into the atmosphere,

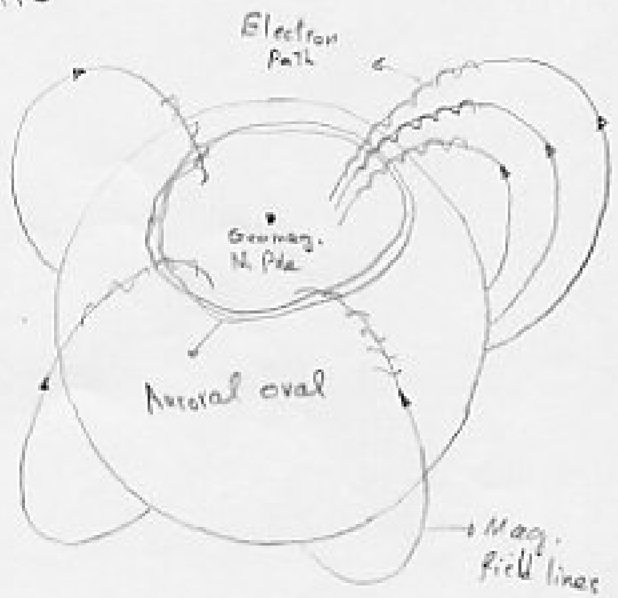
→ they collide with atoms and molecules,

causing → them to emit light (aurora)

Oxygen → green light

Nitrogen → Pink "

Van allen radiation belts



Ex.

The electrons circulating in a mag. field $\bar{B}$ have kinetic energy of $K = 22.5\ eV$. Also, $|\bar{B}| = 4.55\times10^{-4}\ T$

a) $r = ?$ (radius of each electron's path)

b) $f = ?$ c) $T = ?$

Sol.

a) $v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{(2)(22.5\,eV)(1.6\times10^{-19}\,J/eV)}{9.11\times10^{-31}\,kg}} = 2.81\times10^{6}\ m/s$

$r = \frac{mv}{qB}$ $r = \frac{(9.11\times10^{-31}\,kg)(2.81\times10^{6}\,m/s)}{(1.6\times10^{-19}\,C)(4.55\times10^{-4}\,T)} = 3.52\ cm$

b) $f = \frac{qB}{2\pi m} = \frac{(1.6\times10^{-19}\,C)(4.55\times10^{-4}\,T)}{2\pi(9.11\times10^{-31}\,kg)} = 1.27\times10^{7}\ Hz = 12.7\ MHz$

c) $T = \frac{1}{f} = 7.86\times10^{-8}\ s$

Ex.

In the previous example, suppose $\varphi = 65.5°$ (the angle between $\bar{v}$ and $\bar{B}$).

a) $r = ?$ (radius of helical path)

b) $p = ?$ (pitch of the helix)

Sol.

$r = \frac{mv_\perp}{qB} = \frac{mv\sin\varphi}{qB} = \frac{(9.11\times10^{-31}\,kg)(2.81\times10^{6}\,m/s)(\sin 65.5)}{(1.6\times10^{-19}\,C)(4.55\times10^{-4}\,T)}$

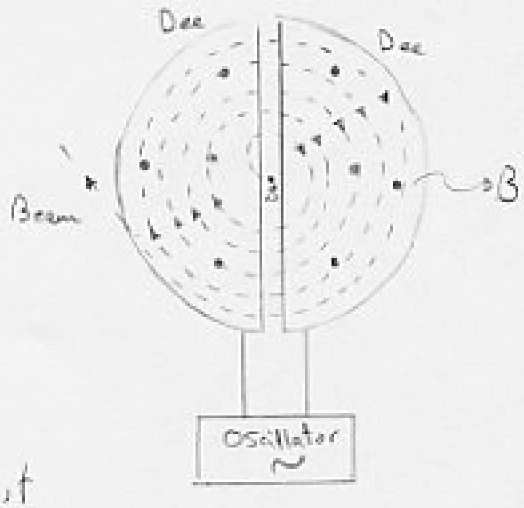
$= 3.20\ cm$

Since T and f are independent of v, they are the same as in the previous example.

$p = v_\parallel T = (v\cos\varphi)\,T = (2.81\times10^{6}\,m/s)(\cos 65.5°)(7.86\times10^{-8}\,s)$

$= 9.16\times10^{-2}\ m = 9.16\ cm$

30-6 Cyclotrons and Synchrotrons:



1) Charged particles originated from S and accelerated by the potential difference between two dees.

2) When these particles enter the interior region of dees, they are screened (don't feel the ΔV, because dees are made of copper and they shield the electric field) But all the time they feel $\vec{B}$.

