Chapter 28

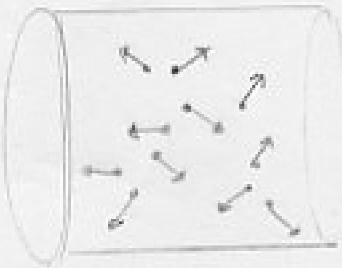
Current and Resistance

28-1 Moving Charges and Electric Currents

The previous 5-chapters deal largely with electrostatic (that is charges at rest). With this chapter we begin to focus on electric currents (that is, charges in motion).

Not all moving charges constitute an electric current:

1- Random motion of elections in an isolated conducting wire with the speed of order of 10^6 m/s doesn't produce a current (net flow $= 0$)



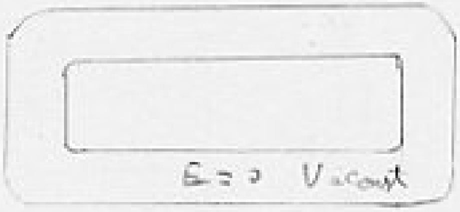
Random motion of elections

2- The flow of water through a garden hose doesn't produce a current (despite there is a net flow, the flow of negative and positive charges are equal).

In this chapter we deal with steady currents of conduction electrons through metallic conductors; $\frac{di}{dt} = 0$

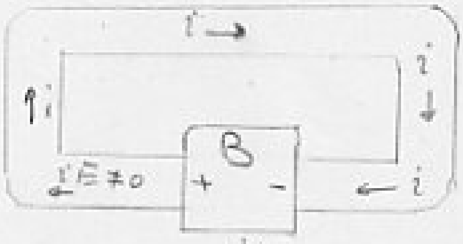
28-2 Electric Current:

In Fig (a) $F_{\text{net on each conduction electron}} = 0$



(a)

In Fig (b) $F \neq 0$



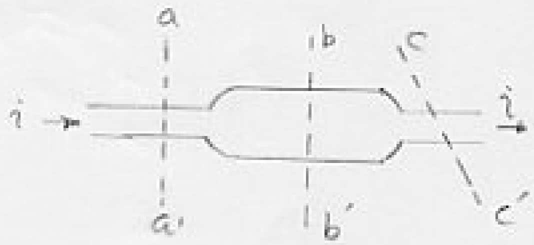
(b)

Within a short time, the electron flow reaches a steady-state cond.

Dir. of **i**: Dir. of positive carriers

$\Delta V \neq 0$

The charge dq passing a hypothetical plane (like aa') in time dt produces a current i



$$i = \frac{dq}{dt} \qquad \text{(def.)}$$

Also;

$$q = \int dq = \int_0^t i\,dt$$

Under steady-state conds.;

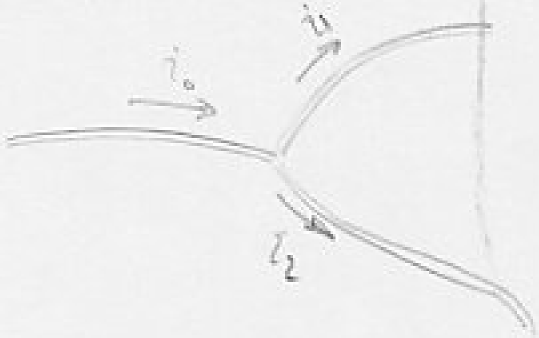
$$i_{aa'} = i_{bb'} = i_{cc'} = i_{\text{all other planes passing completely through the conductor}}$$

from charge conservation if one electron enters the conductor from one end another one leaves from the other end.

Unit $\quad 1 \text{ ampere} = 1\text{A} = \frac{1\,\text{C}}{\text{s}}$

i is a scalar, since q and t are scalars.

So i does not obey the laws of vector addition.



By charge conservation;

$i_0 = i_1 + i_2$ scalar

The Direction of Current:

Historical Convention: The current arrow is drawn in the direction in which positive carriers would move, even if the actual carriers are not positive.

Ex

a) In conductors the negative charges (electrons) are the carriers.

b) " fluorescent lamp both negative and positive charges " " .

A positive charge moving, say, from left to right, has the same external effect as a negative carrier moving from right to left.

$+q$ from (a) $\xrightarrow{to}$ (b)

or $-q$ " (b) $\xrightarrow{to}$ (a)

Ex

water flows through a garden hose at a rate R of $450\ cm^3/s$.

