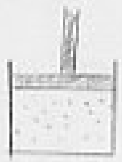
Chapter 27

27-1

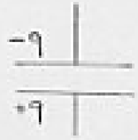
Storing the potential energy:

1 - Stretching a spring



2 - Compressing a gas

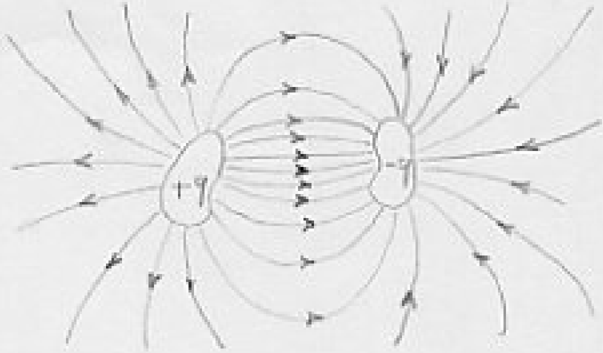
3 - lifting a mass



4 - Capacitor potential energy in an electric field

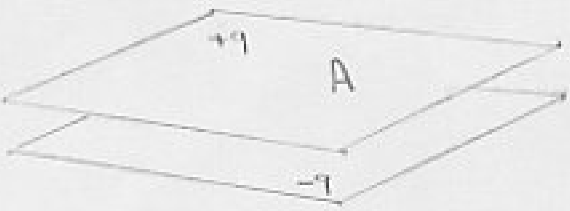
27-2

Capacitor: Two isolated conductors of arbitrary shape carrying equal but opposite charges.



Charge of Capacitor: $|q|$

Net charge of capacitor $= 0$

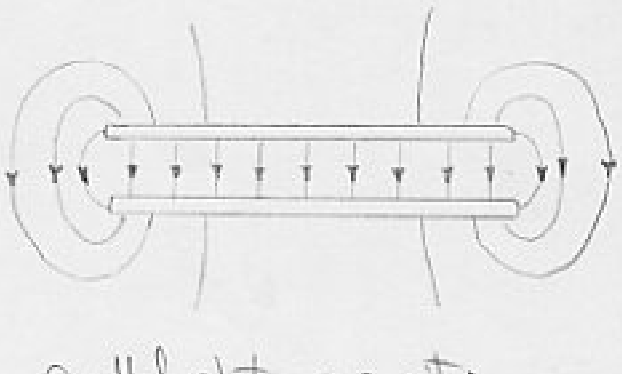


ΔV: Potential difference between the plates

$\Delta V \xrightarrow[\text{is represented by}]{\text{For historical reasons}} V$

$q \sim V$

$\longrightarrow q = CV$

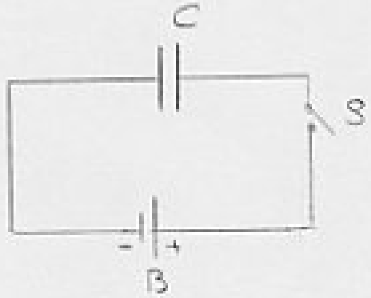


Parallel plate capacitor

$C = C$ (Geometry of the plates, insulators)   Capacitance

C : $\frac{\text{Coulomb}}{\text{Volt}} = 1F$ (farad)   SI unit

Charging a Capacitor:



27-3 Calculating the Capacitance:

We follow 3-steps:

1- Assume a charge q on the plates
2- Calculate the electric field E between the plates
3- Knowing E calculate V and later C.

Calculating the electric field E:

$$\epsilon_0 \oint E \cdot dA = q$$

Calculating the potential difference:

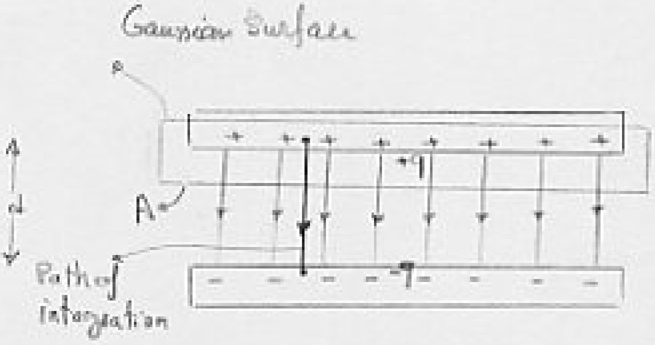
$$V_f - V_i = -\int_i^f E \cdot ds \qquad V = \int_+^- E \cdot ds \qquad (V_f - V_i = -V)$$

(absolute value)

A parallel plate Capacitor:

We neglect the fringing of E at the edges of the plates.

$$\oint E \cdot dA = \frac{q}{\epsilon_0}$$



$$\int_1 + \cdots \int_6 = \frac{q}{\epsilon_0}$$

$$EA = \frac{q}{\epsilon_0} \qquad E = \frac{q}{\epsilon_0 A}$$

$$V = \int_+^- E \cdot dl = E \int ds = Ed$$

$$V = \frac{qd}{\epsilon_0 A} \qquad q = CV \qquad q = C\frac{qd}{\epsilon_0 A} \qquad C = \epsilon_0 \frac{A}{d}$$

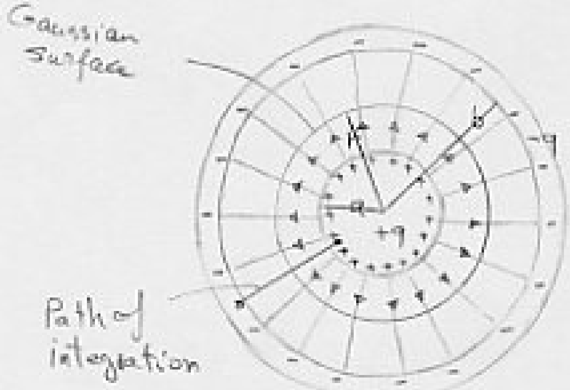
$\longrightarrow$ $C = C$(Geometrical factors)

A cylindrical Capacitor

Assume $L \gg b$ $\longrightarrow$ we neglect the fringing of E at the ends of cylinder

$$\oint E \cdot dA = \frac{q}{\epsilon_0}$$

$$(2\pi r L) E = \frac{q}{\epsilon_0} \qquad E = \frac{q}{2\pi\epsilon_0 L r}$$



$$V = \int_+^- E \cdot dl = \frac{q}{2\pi\epsilon_0 L}\int_a^b \frac{dr}{r}$$

$$= \frac{q}{2\pi\epsilon_0 L} \ln\left(\frac{b}{a}\right)$$

$$C = \frac{q}{V} \qquad C = 2\pi\epsilon_0 \frac{L}{\ln\left(\frac{b}{a}\right)}$$