3) When again these particles emerge into the central gap, assume that ΔV has changed its sign. In this way the particle will be more accelerated.

4) In this way if the frequency of the oscillator is equal with the natural frequency of the charge particles at which they circulate, the particles will get Kinetic energy at each entrance into the gap.

$$f = f_{osc} \quad \text{resonance cond.}$$

$$f = \frac{qB}{2\pi m} \longrightarrow qB = 2\pi m f_{osc}$$

The Proton Synchrotron;

At proton energies above 50 Mev, the conventional cyclotron begins to fail.

Acc. to $T = \frac{2\pi m}{qB}$, T is v-indep.

But as $v \to c$, $m = m(v)$ $m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$

$\to T = T(v)$

Since T_{osf} is fixed, $\to$ the protons gets out of step with the cyclotron oscillator

(Note: m of protons in different r are different)

The second difficulty;

For a 500-GeV proton in a $|\bar{B}| = 1.5$ T $\to$ the path radius $\sim$ 1.1 km

The magnet of this size is expensive.

The proton synchrotron is designed to meet these two difficulties.

f_{osc} and $\bar{B}$ are made to vary with time during the accelerating cycle.

If this is done properly:

1) the frequency of the circulating proton remains in step with the oscillator at all times

2) the protons follow a circular – not a spiral path.

→ the magnet need only extend along that circular path and not over dees area.

$2\pi R = 6.3\,km$ in Fermilab (4×10^5 round-trip → 1 Tev (10^{12} ev))

A larger one is located at the European Center for Particle Physics (CERN)

And a larger one (Superconducting Super Collider, SSC) will be built in Texas ($2\pi R \sim 52$ miles) → 20 Tev

Ex. A syclotron at oscillator frequency $= 12\,\mathrm{MHz}$

and $R_{dee} = 53\,\mathrm{cm}$

a) $|B| = ?$ to accelerate deutrons.

Deutron : $n-p$, $m_d \approx 2m_p$, $q = e$

$$B = \frac{(2\pi)\, m_d\, f_{osc}}{q} = \frac{(2\pi)(2\times 1.67\times 10^{-27}\,\mathrm{kg})(12\times 10^{6}\,\mathrm{s}^{-1})}{1.6\times 10^{-19}\,\mathrm{C}} = 1.57\,\mathrm{T}$$

b) $K = ?$ Kinetic energy

$$V = \frac{RqB}{m_d} = \frac{(0.53\,\mathrm{m})(1.6\times 10^{-19}\,\mathrm{C})(1.57\,\mathrm{T})}{2\times 1.67\times 10^{-27}\,\mathrm{kg}} = 3.99\times 10^{7}\,\mathrm{m/s}$$

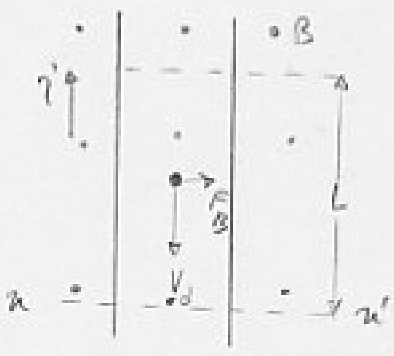
$$K = \tfrac{1}{2} m v^2 = 16.6\,\mathrm{MeV}$$

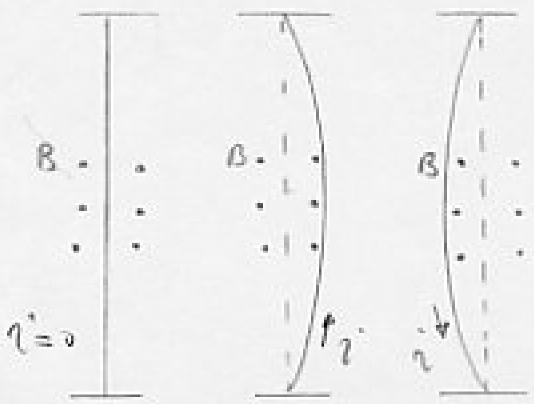
30-7 The Magnetic Force on a Current Carrying Wire;

$\bar{F}_B = q\bar{v}\times\bar{B}$

$F_B = qvB\sin\varphi$, $\varphi = 90$

$F_B = (-e)\, v_d B$ on each electron





$t = \frac{L}{V_d}$ the time required for the electrons to pass the length L.