$i = ?$ (of negative charges)

$$\frac{dN}{dt} = \frac{R\rho N_A}{M} = (450\times10^{-6}\ m^3/s)(1000\ kg/m^3)\frac{(6.02\times10^{23}\ molecules/mol)}{0.018\ kg/mol}$$

$$= 1.51\times10^{25}\ molecules/s$$

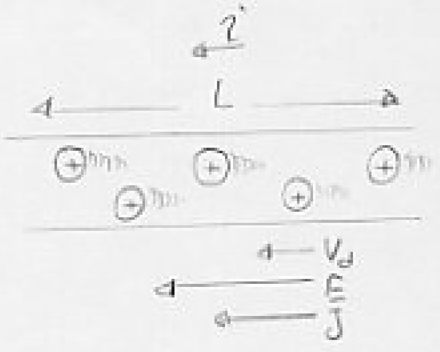
Remark $\begin{cases} \frac{M}{N_A} = \text{mass of each molecule} \\ R\rho = \text{total mass of water passing in one s.} \end{cases}$

H_2O has 10 electrons

$$i = \frac{dq}{dt} = 10e\frac{dN}{dt} = (10\ electron/molecules)(1.6\times10^{-19}\ C/electron)(1.51\times10^{25}\ molecules/s)$$

$$i = 2.42\times10^{7}\ C/s = 2.42\times10^{7}\ A = 24.2\ MA$$

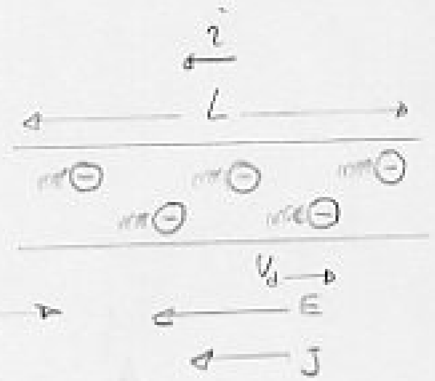
28-3 Current Density

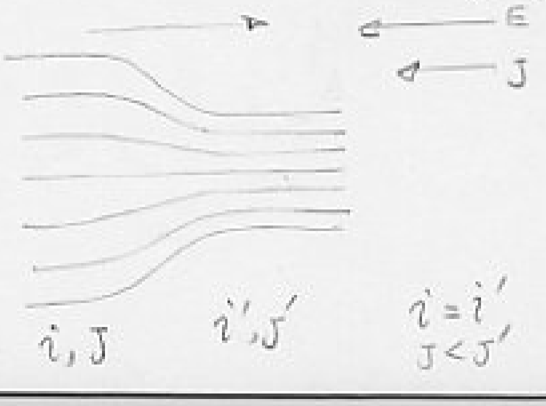


$$J = \frac{i}{A}$$ (for uniformly distributed current)

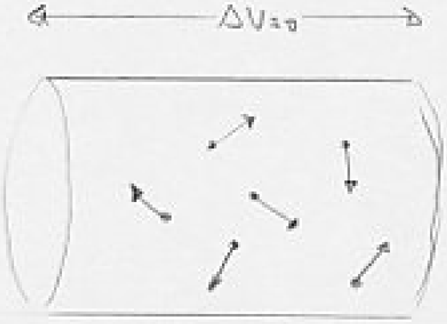
Also

$$i = \int \bar{J}\cdot d\bar{A}$$





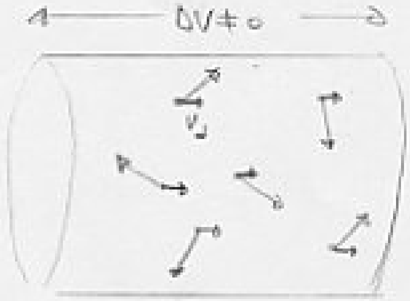
Calculating of Drift speed:



(a)

Random jostling motion

The conduction electrons in a copper have randomly directed velocities with magnitudes of 10^6 m/s



Drift speed is superimposed on the random jostling motion.

The directed flow or drift speed of the conduction electrons is very much smaller. $V_d \sim 10^{-3}$ m/s

In a wire:

$\Delta q = (nAL)e$ n: number of carriers/unit volume

Passes through this volume in a time interval, given by:

$\Delta t = L/v_d$

$$i = \frac{\Delta q}{\Delta t} = \frac{nALe}{L/v_d} = nAev_d \longrightarrow v_d = \frac{i}{nAe} = \frac{J}{ne}$$

or $\bar{J} = (ne)\,\bar{V}_d$

Ex. One end of an Al wire whose diameter is 2.5 mm, is welded to one end of a copper wire whose diameter is 1.8 mm. The composite wire carries a steady current $i = 1.3$ A. $J = ?$ in each wire.

