Chapter 26

Electric potential:

26-1

There is a similarity between Newton's law of gravitation and Coulomb law of electrostatic

In both cases $F \sim \frac{1}{r^2}$ (the only difference: in Coulomb law we have also repulsion)

They are mathematically similar;

In gravitation we defined a quantity, namely, the potential

Here also we can define electric pot. energy.

Def. – $\Delta U = U_f - U_i = -W_{if}$

W_{if} : the work done by the electrostatic force (the field) on test charge.

ΔU : indep of path $\longrightarrow$ electrostatic force: conservative

$\begin{cases} 1 - \text{We choose ref. point (i-point) at } \infty. \\ 2 = \text{We assign } U_i = 0 \end{cases}$ (both arbitrary)

$\rightarrow U = -W_{\infty f}$ (Def.)

Ex. A child's helium-filled ballon, with charge $q = -5.5 \times 10^{-8}$ C, rises vertically into the air by a distance $d = 520$ m.
$E = 150$ N/C the eletric field in atmosphere near the surface of Earth (downward)
$\Delta U = ?$

Sol. $W = \bar{F} \cdot \bar{d}$ $\bar{F} = q\bar{E}$

$$W_{if} = F \cdot d = q\bar{E} \cdot \bar{d} = qEd \cos 180^\circ = -qEd$$
$$= -(-5.5 \times 10^{-8}\,C)(150\,N/C)(520\,m) = 4.3 \times 10^{-3}\,J$$
$$U_f - U_i = -W_{if} = -4.3 \times 10^{-3}\,J \quad \text{(decreasing)}$$

26-2 The Electric Pot. ;

The pot. energy : depends on $\begin{cases} |E| \\ q \end{cases}$

However, the pot. energy per unit charge : has a unique value at any point in an electric field.

(indep. of q at any point)

Def.: $V = \dfrac{U}{q_0}$ electric potential

$$\rightarrow \Delta V = V_f - V_i = \frac{U_f}{q_0} - \frac{U_i}{q_0} = \frac{\Delta U}{q_0}$$

$$\Delta V = V_f - V_i = -\frac{W_{if}}{q_0}$$

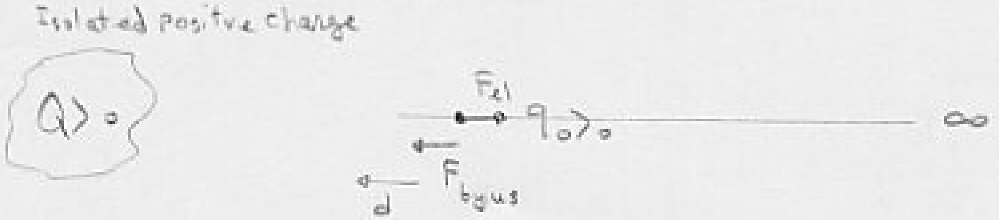
$q_0 \Delta V$: the work done by us to move q_0 from i to f

$$q_0 \Delta V = -W_{if}$$

If $U_i = 0$ at $i \equiv \infty$

$$\longrightarrow \quad V = -\frac{W_{\infty f}}{q_0} \qquad \text{(Def.)}$$

Note:



$W' > 0$ by us to bring $q_0 > 0$ from infinity to a point near the Q.

$$\longrightarrow W_{\infty f} < 0 \longrightarrow V = -\frac{W_{\infty f}}{q_0} > 0$$

In SI unit: V : volt = 1 Joule per Coulomb

Electric potential energy: is an energy of charged object in an external electric field (in Joules)

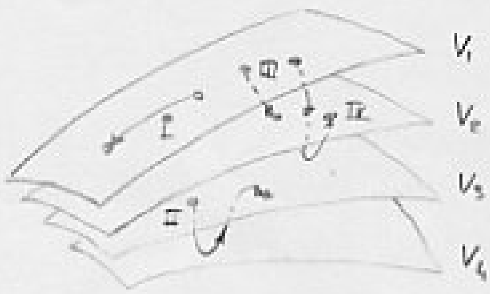
Electric potential: is a property of the field itself, whether or not a charged object has been placed in it (volts)

Note: For E: $1\ N/C = (1\ N/C)\left(\frac{1 V \cdot C}{1 J}\right)\left(\frac{1 J}{1 N \cdot m}\right) = 1\ V/m$ $\quad (1 J = 1 N \cdot m)$ $\quad \left(1V = \frac{1J}{1C}\right)$

Def. - Electron-volt (eV): is an energy equal to the work required to move a single elementary charge $\underline{e}$, through a potential difference of <u>One volt.</u>

$$\Delta V = -\frac{W_{if}}{q_0} \longrightarrow 1\,eV = e(1) = 1.60\times10^{-19}\,C\,(1\,J/C)$$
$$= 1.60\times10^{-19}\,J$$

26-3 Equipotential surfaces



Def. - A locus of points in space that all have the same potential is called an <u>equipotential surface</u>.

i) $V_f - V_i = -\frac{W_{if}}{q_0}$
ii) Path-independence of potential energy

$$\longrightarrow \begin{cases} W_{if} = 0 & \text{for I, II} \\ W_{if} \neq 0 & \text{" III, IV } (W_{if_{III}} = W_{if_{IV}}) \end{cases}$$