A spherical Capacitor

$$\oint E \cdot dA = \frac{q}{\epsilon_0} \qquad (4\pi r^2) E = \frac{q}{\epsilon_0}$$

$$E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2}$$

$$V = \int_{+}^{-} E \cdot dl = \frac{q}{4\pi\epsilon_0} \int_a^b \frac{dr}{r^2} = \frac{q}{4\pi\epsilon_0} \left(\frac{1}{a} - \frac{1}{b} \right)$$

$$V = \frac{q}{4\pi\epsilon_0} \frac{b-a}{ab} \qquad C = 4\pi\epsilon_0 \frac{ab}{b-a}$$

An isolated sphere:

We can assign a capacitance to a <u>single isolated</u> spherical conductor of radius R by assuming that the <u>missing</u> plate is a conducting sphere of infinite radius.

$$C = 4\pi\epsilon_0 \frac{a}{1 - \frac{a}{b}} \quad \begin{cases} b \to \infty \\ a = R \end{cases} \longrightarrow C = 4\pi\epsilon_0 R$$

<u>Ex.</u>

Parallel plate capacitor; $d = 1.0$ mm $\quad C = 1.0F \quad A = ?$

<u>Sol.</u>

$$A = \frac{Cd}{\epsilon_0} = \frac{(1.0F)(1.0 \times 10^{-3} m)}{(8.85 \times 10^{-12} F/m)} = 1.1 \times 10^{8} \ m^2$$

<u>Ex</u>

Coaxial cable, $\quad a = 0.15$ mm $\quad b = 2.1$ mm $\quad \frac{C}{L} = ?$

<u>Sol.</u>

$$\frac{C}{L} = \frac{2\pi\epsilon_0}{\ln(b/a)} = \frac{(2\pi)(8.85 \times 10^{-12})}{\ln(2.1/0.15)} = 21 \times 10^{-12} F/m = 21 \ pF/m$$

Ex.

$C = 55\ fF \qquad V = 5.3\,V \qquad n = ?$ number of electrons

Sol.

$q = CV \qquad n = \frac{q}{e} = \frac{CV}{e} = \frac{(55\times10^{-15})(5.3)}{1.6\times10^{-19}} = 1.8\times10^{6}$ electrons

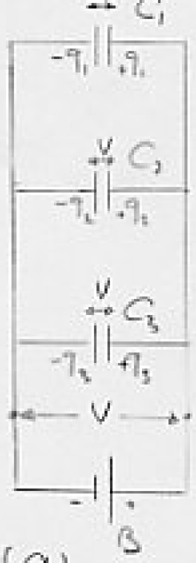
Ex.

$C_{Earth} = ?$ as an isolating conducting sphere of radius 6370 km

Sol.

$C = 4\pi\epsilon_0 R = (4\pi)(8.85\times10^{-12})(6.37\times10^{6}) = 7.1\times10^{-4}\,F = 710\,\mu F$

27-4 Capacitors in parallel and in series:



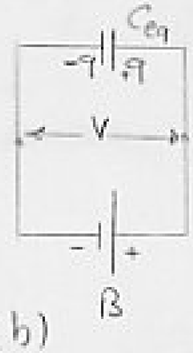
(a)

Capacitors in parallel:

(Fig. a) $\underset{\text{equivalent}}{\approx}$ (Fig. b) (i.e. $q = q_1 + q_2 + q_3$)

$q_1 = C_1 V \quad , \quad q_2 = C_2 V, \quad q_3 = C_3 V$

$q = q_1 + q_2 + q_3 = (C_1 + C_2 + C_3)V$

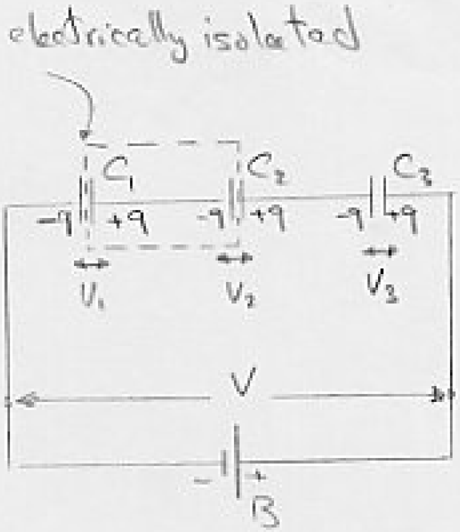


(b)

$C_{eq} = \frac{q}{V} = C_1 + C_2 + C_3$

$C_{eq} = \sum_{j=1}^{n} C_j$

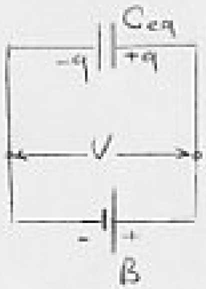
Capacitors in Series:



$$V_1 = \frac{q}{C_1} \qquad V_2 = \frac{q}{C_2} \qquad V_3 = \frac{q}{C_3}$$

$$V = V_1 + V_2 + V_3 = q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)$$

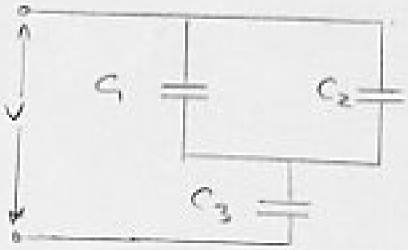
$$C_{eq} = \frac{q}{V} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}}$$



$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

$$\frac{1}{C_{eq}} = \sum_{j=1}^{n} \frac{1}{C_j}$$

Ex. $C_1 = 12\,\mu F$, $C_2 = 5.30\,\mu F$ $C_3 = 4.5\,\mu F$

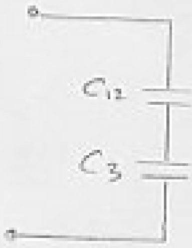


a) $C_{eq} = ?$ b) $V = 12.5$ $q_1 = ?$

Sol.