$$A_{Al} = \pi\left(\frac{d}{2}\right)^2 = \frac{\pi}{4}(2.5\times10^{-3}\,m)^2 = 4.91\times10^{-6}\,m^2$$

$$J_{Al} = \frac{i}{A_{Al}} = \frac{1.3\,A}{4.91\times10^{-6}\,m^2} = 2.6\times10^{5}\,A/m^2$$

$$A_{Cu} = \pi\left(\frac{d'}{2}\right)^2 = 2.54\times10^{-6}\,m^2$$

$$J_{Cu} = \frac{i}{A_{Cu}} = \frac{1.3\,A}{2.54\times10^{-6}\,m^2} = 5.1\times10^{5}\,A/m^2$$

Ex.

What is the drift speed of the conduction electrons in the copper wire in the previous exaple.

Sol.

In Cu there is one conduction electron per atom.

$$n = \frac{N_A \rho}{M} = \frac{(6.02\times10^{23}\,atom/mol)(9\times10^{3}\,kg/m^3)}{64\times10^{-3}\,kg/mol} = 8.47\times10^{28}\,atom/m^3$$

$$v_d = \frac{J}{ne} = \frac{5.1\times10^{5}\,A/m^2}{\left(8.47\times10^{28}\,\frac{electrons}{m^3}\right)\left(1.6\times10^{-19}\,\frac{C}{electron}\right)} = 3.8\times10^{-5}\,m/s = 14\,cm/h$$

Ex.

$w = 3.2\,mm \quad t = 250\,\mu m$

$i = 5.2\,mA \quad n = 1.5\times10^{23}/m^3$

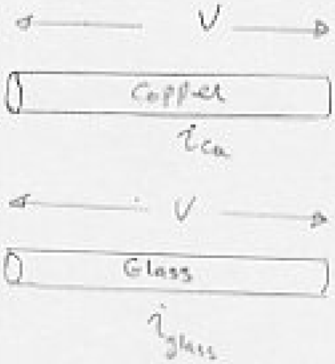
a) $J = ?$ $\quad J = \frac{i}{wt} = \frac{5.2\times10^{-3}\,A}{(3.2\times10^{-3}\,m)(250\times10^{-6}\,m)} = 6500\,A/m^2$ doped semiconductor

b) $V_d = ?$

$$V_d = \frac{J}{ne} = \frac{6500\,A/m^3}{(1.5\times10^{23}/m^3)(1.6\times10^{-19}\,C)} = 0.27\,m/s$$

$$\rightarrow V_{d_{Si}} \gg V_{d_{Cu}}$$

28-4 Resistance and Resistivity:



Applying same V between the ends of Cu and glass, we get;

$$i_{Cu} \neq i_{glass}$$

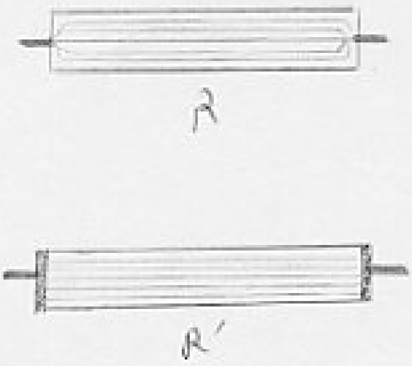
$R = \dfrac{V}{i}$ (def.)

Unit 1 ohm $= 1\,\Omega = 1\,\dfrac{V}{A}$

Resistor: A conductor whose func. in a circuit is to provide a specified resistance.

Resistance: The characteristic of the conductor given by $R = \dfrac{V}{i}$.

The resistance of a conductor depends on the manner in which the potential difference is applied



$R \neq R'$

Alternative approach:

$V \longrightarrow E$, $i \longrightarrow J$, $R \longrightarrow \rho$

$\rho = \dfrac{E}{J}$ (def.) Resistivity

Unit

$E : V/m$

$J : A/m^2$

$\rightarrow \rho : \Omega \cdot m$

In vector form: $\bar{E} = \rho \bar{J}$

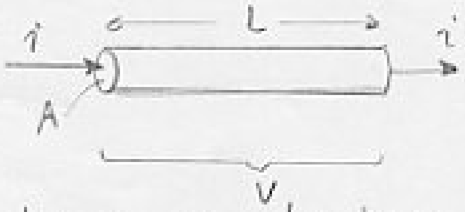
Conductivity: $\sigma = \frac{1}{\rho}$ $\qquad \frac{1}{\Omega \cdot m}$

Calculating the Resistance:

For a uniform and const. current density

$\rightarrow$ E uniform and const.