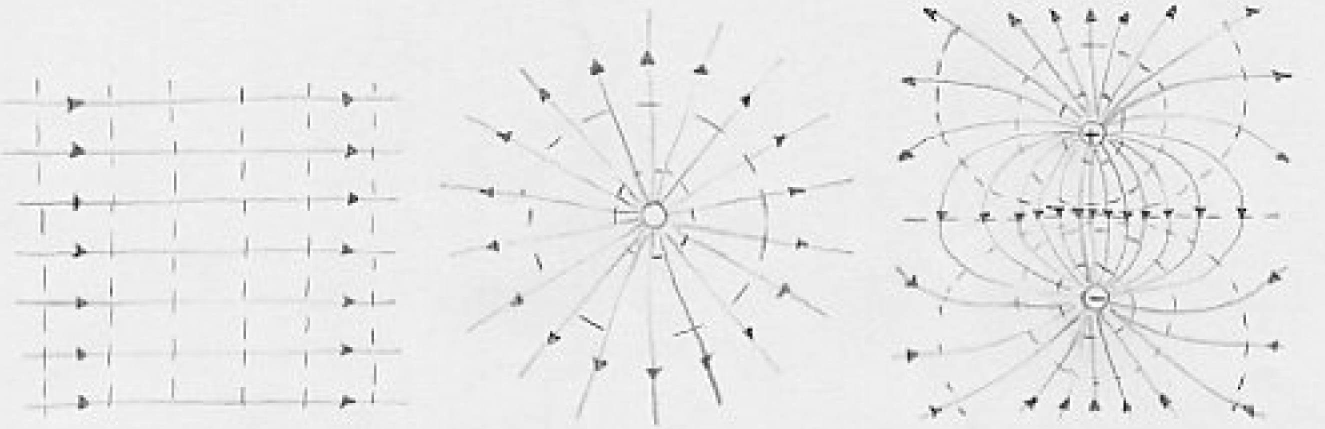
Always; Equipotential surfaces $\perp$ Electric field lines

$\longrightarrow$ " $\perp \overline{E}$

If $E \not\perp$ Equipotential surface

$\longrightarrow$ E would have a component on this surface

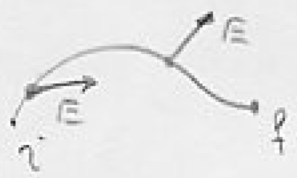
$\longrightarrow$ This component would then do work on a test charge moving on this surface. (but we saw $W_{if} = 0$)



Electric field lines and equipotential surfaces

26-4 Calculating the Potential from the Field:

We can calculate ΔV between i and f <u>if</u> we know $\bar{E}$ at all positions along any path connecting these points.



$dW = \bar{F}\cdot d\bar{s}$ (by $\bar{F}$)

$\bar{F} = q_0\bar{E} \Rightarrow dW = q_0\bar{E}\cdot d\bar{s}$

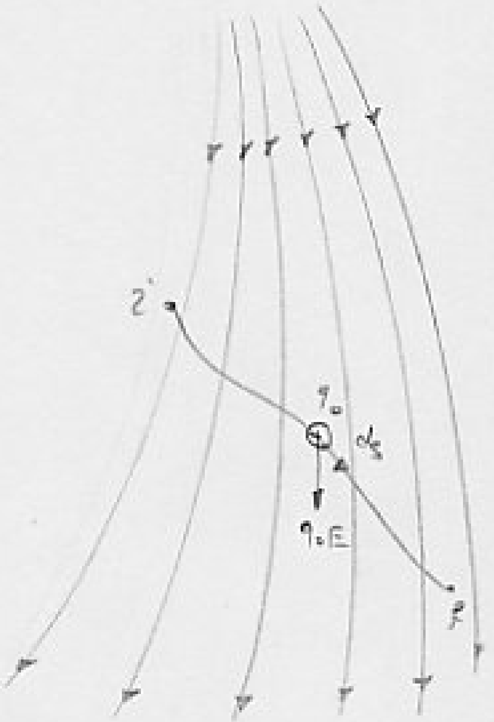
$W_{if} = q_0\int_i^f E\cdot ds \rightarrow V_f - V_i = -\int_i^f E\cdot ds$

Since F_{el} is conservative $\longrightarrow \int_i^f$ path-indep.

If $V_i = 0 \longrightarrow V = -\int_i^f E\cdot ds$

Ex. $\Delta V = ?$ for path (i)

" " (ii)



Sol.

For path (i)

$$V_f - V_i = -\int_i^f \bar{E}\cdot d\bar{s} = -\int_i^f E(\cos 0)\,ds$$

$$= -\int_i^f E\,ds = -E\int_i^f ds = \underline{-Ed}$$

For path (ii)

$$V_f - V_i = -\int_i^f \bar{E}\cdot d\bar{s}$$

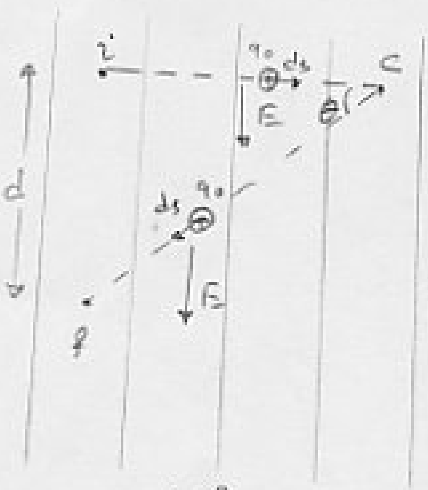
$$= -\int_i^c \bar{E}\cdot d\bar{s} - \int_c^f \bar{E}\cdot d\bar{s}$$

$$= -\int_i^c E\,ds\cos 90^\circ - \int_c^f E\,ds\sin 45$$

$$= 0 - \frac{\sqrt{2}}{2}E\int_i^f ds$$

$$= -\frac{E}{\sqrt{2}}\sqrt{2}\,d = \underline{-Ed}$$

(i)