$C_{12} = C_1 + C_2 = 17.3\,\mu F$

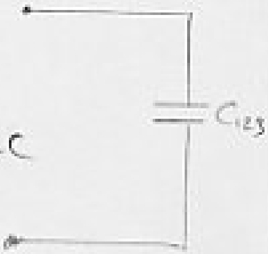
$$\frac{1}{C_{123}} = \frac{1}{C_{12}} + \frac{1}{C_3} = \frac{1}{17.3} + \frac{1}{4.5} = 0.280\,\mu F^{-1}$$



$C_{123} = 3.57\,\mu F$

b) $q_{123} = C_{123} V = (3.57\,\mu F)(12.5\,V) = 44.6\,\mu C$

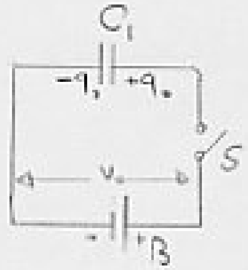
$q_{123} = q_3 = q_{12}$ $\qquad V_{12} = \frac{q_{12}}{C_{12}} = \frac{44.6\,\mu F}{17.3\,\mu C} = 2.58\,V$



$V_{12} = V_1 = V_2$ $\qquad q_1 = C_1 V_1 = (12\,\mu F)(2.58\,V) = 31\,\mu C$

Ex.

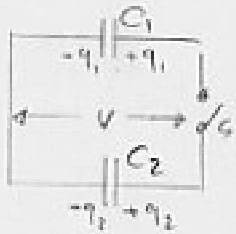
A 3.55 μF capacitor C_1 is charged to a potential difference $V_0 = 6.30\,V$. The battery is then removed and the capacitor is connected to an uncharged 8.95 μF capacitor C_2. What is the common potential difference?



Sol.

$$q_0 = q_1 + q_2 \rightarrow C_1V_0 = C_1V_1 + C_2V_2 \qquad (V_1 = V_2 = V)$$

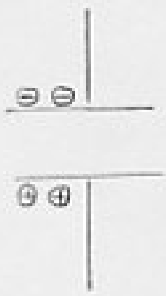
$$V = V_0 \frac{C_1}{C_1 + C_2} = \frac{(6.3\,V)(3.55\,\mu F)}{3.55\,\mu F + 8.95\,\mu F} = 1.79\,V$$



27-5 Storing Energy in an Electric Field:

Work must be done by an external agent to charge a capacitor.

a) Suppose by some way we remove electrons from one plate and transfer them one at time to the other plate.

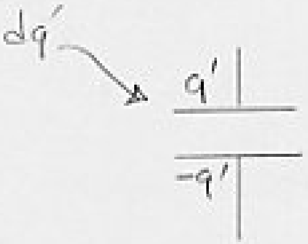


(a)

The E, built up in this way, tends to oppose further transfer.

Then, we have to do increasingly larger amount of work to transfer additional electrons.

Suppose at a given instant the charge of a plate is q'.



$\longrightarrow V' = \frac{q'}{C}$

Now try to transfer extra dq' from one plate to that plate.

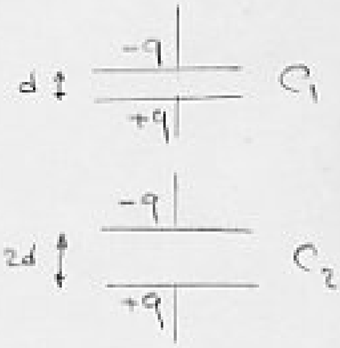
The required work is:

$$dW = V' dq' = \frac{q'}{C} dq'$$

$$W = \int dW = \frac{1}{C}\int_0^q q' dq' = \frac{q^2}{2C}$$

This work is stored as potential energy U in the capacitor:

$$U = \frac{q^2}{2C} \quad \longrightarrow U = \frac{1}{2} C V^2$$



Ex.

C_1, C_2 two parallel plate capacitors with the same A, but C_2 has twice seperation of C_1.

$C = \epsilon_0 \frac{A}{d} \longrightarrow C_1 = 2C_2$

If they have the same charge q; $q = \epsilon_0 E A \longrightarrow E_1 = E_2$

$U = \frac{q^2}{2C} \longrightarrow U_2 = 2U_1$ (Volume of C_2 = 2 Volume of C_1)
(vol. between plates

$\longrightarrow$ The potential energy of a charged capacitor may be viewed as stored in the electric field between its plates.

Energy Density:

$$u = \frac{U}{\text{Volume}} \qquad u = \frac{U}{Ad} = \frac{CV^2}{2Ad}$$

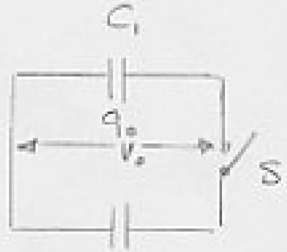
in parallel-plate capacitor.
(neglecting the fringing)
u is uniform

$$C = \frac{\epsilon_0 A}{d} \rightarrow u = \frac{1}{2}\epsilon_0\left(\frac{V}{d}\right)^2$$

But $\frac{V}{d} = E \rightarrow u = \frac{1}{2}\epsilon_0 E^2$ (it hold also in general)

<u>Ex.</u> $C_1 = 3.55\,\mu F \quad C_2 = 8.95\,\mu F$
$V_0 = 6.30$

$U_i = ? \qquad U_f = ?$



(a) $q=0$

(b) q_2

$q_0 = q_1 + q_2$

<u>Sol.</u>

$$U_i = \frac{1}{2}C_1V_0^2 + 0 = \frac{1}{2}(3.55\times10^{-6}F)(6.30)^2$$
$$= 70.4\,\mu J$$

We found before $V = 1.79\,V$

$$U_f = \frac{1}{2}C_1V^2 + \frac{1}{2}C_2V^2 = \frac{1}{2}(C_1+C_2)V^2$$
$$= \frac{1}{2}(3.55\times10^{-6} + 8.95\times10^{-6})(1.79) = 20.0\,\mu J$$

$\longrightarrow U_f < U_i \qquad 72\%$

This is not violation of Energy Cons..

The missing energy appears as radiation in the wires.

Ex.

$R = 6.85\,cm \quad q = 1.25\,nC$

Isolated Conducting Sphere

a) $U = ?$ energy potential stored in the electric field

$$U = \frac{q^2}{2C} = \frac{q^2}{2(4\pi\epsilon_0 R)} = \frac{(1.25\times10^{-9}\,C)^2}{(8\pi)(8.85\times10^{-12}\,F/m)(0.0685\,m)}$$

$= 1.03\times10^{-7}\,J = 103\,nJ$

b) $u = ?$ at the surface.