$\rightarrow E = \frac{V}{L}$ and $J = \frac{i}{A}$



homogeneous and isotropic conductor.

$\rho = \frac{E}{J} = \frac{V/L}{i/A}$ $\qquad$ Since $\frac{V}{i} = R \longrightarrow R = \rho \frac{L}{A}$

Variation with Temperature:

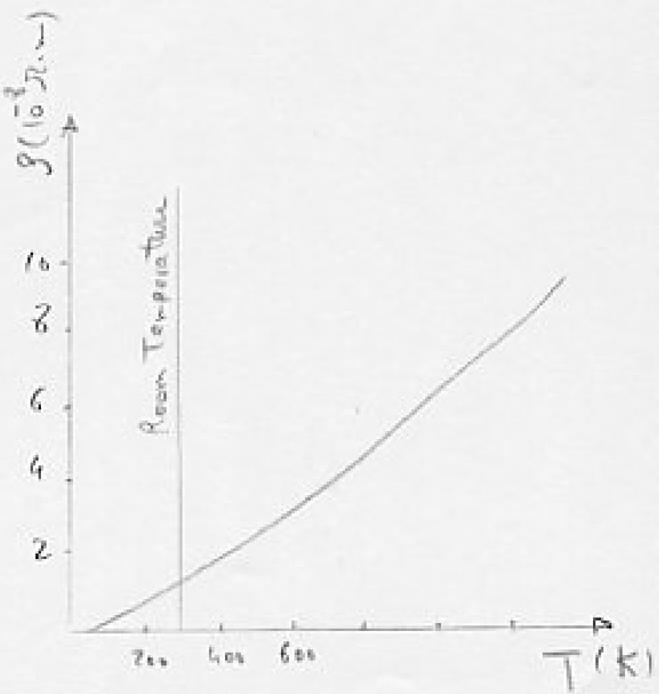
Assuming linear response:

$\rho - \rho_0 = \rho_0 \alpha (T - T_0)$

T_0 : ref. temperature

ρ_0 : resistivity at T_0

$T_0 = 293$ K



One may use either Celsius or Kelvin scale in this eqn.
(since the difference enters and the sizes of the degree on these two scales are identical)

α: temperature coeff. of resistivity.

Since the relation of ρ and T is not completely linear α is chosen to be different from one region to another region.

$$\rho = \rho_0 \, e^{\alpha(T-T_0)} \qquad (\text{exact})$$

Ex. In a copper conductor;

$\rho = 1.69 \times 10^{-8}\ \Omega.m$, $J = 5.1 \times 10^{5}\ A/m^2$

$E = ?$

Sol. $E = \rho J = 8.6 \times 10^{-3}\ V/m$

Ex. In n-type silicon semiconductor;

$\rho = 8.7 \times 10^{-4}\ \Omega.m$, $J = 6500\ A/m^2$

$E = ?$

Sol.

$E = \rho J = 5.7\ V/m$

Ex. A rectangular block of iron has dims. 1.2x1.2x15 cm.

a) R = ? between two square ends?

$\rho_{iron} = 9.68\times10^{-8}\ \Omega\cdot m$

$A = (1.2\times10^{-2} m)^2 = 1.44\times10^{-4} m^2$

$R = \frac{\rho L}{A} = \frac{(9.68\times10^{-8}\Omega\cdot m)(0.15 m)}{1.44\times10^{-4} m^2} = 100 \mu\Omega$

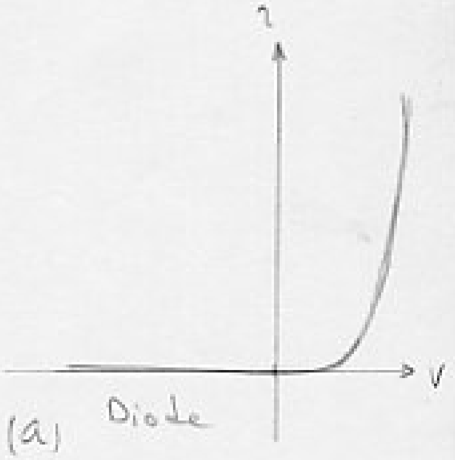
b) R = ? between two opposite rectangular faces?

$A' = (1.2\times10^{-2} m)(0.15 m) = 1.8\times10^{-3} m^2$

$R' = \frac{\rho L'}{A'} = \frac{(9.68\times10^{-8}\Omega\cdot m)(1.2\times10^{-2} m)}{1.80\times10^{-3} m^2} = 0.65 \mu\Omega$

28-5 Ohm's law

a) The relation between i and V is not linear, it depends on the applied potential difference and its polarity.