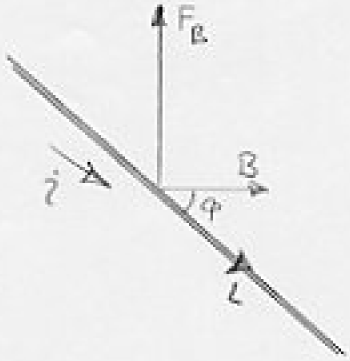
$q = it \quad q = i\left(\frac{L}{V_d}\right)$

$F_B = q V_d B S\varphi = \left(\frac{iL}{V_d}\right)(V_d) B S 90 \quad F_B = iLB$

the force on straight line L

In general $\bar{F}_B = i \bar{L} \times \bar{B}$ force on a current

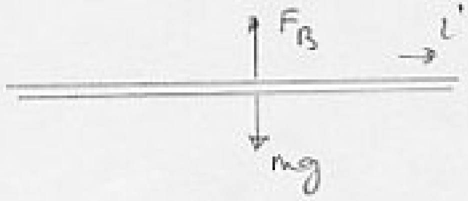
$\bar{L}$ in the dir. of i



If the wire is not straight:

$$d\bar{F}_B = i\, d\bar{L} \times \bar{B}$$

Ex.

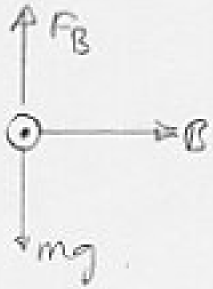


A copper wire carries $i = 28\,A$

$|\bar{B}| = ?$ to float the wire.

$\lambda_{cu} = 46.6\ gr/m$

Sol.

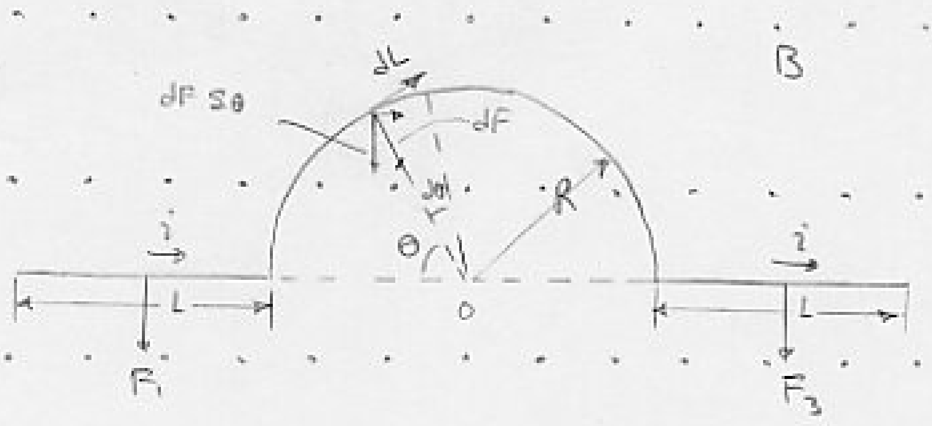


$mg = LiB$

$B = \frac{(m/L)g}{i} = \frac{(46.6\times 10^{-3}\ kg/m)(9.8\ m/s)}{28\,A} = 1.6\times 10^{-2}\ T$

i emerges from the plane

Ex.



$\overline{F}_B = ?$ on wire

Sol.

$F_1 = F_3 = iLB$ on straight sections

On the arc section;

$dF = iB\,dL = iB\,(R\,d\theta)$

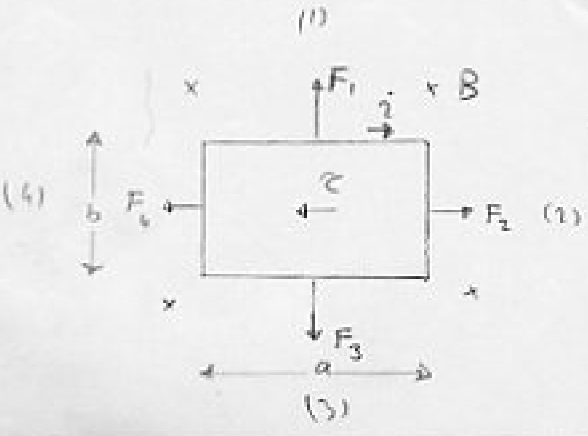
The only effective component is $dF\,S\theta$

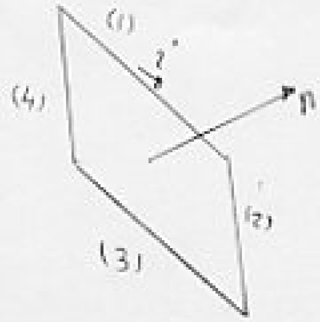
$F_2 = \int_0^\pi dF\,S\theta = \int_0^\pi (iB\,R\,d\theta)\,S\theta = iBR\int_0^\pi S\theta\,d\theta$