$\theta = 45^\circ$

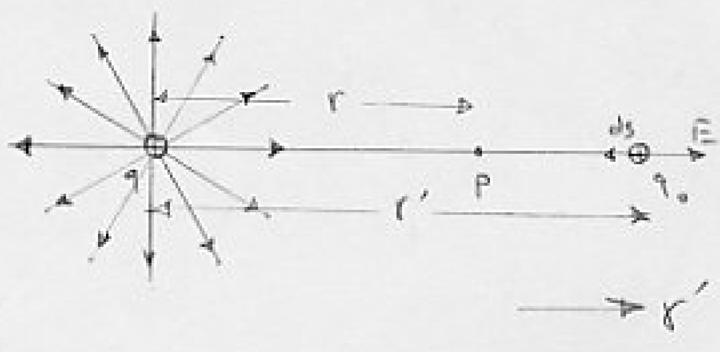
(ii)

26-5 Potential due to a point charge:

$$V = -\int_i^f \bar{E}\cdot d\bar{s}$$

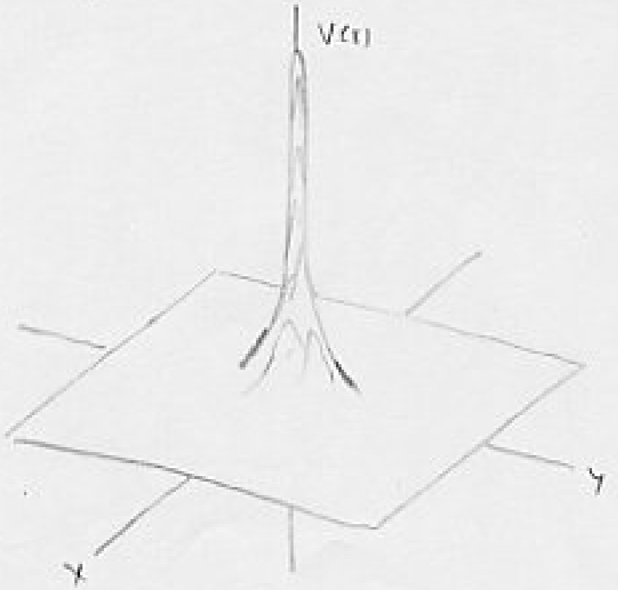
$$\bar{E}\cdot d\bar{s} = E\cos\pi\,(-dr') = E\,dr'$$

$$V = -\int_\infty^r E\,dr'$$

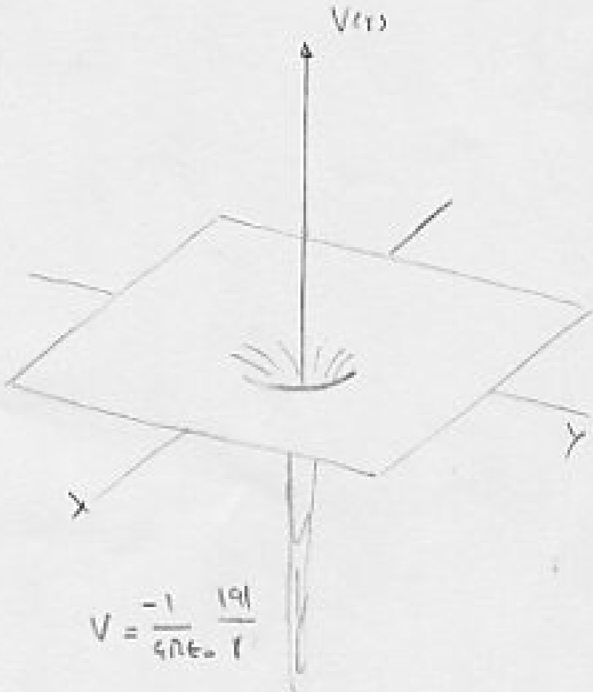


$$E = \frac{1}{4\pi\epsilon_0}\frac{q}{r'^2} \qquad V = -\frac{q}{4\pi\epsilon_0}\int_{\infty}^{r}\frac{1}{r'^2}\,dr' = -\frac{q}{4\pi\epsilon_0}\left[-\frac{1}{r'}\right]_{\infty}^{r}$$

$$V = \frac{1}{4\pi\epsilon_0}\frac{q}{r}$$ Point charge (Positive or negative if q carries the sign)



$$V = \frac{1}{4\pi\epsilon_0}\frac{|q|}{r}$$



$$V = \frac{-1}{4\pi\epsilon_0}\frac{|q|}{r}$$

(V(r) evaluated on xy plane)

Ex.

What is the potential on the surface of a gold nucleus?

$R = 6.2$ fm $\quad z = 79$

Sol.

$$V = \frac{1}{4\pi\epsilon_0}\frac{q}{r} = \frac{1}{4\pi\epsilon_0}\frac{Ze}{R} = 1.8\times10^{7}\ \text{V} = 18\ \text{MV}$$

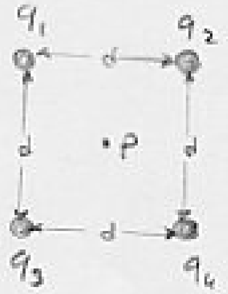
26-6 Potential due to a group of point charges

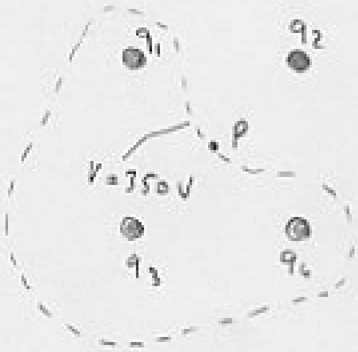
$$V = \sum_{i=1}^{n} V_i = \frac{1}{4\pi\epsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_i}$$