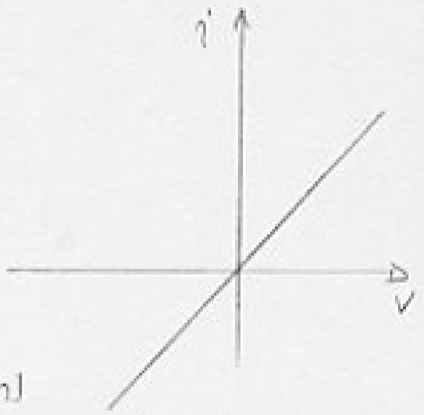


(a) Diode

b) $\frac{i}{V}$ = slope of straight line = const.

is indep. of V and its polarity.

$\frac{i}{V} = R$ resistance.



(b) Resistor

Ohm's law, is an assertion that the current flowing through a device is directly proportional to the potential applied to the device. (i.e. $i \underset{\text{directly}}{\sim} V$)

The device in Fig. (b) obeys Ohm's law, while
" " " (a) doesn't obey " ".

A conducting device obeys Ohm's law, when its resistance is indep. of the <u>magnitude</u> and <u>polarity</u> of the applied potential difference.

$\longrightarrow$ $V = Ri$ <u>is not</u> a statement of Ohm's law.

Alternatively;

$\bar{E} = \rho \bar{J}$

If $\rho \neq \rho(E)$ $\longrightarrow$ The device obeys Ohm's law.

28-6 A Microscopic View of Ohm's law;

Consider a conductor (free-electron model);

The conduction electrons are free to move throughout the volume of the metal.

We assume; the electrons don't collide with each other, but only with the atoms of metal.

Acc. to classical Physics;

speed distribution of the electrons = Maxwellian

$$p(v) = 4\pi \left(\frac{M}{2\pi RT}\right)^{3/2} \frac{v^2}{e^{+\frac{Mv^2}{2RT}}}$$

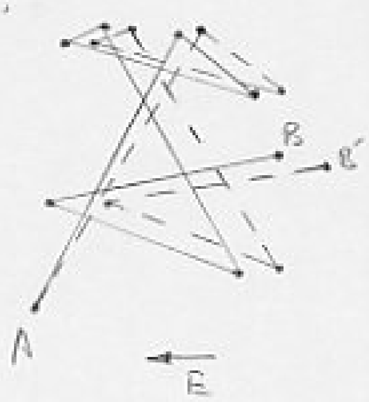
$\rightarrow \bar{V} \sim (T)^{1/2}$

The motion of electrons however is governed by laws of quantum mechanics.

Acc. to quantum theory $\longrightarrow V_{eff} \neq V_{eff}(T)$ for electrons

For copper $V_{eff} = 1.6 \times 10^{6}$ m/s

Applying an electric field E, V_d is appeared:



$V_d << V_{eff}$

(as we have shown in one of the examples $V_d \sim 10^{-5}$ m/s)

The motion of the electrons = Random motion + The effect of E

$\overline{\left(\begin{array}{c}\text{Random motion}\\ \text{effect}\end{array}\right)} = 0$

$\longrightarrow$ V_d is the consequent of E

$$a = \frac{F}{m} = \frac{eE}{m}$$

We assume after a typical collison, the electron completely <u>lose</u> its memory of its previous drift velocity.

$\longrightarrow$ Each electron will then start off <u>fresh</u> after every encounter, moving off in a random dir. -

$V_d = a\tau$ $\quad$ τ: average time between two collisions (mean free time)

$$\begin{cases} V_d = a\tau = \dfrac{eE\tau}{m} \\ J = (ne)V_d \end{cases} \longrightarrow V_d = \frac{J}{ne} = \frac{eE\tau}{m}$$

$$E = \left(\frac{m}{e^2 n\tau}\right) J$$

Comparing with $E = \rho J$ $\longrightarrow$ $\rho = \dfrac{m}{e^2 n \tau}$

If $\rho = \text{const}$ (i.e. $\rho \neq \rho(E)$) $\longrightarrow$ The metal obeys Ohm's law

$\rho = \text{const} \xrightarrow{\text{must}} \tau = \text{const}.$

It is so, because $\underset{\substack{\uparrow \\ \text{by } \overline{E}}}{V_d} << V_{eff}$

Ex. a) $\tau = ?$ in copper

$\lambda = ?$ mean free path if $\begin{cases} V_{eff} = 1.6\times10^{6}\ m/s \\ n = 8.47\times10^{28}\ \dfrac{electrons}{m^3} \end{cases}$