$u = \frac{1}{2}\epsilon_0 E^2 \qquad$ Since $E = \frac{1}{4\pi\epsilon_0}\frac{q}{R^2} \qquad u = \frac{1}{2}\epsilon_0 E^2 = \frac{q^2}{32\pi^2\epsilon_0 R^4}$

$$u = \frac{(1.25\times10^{-9}\,C)^2}{(32\pi^2)(8.85\times10^{-12}\,C^2/N.m^2)(0.0685\,m)^4} = 2.54\times10^{-5}\,J/m^3 = 25.4\,\mu J/m^3$$

c) What is the radius R_0 of an imaginary spherical surface such that one-half of the stored potential energy lies within it?

$$\int_R^{R_0} dU = \frac{1}{2}\int_R^{\infty} dU$$

$$\begin{cases} dU = (u)(4\pi r^2)(dr) \\ u = \frac{1}{2}\epsilon_0 E^2 \quad \text{and } E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2} \end{cases} \longrightarrow dU = \frac{q^2}{8\pi\epsilon_0}\frac{dr}{r^2}$$

$$\rightarrow \int_R^{R_0} \frac{dr}{r^2} = \frac{1}{2}\int_R^{\infty}\frac{dr}{r^2} \quad \rightarrow \quad \frac{1}{R} - \frac{1}{R_0} = \frac{1}{2R}$$

$R_0 = 2R = (2)(6.85\,cm) = 13.7\,cm$

27-6 Capacitor with a Dielectric

1837, Michael Faraday (to whom the concept of capacitance is due, and for whom SI unit of capacitance is named)

found that:

The capacitance increased by a factor k (dielectric const.) of the introduced material.

C_1

C_2

$C_2 = k C_1$

We have found before that C is always written in the form of

$$C = \epsilon_0 \mathcal{L} \qquad \mathcal{L}: \text{has dimensions of length}$$

For example $\mathcal{L} = \frac{A}{d}$ for parallel plate capacitor

$$\mathcal{L} = \frac{4\pi ab}{b-a} \text{ for spherical capacitor}$$

Faraday's discovery:

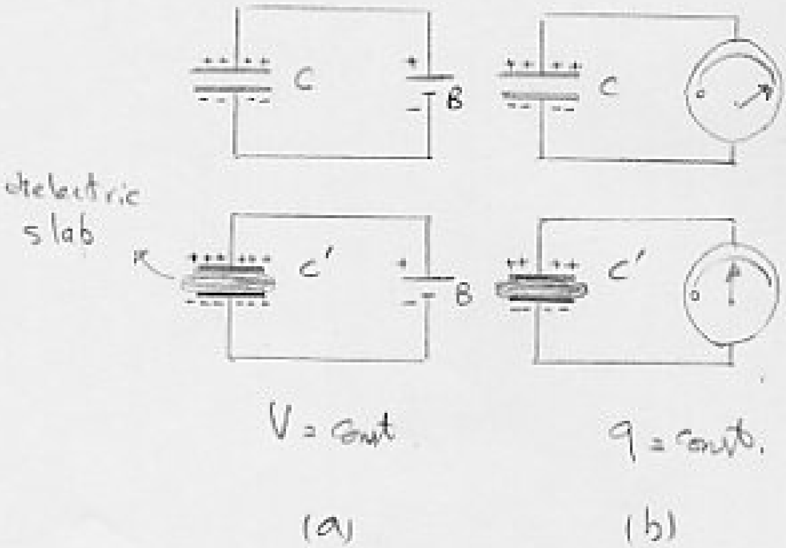
$$C = k\epsilon_0 \mathcal{L} = k\, C_{\text{vacuum}}$$

for capacitors with dielectric

$\mathcal{L} = \mathcal{L}(\text{Geometry})$

Faraday's Experiment:

a) $V = \text{const.}$



$C' = KC$

$V = \frac{q}{C} \quad V' = \frac{q'}{C'}$

$V = V' \rightarrow \frac{q}{C} = \frac{q'}{C'}$

$\frac{q}{C} = \frac{q'}{kC} \rightarrow q' = kq$

b) $q = \text{const.}$

$q = CV \quad q' = C'V'$

$q' = q \rightarrow CV = C'V' \quad CV = kCV' \quad V' = \frac{V}{K}$

Both experiments are consistant with $C' = kC$

Thus:

In a region completely filled by a dielectric, all electrostatic equations containing the permitivity const. ϵ_0 are to be modified by replacing that const. to $k\epsilon_0$

$$\epsilon_0 \longrightarrow k\epsilon_0$$

For example: $E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2} \longrightarrow E = \frac{1}{4\pi k\epsilon_0}\frac{q}{r^2}$ (Point charge)

Ex. A parallel-plate capacitor whose capacitance C is 13.5PF has a potential difference V = 12.5V between its plates. The battery is now disconnected and a porcelain slab (K = 6.50) is slipped between the plates. What is the potential energy of the device, both before and after the slab is introduced.

Sol.

$$U_i = \frac{1}{2}CV^2 = \frac{1}{2}(13.5\times10^{-12}\,F)(12.5V)^2 = 1.055\times10^{-9}\,J \simeq 1055\,PJ$$

$$U_i = \frac{q^2}{2C} \longrightarrow U_f = \frac{q^2}{2C'} = \frac{q^2}{2kC} = \frac{U_i}{k} = \frac{1055\,PJ}{6.50} = 162\,PJ$$

U_f is smaller by a factor of $\frac{1}{K}$

Where has gone the missing energy?

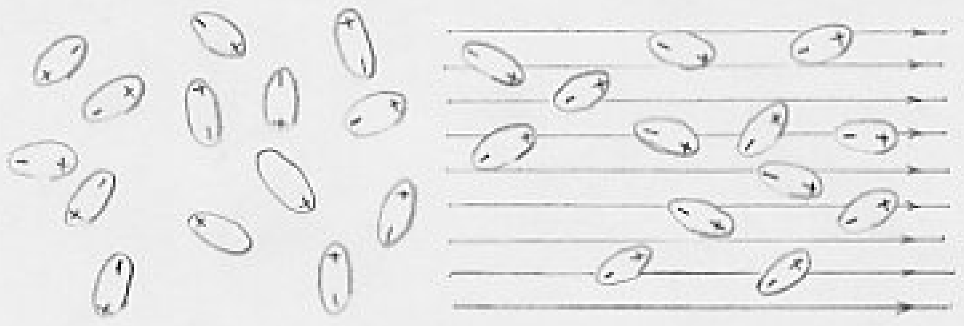
This energy, in principle, would be apparent to the person who introduced the slab.

$$W = U_i - U_f = (1055 - 162) = 893\,PJ$$

If the slab were allowed to slide between the plates with no restraint and if there were no friction, the slab would oscillate back and forth between the plates, with a const. mechanical energy 893PJ.

27-7 Dielectrics: An Atomic View:

1- Polar dielectrics: The molecules of some dielectrics, like water have permanent electric dipole moments.