Ex. $q_1 = +12\,nC \quad q_3 = +31\,nC$

$q_2 = -24\,nC \quad q_4 = +17\,nC$

$V_P = ? \quad d = 1.3\,m$





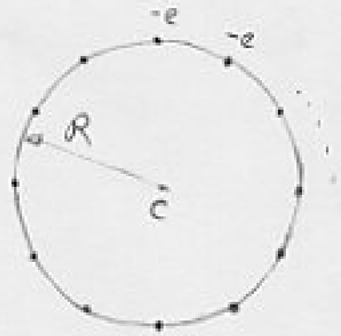
Sol.

$$V = \sum_{i=1}^{4} V_i = \frac{1}{4\pi\epsilon_0} \frac{q_1+q_2+q_3+q_4}{r}$$

$$r = \frac{d}{\sqrt{2}} \qquad V \simeq 350\,V$$

Ex. a) 12-Electrons (charge $-e$)

$V_c = ? \quad E_c = ?$



(a)

Sol.

$$V = \sum_{i=1}^{12} V_i = -12 \frac{1}{4\pi\epsilon_0} \frac{e}{R} \quad \text{(scalar)}$$

But $E_c = 0$ (vector) (from the symmetry of the prob.)

b) 12-Electrons, distributed non-uniformally.

$V_c = ?$, How does E_c change?

Sol.

$$V = -12 \frac{1}{4\pi\epsilon_0} \frac{e}{R} \quad \text{as before}$$

$E_c \neq 0$ directed to the charge-distribution.

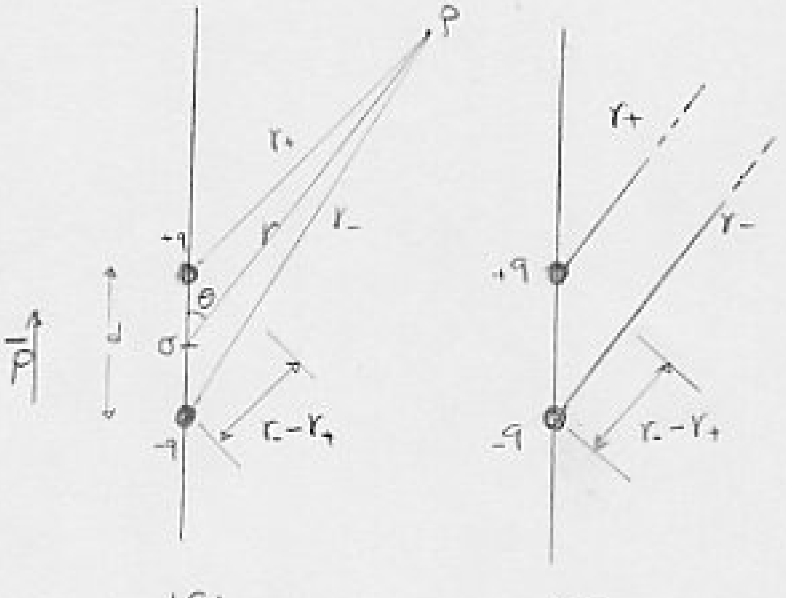
(b)

26-7 Potential due to an Electric-Dipole;

$$V = \sum_{i=1}^{2} V_i = V_+ + V_-$$

$$V = \frac{1}{4\pi\epsilon_0}\left(\frac{q}{r_+} + \frac{-q}{r_-}\right)$$

$$= \frac{q}{4\pi\epsilon_0}\,\frac{r_- - r_+}{r_+ r_-}$$



P : far
$\rightarrow (\bar{r}_- - \bar{r}_+) \perp \bar{r}_-$

For $r \gg d$

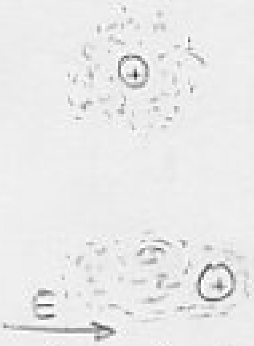
$$\begin{cases} r_- - r_+ \simeq d\cos\theta \\ r_- r_+ \simeq r^2 \end{cases}$$

$$\rightarrow V = \frac{q}{4\pi\epsilon_0}\,\frac{d\cos\theta}{r^2} \quad \rightarrow \quad V = \frac{1}{4\pi\epsilon_0}\,\frac{p\cos\theta}{r^2}$$

Many molecules like H_2O have permanent electric dipole. In some others since the centers of negative and positive charges coinside (nonpolar molecules), and thus, no dipole moment is set up. But in the presence of $\bar{E}$ they are polarized, and they obtain electric moment (induced moment).

(a) $P = 0$

(b) $P \neq 0$



induced electric dipole moment

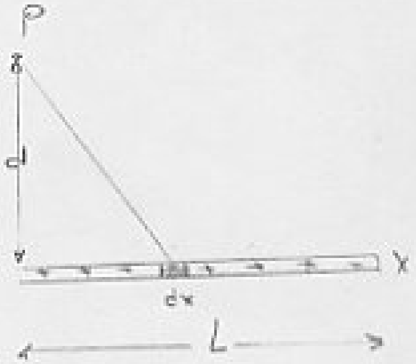
In TV and Radio antennas, the electrons oscillate from one end to the other end. Such a antenna is called an <u>oscillating</u> electric dipole antenna.