Sol.

a) $\tau = \dfrac{m}{ne^2\rho} = \dfrac{9.1\times10^{-31}\ kg}{(8.47\times10^{28}\ \frac{electron}{m^3})(1.6\times10^{-19}\ C)^2(1.69\times10^{-8}\ \Omega.m)}$

$\tau = \dfrac{9.1\times10^{-31}\ kg}{3.66\times10^{-17}\ C^2.\Omega/m^2} = \dfrac{9.1\times10^{-31}\ kg}{3.66\times10^{-17}\ kg/s} = 2.5\times10^{-14}\ s$

$\dfrac{C^2.\Omega}{m^2} = \dfrac{C^2.V}{m^2.A} = \dfrac{C^2.\ J/C}{m^2.\ C/s} = \dfrac{kg.m^2/s^2}{m^2/s} = \dfrac{kg}{s}$

b) $\lambda = \tau V_{eff} = (2.5\times10^{-14}\ s)(1.6\times10^{6}\ m/s) = 4\times10^{-8}\ m$

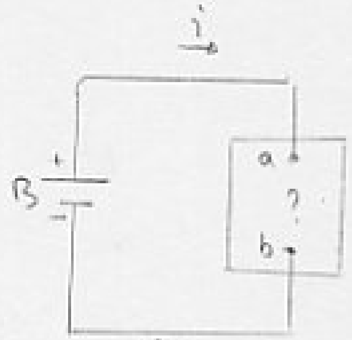
28-7 Energy and power in Electric Circuits

$dU = dq\ V = i\,dt\ V$

dq: the charge that moves between the terminals of a battery in a ciruit in time dt

$P = \dfrac{dU}{dt} = iV$ Power (rate of electrical energy transfer)

Unit: $1 V.A = (1\frac{J}{C})(1\frac{C}{s}) = 1\frac{J}{s} = 1W$



$\begin{cases} P = Ri^2 \\ P = \dfrac{V^2}{R} \end{cases}$ For resistors

$P = iv$ general

Ex. $R = 72\,\Omega$

a) $P = ?$ if $V = 120\,V$

b) $P = ?$ the wire is cut in half, and $V = 120\,V$ is applied across the length of each half.

a) $P = \frac{V^2}{R} = \frac{(120\,V)^2}{72\,\Omega} = 200\,W$

b) $P' = \frac{(120\,V)^2}{36\,\Omega} = 400$ $\qquad P' = 2 \times 400 = 800\,W$

Ex.

For a wire; $L = 2.35\,m$, $d = 1.63\,mm$ $\quad i = 1.24\,A$

$P = 48.5\,mW$

Material of the wire $= ?$

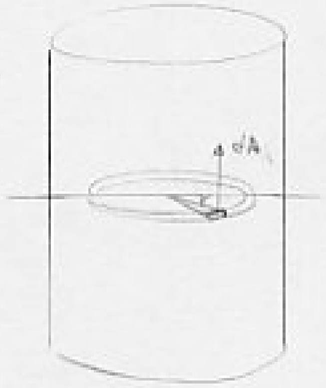
Sol.

$$P = i^2 R = \frac{i^2 \rho L}{A} = \frac{4 i^2 \rho L}{\pi d^2} \qquad \left(A = \pi \left(\frac{d}{2}\right)^2\right)$$

$$\rho = \frac{\pi P d^2}{4 i^2 L} = 2.80 \times 10^{-8}\,\Omega.m \longrightarrow Al$$

15P The current density across a cylindrical conductor of radius R;

a) $$J = J_0\left(1 - \frac{r}{R}\right)$$



$i = ?$

$$i = \int \bar{J} \cdot d\bar{A} = \int J_0\left(1-\frac{r}{R}\right)(r\,d\theta)\,dr$$

$$i = J_0 \int_0^{2\pi} d\theta \int_0^{R} \left(1 - \frac{r}{R}\right) r\,dr = 2\pi J_0 \left\{ \int_0^R r\,dr - \frac{1}{R}\int_0^R r^2\,dr \right\}$$

$$i = 2\pi J_0 \left\{ \frac{R^2}{2} - \frac{1}{R}\,\frac{R^3}{3} \right\} = 2\pi J_0 \frac{R^2}{6} = \frac{1}{3}\pi J_0 R^2$$

b) If $J = J_0 \frac{r}{R}$ $\quad i = ?$

$$i = \int \bar{J} \cdot d\bar{A} = \int \left(J_0 \frac{r}{R}\right)(r\,d\theta)\,dr = \frac{J_0}{R}\int_0^{2\pi} d\theta \int_0^R r^2\,dr$$