(a)

$E = 0$

The polar molecules have random direction

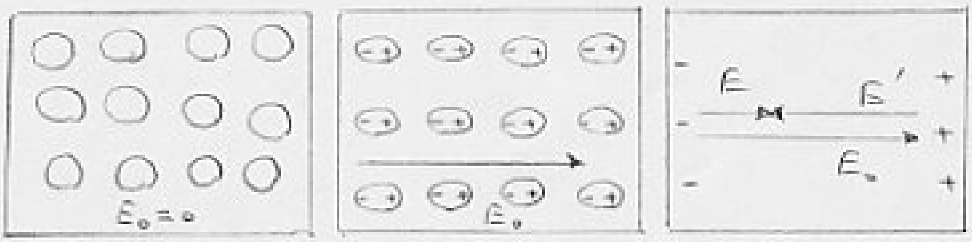
(b)

$E \neq 0$

The polar molecules tend to line up with E. Alignment increases with increasing E or decreasing $T°$ (thermal agitation decreasing)

2- Nonpolar dielectrics:

They aquire dipole moments in an external field.



(a)

neutral molecales

(b)

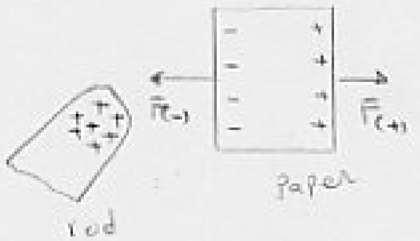
The presence of E_0 stretch out the molecules and give the polarity

(c)

E' is produced by the molecales.

E: resultant electric field

This is the origin of attraction of paper by the charged rod.

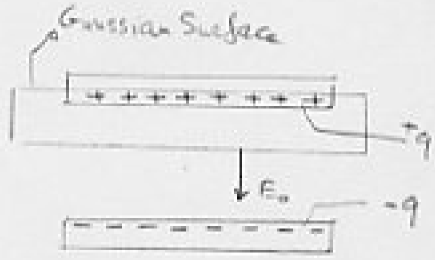


27-8 Dielectrics and Gauss' law

(a) $\epsilon_0 \oint E \cdot dA = \epsilon_0 E_0 A = q$

$$E_0 = \frac{q}{\epsilon_0 A}$$

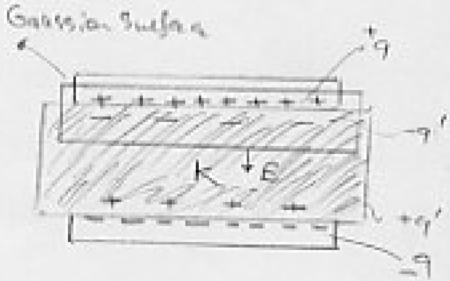
(a)



(b) $\epsilon_0 \oint E \cdot dA = \epsilon_0 E A = q - q'$

$$E = \frac{q - q'}{\epsilon_0 A} \qquad (1)$$

(b)



q': induced (bound) surface charge
q: free charges on the plates

Since $\epsilon_0 \longrightarrow k\epsilon_0 \quad \Rightarrow E = \dfrac{q}{k\epsilon_0 A} = \dfrac{E_0}{k} \qquad (2)$

$(1),(2) \longrightarrow \quad \dfrac{q-q'}{\epsilon_0 A} = \dfrac{q}{k\epsilon_0 A} \qquad q - q' = \dfrac{q}{k} \quad \Rightarrow \begin{cases} q' < q \\ q' = 0 \text{ if } k=1 \end{cases}$

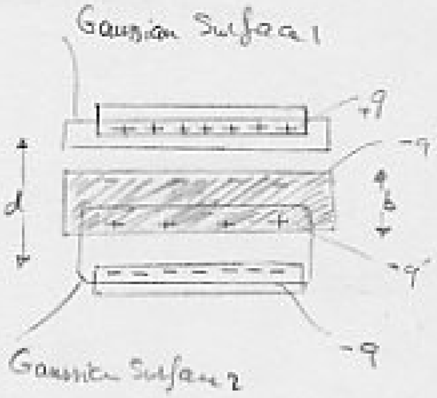
$\epsilon_0 \oint k E \cdot dA = q$ (Also valid generally)

$k = k(r)$ in general

q: free charge only

Ex.

The capacitor is charged and the battery is then disconnected and a dielectric slab is placed between the plates.



$A = 115\,cm^2 \qquad d = 1.24\,cm$

$b = 0.780\,cm \qquad k = 2.61 \qquad V_0 = 85.5\,V$

a) $C_0 = ?$

$$C_0 = \frac{\epsilon_0 A}{d} = \frac{(8.85\times10^{-12}\,F/m)(115\times10^{-4}\,m^2)}{1.24\times10^{-2}\,m} = 8.21\times10^{-12}\,F = 8.21\,pF$$

b) $q = ?$ free charge

$$q = C_0 V_0 = (8.21\times10^{-12}\,F)(85.5\,V) = 7.02\times10^{-10}\,C = 702\,pC$$

c) $E_0 = ?$ in the gaps between the plates and the dielectric slab

$$\epsilon_0 \oint_{\text{Surface 1}} k\vec{E}\cdot d\vec{A} = q \qquad \epsilon_0 (1) E_0 A = q$$

$$E_0 = \frac{q}{\epsilon_0 A} = \frac{7.02\times10^{-10}\,C}{(8.85\times10^{-12}\,F/m)(115\times10^{-4}\,m^2)} = 6900\,V/m$$

E_0 remains unchanged in the gap as the dielectric introduced.

d) $E' = ?$ in the dielectric slab.

$$\epsilon_0 \oint_{\text{Surface 2}} k\vec{E}\cdot d\vec{A} = q \qquad -k\epsilon_0 E' = q$$

$$E' = \frac{q}{K\epsilon_0 A} = \frac{E_0}{K} = \frac{6900\ V/m}{2.61} = 2640\ V/m$$

e) $V' = ?$

$$V' = +\int_{+}^{-} \bar{E}\cdot ds = \int_{+}^{-} E\,ds = E_0(d-b) + E'b$$

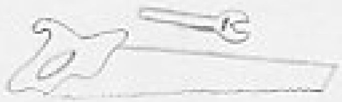
$$= (6900\ V/m)(0.0124m - 0.00780m) + (2640\ V/m)(0.00780m) = 52.3V$$

f) $C' = ?$

$$C' = \frac{q}{V'} = \frac{7.02\times10^{-10}C}{52.3V} = 1.34\times10^{-11}F = 13.4pF$$

2E

The two metal objects in Fig. have net charges of +70 pC and -70 pC and this results a 20 V potential difference between them.



a) $C = ?$

b) If the charges are changed to +200 pC and -200 pC, what does the capacitance become?

c) What does the potential difference become?