26-8 Potential due to a Continuous Charge Distribution:

$$dV = \frac{1}{4\pi\epsilon_0}\frac{dq}{r} \qquad \Sigma \longrightarrow \int$$

$$V = \int dV = \frac{1}{4\pi\epsilon_0}\int \frac{dq}{r}$$

Line Charge



$$dq = \lambda\, dx \qquad r = (x^2+d^2)^{1/2}$$

$$dV = \frac{1}{4\pi\epsilon_0}\frac{dq}{r} = \frac{1}{4\pi\epsilon_0}\frac{\lambda\, dx}{(x^2+d^2)^{1/2}}$$

$$V = \int dV = \int_0^L \frac{1}{4\pi\epsilon_0}\frac{\lambda}{(x^2+d^2)^{1/2}}\, dx$$

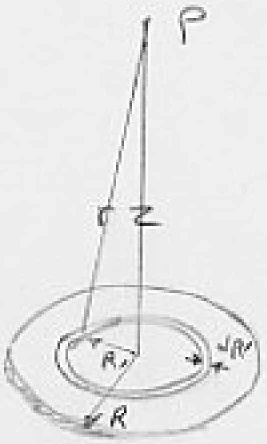
$$= \frac{\lambda}{4\pi\epsilon_0}\int_0^L \frac{dx}{(x^2+d^2)^{1/2}}$$

Since $\displaystyle\int \frac{dx}{\sqrt{x^2+a^2}} = \ln\left(x + \sqrt{x^2+a^2}\right)$

$$V = \frac{\lambda}{4\pi\epsilon_0}\ln\left[x + (x^2+d^2)^{1/2}\right]_0^L$$

$$= \frac{\lambda}{4\pi\epsilon_0}\left(\ln\left[L + (L^2+d^2)^{1/2}\right] - \ln d\right) = \frac{\lambda}{4\pi\epsilon_0}\ln\left[\frac{L+(L^2+d^2)^{1/2}}{d}\right]$$

Charged Disk:



$dq = \sigma(2\pi R')\,dR'$

$$dV = \frac{1}{4\pi\epsilon_0}\frac{dq}{r} = \frac{1}{4\pi\epsilon_0}\frac{\sigma(2\pi R')\,dR'}{\sqrt{z^2+R'^2}}$$

$$V = \int dV = \frac{\sigma}{2\epsilon_0}\int_0^R (z^2+R'^2)^{-1/2} R'\,dR'$$

$z^2 + R'^2 = u^2 \rightarrow R'dR' = u\,du$

$$V = \frac{\sigma}{2\epsilon_0}\int_z^{\sqrt{z^2+R^2}} du = \frac{\sigma}{2\epsilon_0}[u]_z^{\sqrt{z^2+R^2}} = \frac{\sigma}{2\epsilon_0}\left(\sqrt{z^2+R^2} - z\right)$$

$z \geq 0$

Ex.

The potential at the center of a uniformly charged circular disk of radius $R = 3.5$ cm is $V_0 = 550$ V

a) What is the total charge q on the disk?

$$V = \frac{\sigma}{2\epsilon_0}\left(\sqrt{z^2+R^2} - z\right) \quad z = 0 \rightarrow V = \frac{\sigma R}{2\epsilon_0}$$

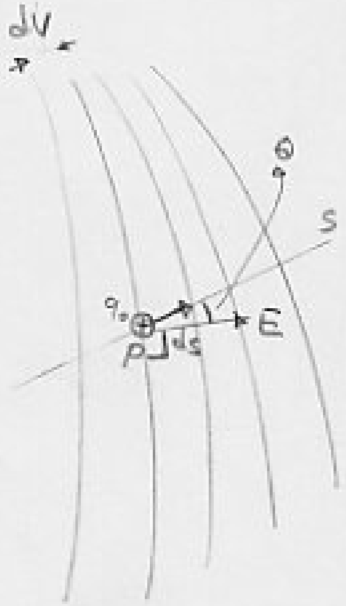
$$\sigma = \frac{2\epsilon_0 V_0}{R} \qquad q = \sigma(\pi R^2) = 2\pi\epsilon_0 R V_0 = 1.1\times10^{-9}\ \text{C}$$

b) what is the potential at a point on the axis of the disk a distance $z = 5.0R$?

$$V = \frac{\sigma}{2\epsilon_0}\left(\sqrt{(5.0R)^2+R^2} - 5.0R\right) \qquad \sigma = \frac{2\epsilon_0 V_0}{R}$$

$$\rightarrow V = \frac{V_0}{R}\left(\sqrt{26R^2} - 5.0R\right) = V_0\left(\sqrt{26} - 5\right) = 550(9.99) = 54\ \text{V}$$

26-9 Calculating the Field from the Potential;



$$\Delta V = V_f - V_i = -\frac{W_{if}}{q_0}$$

$$\rightarrow dW_{if} = -q_0\, dV$$ the work done by $\bar{E}$

$$dW = q_0 \bar{E} \cdot d\bar{s} = q_0 E \cos\theta\, ds$$

$$\rightarrow \quad -q_0\, dV = q_0 E \cos\theta\, ds$$

$$E\cos\theta = -\frac{dV}{ds} \quad \rightarrow E_s = -\frac{\partial V}{\partial s}$$

$$\rightarrow E_x = -\frac{\partial V}{\partial x} \;;\; E_y = -\frac{\partial V}{\partial y} \;,\; E_z = -\frac{\partial V}{\partial z}$$

If we know $V(x,y,z)$ $\xrightarrow{\text{then}}$ $E(x,y,z)$ can be obtained.

<u>Ex.</u>

$$V = \frac{\sigma}{2\epsilon_0}\left[(z^2+R^2)^{1/2} - z\right]$$ for a charged disk along any point on the axis

$E = ?$ at any point on axis.