$$i = 2\pi \frac{J_0}{R}\,\frac{R^3}{3} = \frac{2}{3}\pi J_0 R^2$$

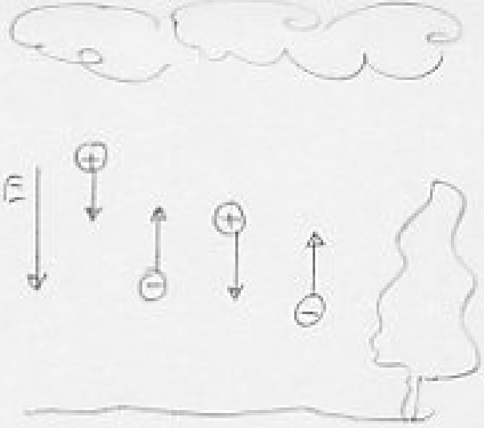
39P In the lower atmosphere of the Earth; there are negative and positive ions, created by radioactive elements in the soil and cosmic rays from space.

If $E = 120\ V/m \downarrow$ and $n_+ = 620 \frac{1}{cm^3}$ $\quad n_- = 550 \frac{1}{cm^3}$

$\sigma = 2.70 \times 10^{-14} \frac{1}{\Omega \cdot m}$ (measured conductivity) $\qquad$ E: Atmospheric electric field

$V_{d\,{+e \atop -e}} = ?$ (assume the same for + and −) $\quad J = ?$

$$V_d = a\tau = \frac{eE\tau}{m}$$



$$\rho = \frac{m}{e^2 n\tau} \qquad \sigma = \frac{e^2 n\tau}{m}$$

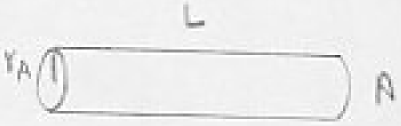
$$\rightarrow \frac{\tau}{m} = \frac{\sigma}{ne^2}$$

$$V_d = eE\frac{\sigma}{ne^2} = E\frac{\sigma}{ne} = 120\,\frac{2.70\times10^{-14}}{(550+620)\times10^{6}\times(1.6\times10^{-19})} = 1.7\times10^{-2}\ \mathrm{m/s}$$

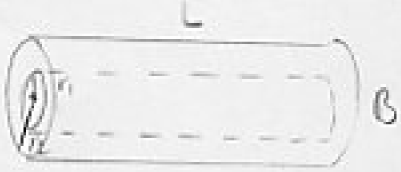
$$V_d = \frac{J}{ne} \qquad J = (ne)V_d = 3.24\times10^{-18}\ \mathrm{A/m^2}$$

30P

$r_A = 0.5\ \mathrm{mm}$ $\quad r_1 = 0.5\ \mathrm{mm}$ $\quad r_2 = 1.\ \mathrm{mm}$



$$\frac{R_A}{R_B} = ?$$

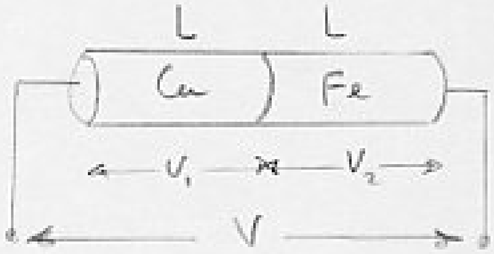


$$\frac{R_A}{R_B} = \frac{\rho\frac{L}{\pi r_A^2}}{\rho\frac{L}{\pi(r_2^2-r_1^2)}} = \frac{r_2^2-r_1^2}{r_A^2} = \frac{1^2-(0.5)^2}{(0.5)^2} = 3$$

Ex.

$V = 100\mathrm{V}$ $\quad L = 10\ \mathrm{m}$ $\quad d = 2\ \mathrm{mm}$

$\rho_{Cu} = 1.7\times10^{-8}\ \Omega.\mathrm{m}$ $\quad \rho_{Fe} = 10\times10^{-8}\ \Omega.\mathrm{m}$



a) $V_1 = ?$ $\quad V_2 = ?$

$$R_{Cu} = \rho_{Cu}\frac{L}{A} \qquad R_{Fe} = \rho_{Fe}\frac{L}{A}$$

$$V = V_1 + V_2 = i(R_{Cu} + R_{Fe})$$

$$\rightarrow i = \frac{V}{\frac{L}{A}(\rho_{Cu}+\rho_{Fe})}$$

$$V_1 = i R_{Cu} = \frac{VA}{L(\rho_{Cu}+\rho_{Fe})} \rho_{Cu} \frac{L}{A} = \frac{\rho_{Cu}}{\rho_{Cu}-\rho_{Fe}} V = \frac{1.7\times10^{-8}}{(1.7+10)\times10^{-8}} \times 100$$