Sol.

a) $q = CV \qquad C = \frac{q}{V} = \frac{70 \times 10^{-12}}{20} = 3.5 \times 10^{-12}$ F

b) The same

c) $V = \frac{q}{C} = \frac{200 \times 10^{-12}}{3.5 \times 10^{-12}} = 57$ V

4E Show that the unit of ϵ_0, F/m and $C^2/N.m^2$ are equivalent.

Sol.

$$1F = 1\frac{C}{V} \rightarrow \frac{F}{m} = \frac{C}{V.m}$$

But $1\frac{N}{C} = 1\frac{V}{m} \rightarrow V = \frac{N.m}{C}$

$$\rightarrow \frac{F}{m} = \frac{C^2}{N.m^2}$$

10E Two sheets of Al foil have a separation of 1.0 mm and a capacitance of 10 pF, and are charged to 12 V. a) calculate $A=?$

b) d is now changed to 0.1 mm with $q=$ const. $C_{new}=?$

c) $V_{new}=?$

Sol.

a) $C=\epsilon_0 \frac{A}{d}$ $\quad A=\frac{Cd}{\epsilon_0}=\frac{(10\times10^{-12})(1\times10^{-3})}{8.85\times10^{-12}}=1.13\times10^{-3}\ m^2$

b) $C'=\epsilon_0 \frac{A}{d'}=8.85\times10^{-12}\ \frac{1.13\times10^{-3}}{0.1\times10^{-3}}=1\times10^{-10}\ F=100\ pF$

c) $q=CV=10\times10^{-12}\times12=1.2\times10^{-10}\ C$

$q=C'V'$ $\quad V'=\frac{q}{C'}=\frac{1.2\times10^{-10}}{1\times10^{-10}}=1.2\ V$

12P For a cylindrical capacitor we have, $C=2\pi\epsilon_0\frac{L}{\ln\frac{b}{a}}$. show that the capacitance approaches that of a parallel-plate capacitor when the spacing between the two cylinders is small.

Sol.

$\ln(1+x)=x-\frac{1}{2}x^2+\frac{1}{3}x^3-\cdots \qquad |x|<1$

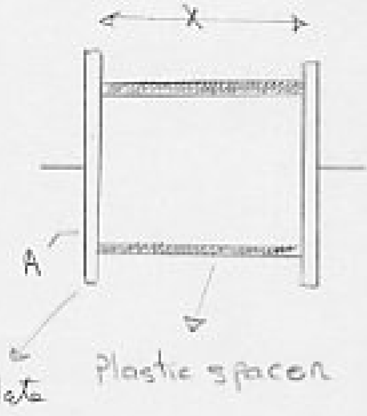
$\rightarrow \ln(1+x)\approx x \qquad |x|<<1$

$C=2\pi\epsilon_0\frac{L}{\ln\frac{b}{a}}=2\pi\epsilon_0\frac{L}{\ln(\frac{a+d}{a})}=2\pi\epsilon_0\frac{L}{\ln(1+\frac{d}{a})}$

$C=2\pi\epsilon_0\frac{L}{\frac{d}{a}} \quad (\frac{d}{a}<<1) \qquad C=\epsilon_0\frac{(2\pi a)L}{d}=\epsilon_0\frac{A}{d}$

14P.

Show that $\frac{dc}{dT} = C\left(\frac{1}{A}\frac{dA}{dT} - \frac{1}{x}\frac{dx}{dT}\right)$



(T: temp.)

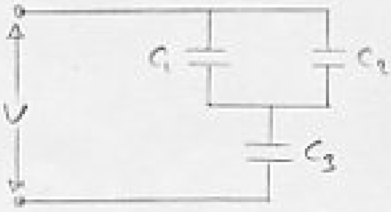
For parallel plate capacitor;

$$C = \epsilon_0 \frac{A}{x} \rightarrow \frac{dc}{dT} = \epsilon_0\left(\frac{1}{x}\frac{dA}{dT} - \frac{A}{x^2}\frac{dx}{dT}\right)$$

$$\frac{dc}{dT} = \epsilon_0 \frac{A}{x}\left(\frac{1}{A}\frac{dA}{dT} - \frac{1}{x}\frac{dx}{dT}\right) = C\left(\frac{1}{A}\frac{dA}{dT} - \frac{1}{x}\frac{dx}{dT}\right)$$

$$\frac{1}{A}\frac{dA}{dT} - \frac{1}{x}\frac{dx}{dT} = 0$$ Cond. fo $C = $ cont. (with changing T)

16E $C_1 = 10.0\,\mu F$ $C_2 = 5.0\,\mu F$ $C_3 = 4.0$

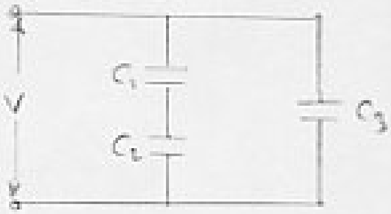


$C_{equ} = ?$

$C_{12} = C_1 + C_2 = 15\,\mu F$ $\frac{1}{C_{eq}} = \frac{1}{C_{12}} + \frac{1}{C_3}$

$\rightarrow C_{eq} = 3.16\,\mu F$

17E. $C_1 = 10.0\,\mu F$ $C_2 = 5.0\,\mu F$ $C_3 = 4.0$



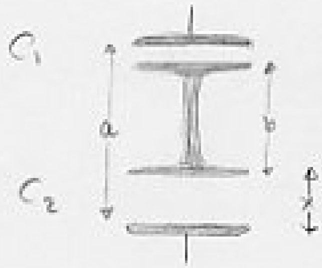
$C_{equ} = ?$

$\frac{1}{C_{12}} = \frac{1}{C_1} + \frac{1}{C_2}$ $C_{12} = 3.33\,\mu F$

$C_{equ} = C_{12} + C_3 = 7.33\,\mu F$

26P

Show that $C = \frac{\epsilon_0 A}{a-b}$ indep. of position of the center section.