<u>Sol.</u>

$$E_z = -\frac{\partial V}{\partial z} = \frac{\sigma}{2\epsilon_0}\left(1 - \frac{z}{\sqrt{z^2+R^2}}\right)$$ (as we obtained it before)

26-10 Electric Potential Energy due to a System of Point Charges

Def. – The electric potential energy of a system of fixed point charges is equal to the work that must be done by an external agent to assemble the system, bringing each charge in from an infinite distance.

Ex. Two point charge q_1 and q_2 are brought from infinity to a distance r from each other,

Potential Energy = ?

(i)

No work is done for bringing q_1 from infinity
(since there is no $\vec{E}$)

(ii)

$$V = -\frac{W_{\infty f}}{q_0} \rightarrow W_{\infty f} = -q_0 V \rightarrow W = q_0 V \quad \text{done by us}$$

here $W = q_2 V$

But $V = \frac{1}{4\pi\epsilon_0}\frac{q_1}{r}$

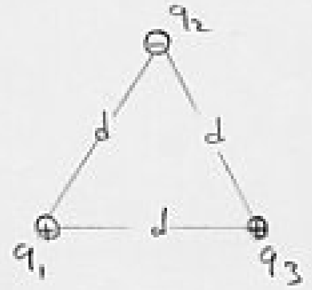
$$\rightarrow U = W = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r} \quad \text{Potential energy of two charges}$$

Ex.

$q_1 = +q \qquad q_2 = -4q \qquad q_3 = +2q$

$q = 150\ nC \qquad d = 12\ cm$

$U = ?$ Potential energy of the sys.



Sol.

$W_1 = 0$ the work to bring q_1 from ∞.

$W_2 = U_{12}$ " " " q_2 " "

$$U_{12} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{d}$$

$W_3 = U_{13} + U_{23}$ " " " q_3 " "

$= q_3 V_1 + q_3 V_2$

$$U_{13} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_3}{d} \qquad U_{23} = \frac{1}{4\pi\epsilon_0}\frac{q_2 q_3}{d}$$

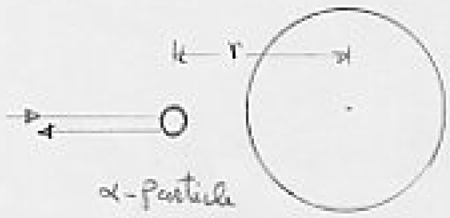
$$U = U_{12} + U_{13} + U_{23} = \frac{1}{4\pi\epsilon_0}\frac{1}{d}\left(q_1 q_2 + q_1 q_3 + q_2 q_3\right) = -1.7\times10^{-2}\ J$$

Ex.

$Z = 79$, $N = 118$ for Gold $\quad r = 9.23\ fm$

α-particle coming from ∞ stops at r

$K_{initial} = ?$ (α-particle)



Sol.

outside the atom $E = 0$ (because $\underset{\text{Protons}}{Ze} - \underset{\text{electrons}}{Ze} = q = 0$)

$E = E_\alpha + E_{Au} =$ const mechanical energy

$K_{i\alpha} + U_{i\alpha} = K_{f\alpha} + U_{f\alpha} \qquad U_{i\alpha} = K_{f\alpha} = 0 \qquad K_{i\alpha} = U_{f\alpha} \qquad U_{f\alpha} = q_\alpha V_{gold}$

$$K_{i\alpha} = (2e)\frac{1}{4\pi\epsilon_0}\frac{79e}{9.23\times10^{-15}} = 24.6\ \text{MeV}$$

26-11 An Isolated Conductor:

We saw

$E = 0$ inside the conductor

$\longrightarrow$ An excess charge is distributed on the surface of conductor (even if the conductor has a cavity)

Now using $E = 0$ inside a conductor, we prove;

An excess charge placed on an isolated conductor will distribute itself on the surface of that conductor, so that all points of the conductor, whether on the surface or inside, come to the same potential -

This is true whether or not the conductor has an internal cavity.

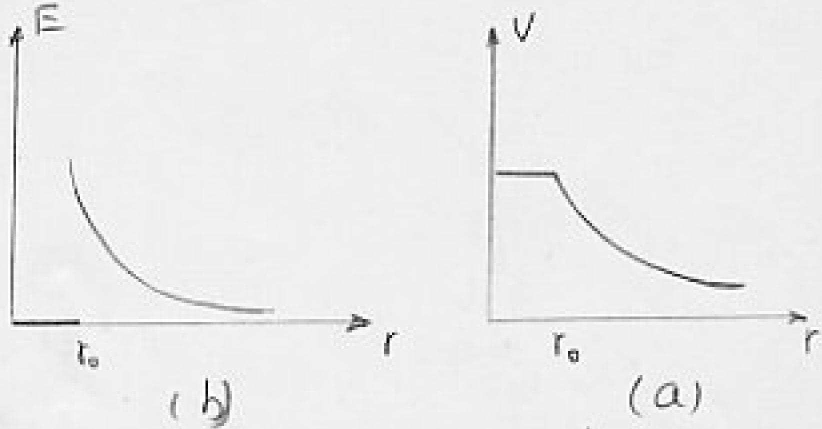
Proof:

$$V_f - V_i = -\int_i^f E \cdot ds$$

Since $E = 0$ inside $\longrightarrow$ $V_f - V_i = 0$ $V_f = V_i$ for any pair

From outside when we reach at a distance r_0 from the center of sphere, if we try to push a test charge through a small hole in the surface to its center, no extra work is needed to do this (because $E = 0$)

$\longrightarrow$ No change in V (it remains const)



For a conducting sphere

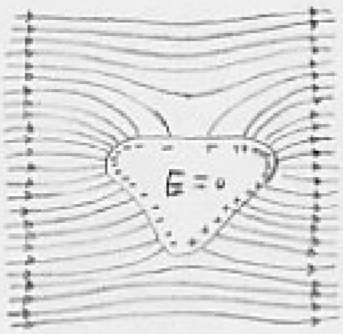
$$\text{Fig. (a)} = -\int \text{Fig (b)}\, dr$$

$$\text{Fig(b)} = -\frac{\partial\, \text{Fig(a)}}{\partial r}$$

Except for spherical conductors, the surface charge density does not distribute itself uniformly.