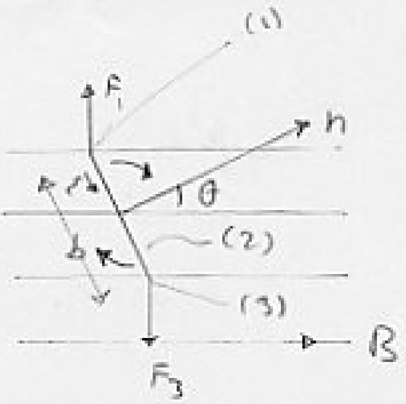
$F_2 = 2iBR$

$F = F_1 + F_2 + F_3 = iLB + 2iBR + iLB = 2iB(L+R)$

30-8 Torque on a Current loop;







$F_1 = F_3 = iaB \qquad (\varphi = 90^\circ)$

$F_2 = F_4 = ibB \sin(90-\theta) = ibB\cos\theta$

$\tau = r \times F \qquad |\tau| = rF\sin\theta$

$\tau_1 = \tau_3 = (iaB)(\frac{b}{2})\sin\theta) \qquad \tau = \tau_1 + \tau_3 = iabB\sin\theta$

$\tau_2 = \tau_4 = 0$

For N turns; $\tau' = N\tau = NiabB\sin\theta$

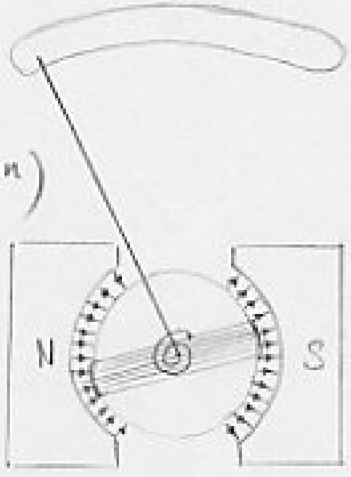
$\tau' = (NiA)B\sin\theta \qquad (A = ab) \qquad (1)$

the properties of coil

This equ holds for all plane loops, no <u>matter</u> what their shape.

Equ. (1) tells us that the current carrying coil in $\bar{B}$ tends to rotate so that $\hat{n}$ points in $\bar{B}$ dir.

Ex.



$(\text{Net } \bar{B}) \perp \hat{n}$ → $\theta = 90^\circ$ (for any orientation of the coil)

$B = 0.23\,T$

$i = 100\,\mu A \longrightarrow \varphi = 28^\circ$ deflection of needle

$\tau_{spring} = K\varphi$ counter torque (1)

$N = 250$ Turns $A = 2.52\times10^{-4}\,m^2$

$\tau = NiAB\,S\theta$ (2) $\tau = \tau_{spring} \rightarrow K = \dfrac{NiABS\theta}{\varphi}$

$= (250)(100\times10^{-6}A)(2.52\times10^{-4}m^2)\dfrac{(0.23\,T)(S\,90^\circ)}{28^\circ} = 5.2\times10^{-8}\,\dfrac{N.m}{degree}$

30-9 The Magnetic Dipole

$\mu = NiA$ mag. moment

$\tau = (NiA)BS\theta \longrightarrow \mu B S\theta$

$\bar{\tau} = \bar{\mu}\times\bar{B}$ $\bar{\mu}$ in the dir of $\hat{n}$

(remember $\bar{\tau} = \bar{P}\times\bar{E}$)

In analogy with $U(\theta) = -\bar{P} \cdot \bar{E}$

$U(\theta) = -\bar{\mu} \cdot \bar{B}$ magnetic potential energy of a mag. dipole

$U_{min} = -\mu B \cos 0 = -\mu B$ $\quad \bar{\mu} \parallel \bar{B}$

$U_{max} = -\mu B \cos 180 = +\mu B$ $\quad \bar{\mu}$ antiparallel with $\bar{B}$

$\Delta U = U(180) - U(0) = 2\mu B$

This much energy must be done by an external agent to turn a magnetic dipole through 180°, starting when it is lined up with $\bar{B}$.

A small bar magnet	5 J/T
The Earth	8×10^{22} "
Proton	1.4×10^{-26} "
Electron	9.3×10^{-24} "

Ex.

$N = 250 \quad i = 100\,\mu A \quad A = 2.52 \times 10^{-4}\, m^2$

a) $\mu = ?$ $\quad \mu = NiA = 6.3 \times 10^{-6}$ J/T

b) $|B| = 0.85$ T

If $\bar{B} \parallel \bar{\mu}$, $\quad W = ?$ to change the orientation from $\theta = 0$ to $\theta = \pi$

$W = \Delta U = 2\mu B = 10.7 \times 10^{-6}$ J