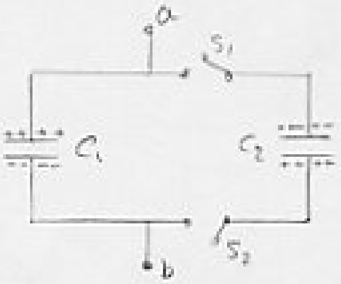


$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} \qquad C_1 = \epsilon_0 \frac{A}{a-b-x} \qquad C_2 = \epsilon_0 \frac{A}{x}$$

$$\frac{1}{C} = \frac{a-b-x}{\epsilon_0 A} + \frac{x}{\epsilon_0 A} \qquad \frac{1}{C} = \frac{1}{\epsilon_0 A}(a-b)$$

$$C = \epsilon_0 \frac{A}{a-b}$$

28P



$C_1 = 1.0\,\mu F$ and $C_2 = 3.0\,\mu F$ are each charged to a potential $V = 100\,V$ but with opposite polarity. Switches S_1 and S_2 are now closed.

a) $V_a - V_b = ?$ b) $q_1 = ?$ c) $q_2 = ?$

Sol.

$$q_1 = C_1 V_1 \qquad q_1 = 1 \times 10^{-6} \times 100 = 10^{-4}\ C$$

$$q_2 = C_2 V_2 \qquad q_2 = 3 \times 10^{-6} \times 100 = 3 \times 10^{-4}\ C$$

$$q = q_2 - q_1 = 2 \times 10^{-4}\ C$$

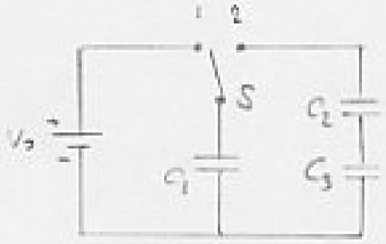
$$\begin{cases} V' = \frac{q_1'}{C_1} \\ V' = \frac{q_2'}{C_2} \end{cases} \rightarrow \frac{q_1'}{C_1} = \frac{q_2'}{C_2} \rightarrow q_1' = \frac{q_2'}{3}$$

$$\begin{cases} q_2' = 3q_1' \\ q_1' + q_2' = 2 \times 10^{-4} \end{cases} \longrightarrow \begin{matrix} q_1' = 0.5 \times 10^{-4}\ C \\ q_2' = 1.5 \times 10^{-4}\ C \end{matrix}$$

29P

C_1 is first charged. Then the S is connected to terminal 2.

What is final $q_1' = ?$, $q_2' = ?$, $q_3' = ?$.



Sol.

$$q_1 = C_1 V_0$$

$$\frac{1}{C_{23}} = \frac{1}{C_2} + \frac{1}{C_3} \rightarrow C_{23} = \frac{C_2 C_3}{C_2 + C_3}$$

$$\begin{cases} q_{23} = C_{23} V' \\ q_1' = C_1 V' \end{cases} \rightarrow \begin{cases} C_1 q_{23} = C_{23} q_1' \\ q_{23} + q_1' = q_1 = C_1 V_0 \end{cases} \qquad C_1 q_{23} = C_{23}(C_1 V_0 - q_{23})$$

$$q_{23} + q_1' = q_1$$

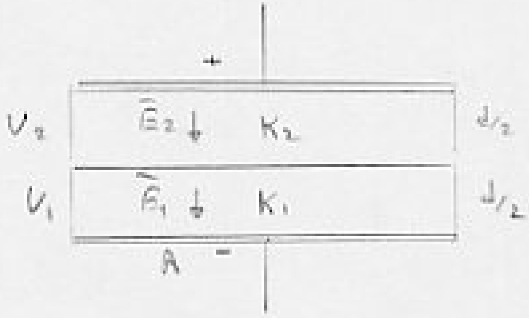
$$q_{23} = \frac{C_{23} C_1 V_0}{C_1 + C_{23}} \rightarrow q_{23} = \frac{\frac{C_1 C_2 C_3}{C_2 + C_3} V_0}{C_1 + \frac{C_2 C_3}{C_2 + C_3}} \rightarrow q_{23} = q_2 = q_3$$

$$q_1' = C_1 V_0 - q_{23} = C_1 V_0 - \frac{\frac{C_1 C_2 C_3}{C_2 + C_3} V_0}{C_1 + \frac{C_2 C_3}{C_2 + C_3}}$$

6.4 P

a) Show that:

$$C = \frac{2\epsilon_0 A}{d}\left(\frac{k_1 k_2}{k_1+k_2}\right)$$



b) Can you justify this arrangement as being two capacitors in series?

Sol.

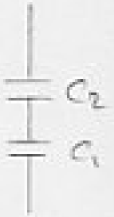
a) $E_1 = \frac{\sigma}{\epsilon_0 k_1} = \frac{q}{\epsilon_0 A k_1}$ $\quad E_2 = \frac{\sigma}{\epsilon_0 k_2} = \frac{q}{\epsilon_0 A k_2}$

$V = \int E \cdot dl$ $\quad V_1 = E_1 \frac{d}{2}$ $\quad V_2 = E_2 \frac{d}{2}$

$$V = V_1 + V_2 = (E_1+E_2)\frac{d}{2} = \frac{qd}{2\epsilon_0 A}\left(\frac{1}{k_1}+\frac{1}{k_2}\right) = \frac{qd}{2\epsilon_0 A}\left(\frac{k_1+k_2}{k_1 k_2}\right)$$

$$C = \frac{q}{V} = \frac{2\epsilon_0 A}{d}\left(\frac{k_1 k_2}{k_1+k_2}\right)$$

b) Yes, since if we calculate;



$C_1 = \frac{q}{V_1} = \frac{2\epsilon_0 A k_1}{d}$ $\quad C_2 = \frac{2\epsilon_0 A k_2}{d}$

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} \longrightarrow C = \frac{2\epsilon_0 A}{d}\left(\frac{k_1 k_2}{k_1+k_2}\right)$$

If $k_1 = k_2 = k$ $\quad C = \epsilon_0 k \frac{A}{d}$

If $k_1 = k_2 = 1 \longrightarrow C = \epsilon_0 \frac{A}{d}$

65P $C_{eq} = ?$

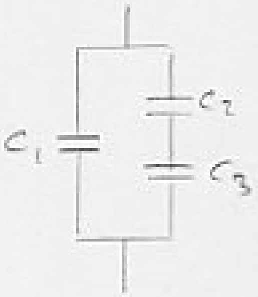
Sol. $E = \frac{\sigma}{\epsilon_0 k} = \frac{q}{\epsilon_0 A k}$ $\quad V = \int E \cdot dl$

$V = \frac{qd}{\epsilon_0 A k}$ $\quad C = \frac{q}{V} = \frac{\epsilon_0 A k}{d}$

$C_1 = \frac{\epsilon_0 k_1 A/2}{2d}$ $\quad C_2 = \frac{\epsilon_0 k_2 A/2}{d}$ $\quad C_3 = \frac{\epsilon_0 k_3 A/2}{d}$

$C_{23} = \frac{C_2 C_3}{C_2 + C_3}$ $\quad C_{23} = \frac{\epsilon_0 A}{2d}\left(\frac{k_2 k_3}{k_2 + k_3}\right)$

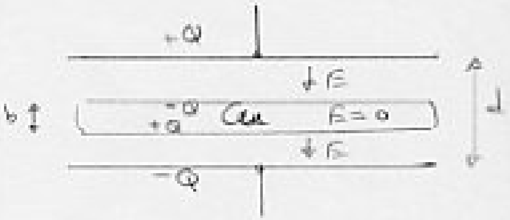
$C_{eq} = C_{23} + C_1$ $\quad C = \frac{\epsilon_0 A}{2d}\left(\frac{k_1}{2} + \frac{k_2 k_3}{k_2 + k_3}\right)$



61P

a) $C = ?$ after the copper slab is introduced.

b) If a charge q is maintained on the plate, find $\frac{U_0}{U} = ?$ $\quad$ c) Work done on slab = ?