$$= 14.5 \text{ V}$$

$$V_2 = V - V_1 = 85.5 \text{ V}$$

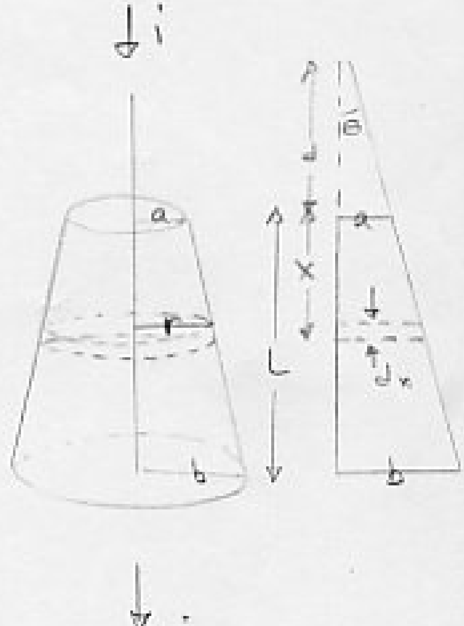
b) $J = ?$

$$J = \frac{i}{A} = \frac{V}{L(\rho_{Cu}+\rho_{Fe})} = \frac{100}{10\,(1.7+10)\times10^{-8}} \qquad J = 8.5\times10^{7} \text{ A/m}^2$$

c) $E_{Cu} = ?$ $E_{Fe} = ?$

$$E_{Cu} = \frac{V_1}{L} = \frac{14.5}{10} = 1.5 \text{ V/m}$$

$$E_{Fe} = \frac{V_2}{L} = \frac{85.5}{10} = 8.6 \text{ V/m}$$



4.2P

$$dR = \rho \frac{dx}{\pi r^2}$$

$$\tan\theta = \frac{a}{d} = \frac{r}{x+d} = \frac{b}{L+d} = \frac{b-a}{L}$$

$$\longrightarrow r = \frac{b-a}{L} x + a$$

$$dR = \frac{\rho}{\pi} \frac{dx}{\left[\frac{b-a}{L}x+a\right]^2} \qquad \left(\frac{b-a}{L}x+a\right) = u$$

$$du = \frac{b-a}{L} dx$$

$$R = \frac{\rho}{\pi} \frac{L}{b-a} \int_a^b \frac{du}{u^2} \qquad \int_0^L dx \longrightarrow \int_a^b$$

$$R = \frac{\rho}{\pi} \frac{L}{b-a} \left[-\frac{1}{u}\right]_a^b = \frac{\rho}{\pi} \frac{L}{b-a} \left(\frac{1}{a} - \frac{1}{b}\right) = \frac{\rho}{\pi} \frac{L}{ab}$$

If $a=b \rightarrow R = \rho \frac{L}{\pi a^2} = \rho \frac{L}{A}$

Ex.

$\rho_1 = 10\times10^{-8}\ \Omega.m \qquad \alpha_1 = 5\times10^{-3}$

$\rho_2 = 3.5\times10^{-5}\ \Omega.m \qquad \alpha_2 = -5\times10^{-4}$

$\frac{l_1}{l_2} = ?$ to have $R_{eq} =$ indep of (T)

Sol.

$R = \rho \frac{l}{S} \rightarrow R_{eq} = \rho_1 \frac{l_1}{S} + \rho_2 \frac{l_2}{S}$

$\rho = \rho_0 [1 + \alpha (T - T_0)] l$

$R_{eq} = \rho_{01} [1 + \alpha_1 (T - T_0)] l_1 + \rho_{02} [1 + \alpha_2 (T - T_0)] l_2$

$\frac{\partial R_{eq}}{\partial T} = 0 \qquad \rho_{01} \alpha_1 l_1 + \rho_{02} \alpha_2 l_2 = 0$

$\frac{l_1}{l_2} = - \frac{\rho_{02} \alpha_2}{\rho_{01} \alpha_1} = - \frac{3.5\times10^{-5} (-5\times10^{-4})}{10\times10^{-8} (5\times10^{-3})} = 35$

i.e. $R_{eq} = \rho_0 [1 + \alpha (T - T_0)] l \rightarrow \alpha = 0$