→ At sharp points or edges charge density and thus E → may reach very high values

→ the air around such sharp points may become ionized → producing the corona discharge



Note: If an isolated conductor is placed in an external electric field, all points of the conductor still come to a single potential (whether or not the conductor has an excess charge)

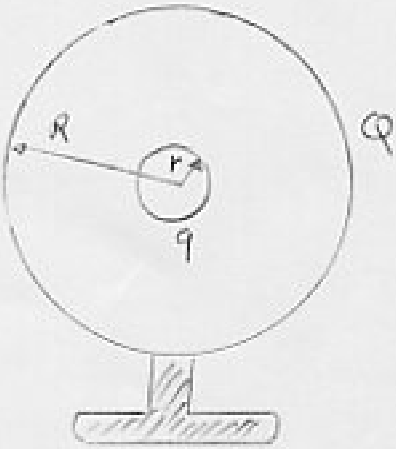
The free conduction electron distribute themselves on the surface in such a way that $\begin{cases} E_{int} + E_{ext} = 0 \text{ (inside the sphere and close to surface)} \\ E_{ext} \perp \text{surface} \end{cases}$

If the conductor could be somehow removed, leaving the surface charges frozen in place, the pattern of the electric field would remain absolutely unchanged, for both exterior and interior points.

26-12 The Van De Graaff Accelerator;

Inner conducting shell has charge q

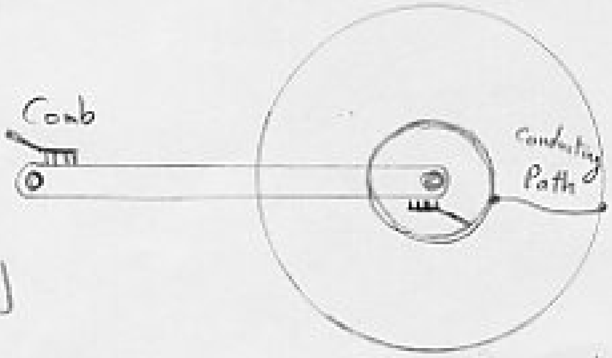
Outer " " " " = Q



They are connected by a conducting path $\xrightarrow{\text{the}}$ charge q is transfers to the outer shell and its potential increases.

In practice the charge is carried into the inner shell as shown in the fig.

The potential produced = several MV



charged particles are accelerated by this potential.

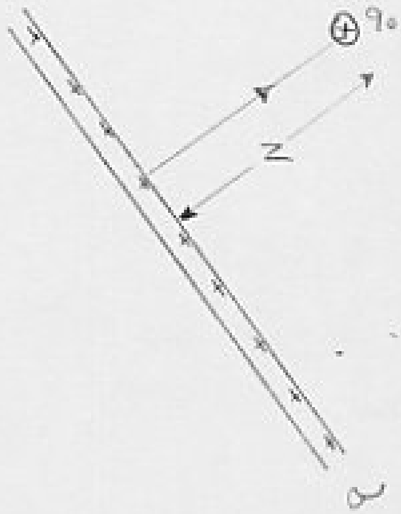
charge is spryed onto the belt by a comb.

× 6E –

a) $W_E = ?$ (from $z=0$ to Z on q_0)

b) Show that $V = V_0 - \left(\frac{\sigma}{2\epsilon_0}\right) Z$

V_0: potential at the surface of sheet.



Infinite nonconducting sheet

Sol.

a) $E = \frac{\sigma}{2\epsilon_0}$ for an infinite sheet

$|F| = q_0 E = q_0 \frac{\sigma}{2\epsilon_0}$

$W_E = \int_0^Z F \cdot dx = q_0 \frac{\sigma}{2\epsilon_0} Z$

b) $V_f - V_i = -\int_i^f E \cdot ds$ $\quad V = V_0 - \int_i^f E \cdot ds$

$V = V_0 - \left(\frac{\sigma}{2\epsilon_0}\right) Z$

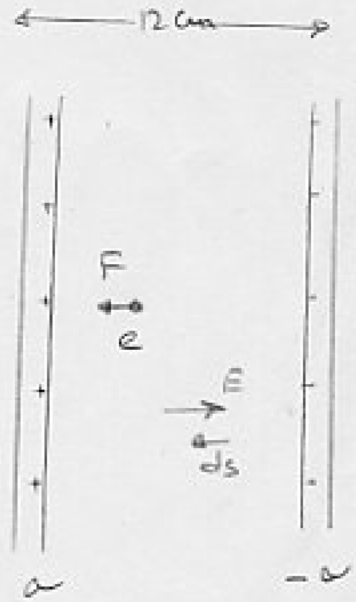
8E –

$F_e = 3.9 \times 10^{-15}$ N

(Neglect fringing)

a) $E = ?$ at the place of e

b) $\Delta V = ?$ between Plates



Two large conducting plates

Sol.