Sol. a) $C = \frac{\epsilon_0 A}{D}$ $\quad C_1 = C_2 = \frac{\epsilon_0 A}{\frac{d-b}{2}}$

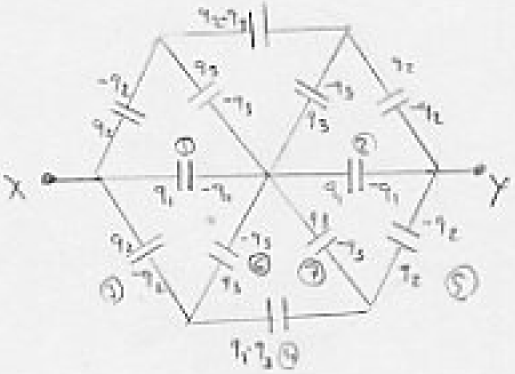
$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$ $\quad C_{eq} = \frac{\epsilon_0 A}{d-b}$

b) $\begin{cases} U_0 = \frac{1}{2}\frac{q^2}{C} & C = \frac{\epsilon_0 A}{d} \rightarrow U_0 = \frac{d q^2}{2\epsilon_0 A} \\ U = \frac{1}{2}\frac{q^2}{C_{eq}} = \frac{(d-b) q^2}{2\epsilon_0 A} \end{cases}$

$\frac{U_0}{U} = \frac{d}{d-b}$

c) $U - U_0 = -\frac{b q^2}{2\epsilon_0 A}$ $\quad$ Since $U < U_0 \longrightarrow$ The slab is sucked (system tends to reach Min. of energy)

$C_{xy} = ?$ $\quad C_1 = C_2 = \cdots C_{12} = 10\,\mu F$



$$V_{xy} = \frac{q_1}{C_1} + \frac{q_1}{C_2} = \frac{2q_1}{10} = \frac{q_1}{5}$$

$$V_{xy} = \frac{q_2}{C_3} + \frac{q_2 - q_3}{C_4} + \frac{q_2}{C_5} = \frac{3q_2}{10} - \frac{q_3}{10}$$

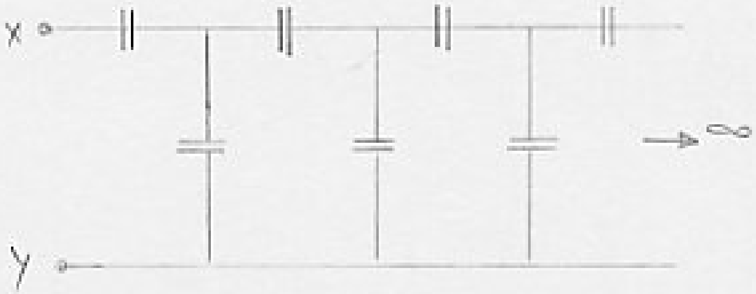
$$V_{xy} = \frac{q_2}{C_3} + \frac{q_3}{C_6} + \frac{q_1}{C_2} = \frac{q_2}{10} + \frac{q_3}{10} + \frac{q_1}{10}$$

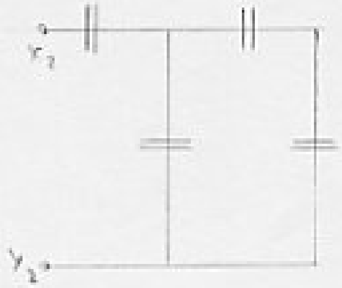
$$\begin{cases} q_1 = 5V_{xy} \\ 3q_2 - q_3 = 10V_{xy} \\ q_1 + q_2 + q_3 = 10V_{xy} \end{cases} \qquad \begin{cases} q_1 = 5V_{xy} \\ q_2 = \frac{5}{2}V_{xy} \\ q_3 = \frac{5}{2}V_{xy} \end{cases}$$

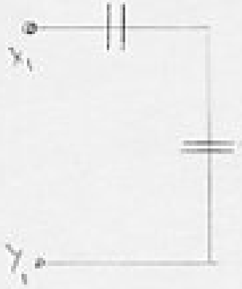
$$Q = q_1 + 2q_2 = 10V_{xy} \qquad C_{xy} = \frac{Q}{V_{xy}} = \frac{10V_{xy}}{V_{xy}} = 10\,\mu F$$

Ex.

$C_{xy} = ?$





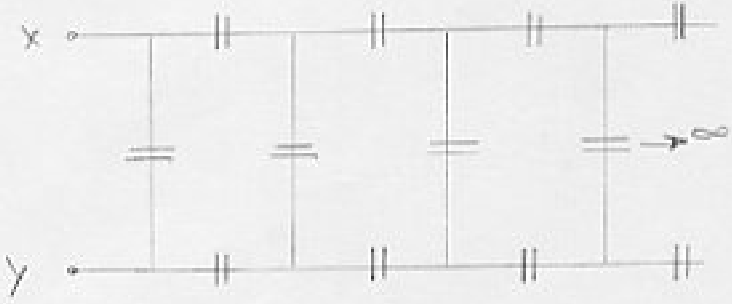


$C_{x_3y_3} = \frac{8}{13}C$ $\qquad$ $C_{x_2y_2} = \frac{3}{5}C$ $\qquad$ $C_{x_1y_1} = \frac{C}{2}$

$$\frac{1}{2}C < \frac{3}{5}C < \frac{8}{13}C < \cdots < C$$

$$C_{xy} = C$$

$n \to \infty$



E $C_{xy} = ?$

Similarly:

$$\frac{C}{3} < \frac{4C}{11} < \frac{15C}{41} < \cdots < C$$

$$C_{xy} = C + C = 2C$$