a) $F = eE \rightarrow E = \frac{F}{e} = \frac{3.9 \times 10^{-15}}{1.6 \times 10^{-19}} = 24375$ N/C

b) $V_f - V_i = -\int_i^f E \cdot ds$

$\Delta V = -\int_{i(-)}^{f(+)} E \cdot ds = -\int E\, ds\, \cos\pi = + E S$

$\Delta V = 24375 (0.12) = 2925$ V

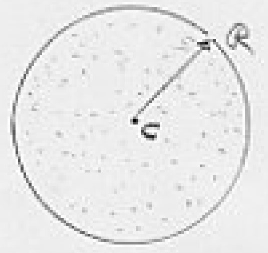
× 11P —

$$E(r) = \frac{qr}{4\pi\epsilon_0 R^3} \qquad q: \text{total charge}$$

a) If $V_c = 0$ $\quad V(r) = ?$ $\quad r < R$

b) $V_R - V_c = ?$

c) If $q > 0$ $\quad V_R \overset{?}{>} V_c$ or $V_R \overset{?}{<} V_c$



uniformly charged nonconducting sphere

sol.

a) $$V(r) - V_c = -\int_c^r E \cdot dr' = -\frac{q}{4\pi\epsilon_0 R^3}\int_0^r r'\,dr' = -\frac{q}{4\pi\epsilon_0 R^3}\left[\tfrac{1}{2}r'^2\right]_0^r$$

(if $q > 0$ $\quad E$ and dr' in the same dir)

$$V(r) = -\frac{q}{8\pi\epsilon_0 R^3} r^2 \qquad (q>0)$$

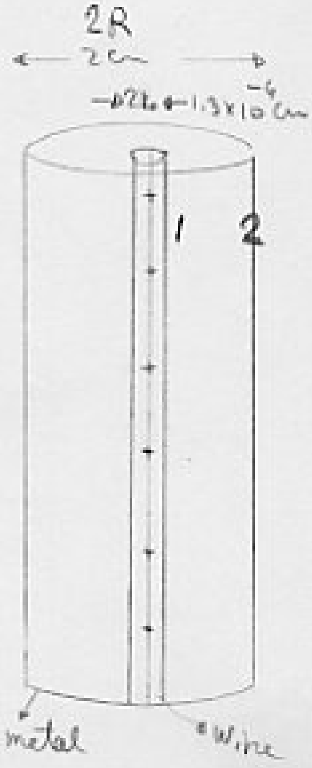
$$V(r) - V_c = -\int_c^r E \cdot dr' = -\frac{|q|}{4\pi\epsilon_0 R^3}\int_0^r r'\,dr'(\cos 180) = \frac{|q|}{8\pi\epsilon_0 R^3} r^2 \qquad (q<0)$$

b) $$V(R) = -\frac{q}{8\pi\epsilon_0 R^3}R^2 = -\frac{q}{8\pi\epsilon_0 R}$$

c) If $q > 0$ $\quad V_R < V_c$

12P — Geiger Counter;

$\Delta V = 850$ $\quad E_{\text{on wire}} = ?$ $\quad E_{\text{on cylinder}} = ?$



sol. $$\oint E \cdot ds = \frac{\lambda L}{\epsilon_0} \qquad E = \frac{\lambda}{2\pi\epsilon_0 r}$$

$$\Delta V = V_1 - V_2 = -\int_2^1 E \cdot d\ell = -\int_2^1 E\,d\ell\cos\pi = \int_2^1 E\,d\ell = -\int_2^1 E\,dr$$

$$\Delta V = +\int_1^2 E\,dr = \frac{\lambda}{2\pi\epsilon_0}\int_{r_0}^R \frac{dr}{r} = \frac{\lambda}{2\pi\epsilon_0}\ln R/r_0 \qquad \lambda = \frac{2\pi\epsilon_0 \Delta V}{\ln\frac{R}{r_0}}$$

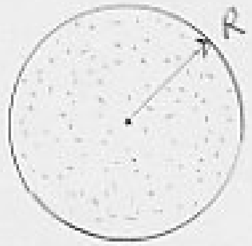
$$E = \frac{\Delta V}{\ln\left(\frac{R}{r_0}\right)}\frac{1}{r} \qquad E_{r_0} = \frac{850}{\ln\left(\frac{2}{1.3\times10^{-4}}\right)}\frac{1}{\frac{1.3}{2}\times10^{-6}} = 1.36\times10^{8}\ \text{V/m}$$

$$E_R = \frac{850}{\ln\left(\frac{2}{1.3\times10^{-4}}\right)}\frac{1}{\frac{2}{2}\times10^{-2}} = 8.82\times10^{3}\ \text{V/m}$$

13P — setting $V=0$ at $r\to\infty$

Show that $V(r)=\dfrac{q(3R^2-r^2)}{8\pi\epsilon_0 R^3}$ $\quad r<R$

$V(R)-V(0)=?$



charge q uniformly distributed throughout the sphere.

Sol.

$$E=\frac{1}{4\pi\epsilon_0}\frac{q}{R^3}r \quad r\leq R \quad , \quad E=\frac{1}{4\pi\epsilon_0}\frac{q}{r^2} \quad r>R$$

$$V(r)-V_\infty=-\int_\infty^r \vec{E}\cdot d\vec{l}=-\int_\infty^r E\,dl\cos\pi$$

$$V(r)=\int_\infty^r E\,dl=-\int_\infty^r E\,dr$$

$$V(r)=-\int_\infty^R \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}dr-\int_R^r \frac{1}{4\pi\epsilon_0}\frac{q}{R^3}r\,dr$$

$$V(r)=-\frac{q}{4\pi\epsilon_0}\left[-\frac{1}{r}\Big|_\infty^R+\frac{r^2}{2R^3}\Big|_R^r\right]=\frac{q(3R^3-r^2)}{8\pi\epsilon_0 R^3}$$

This result is different from the result obtained in Prob. 11P (because of different references)

$$V(R)-V(0)=\frac{q(3R^3-R^2)}{8\pi\epsilon_0 R^3}-\frac{q(3R^3)}{8\pi\epsilon_0 R^3}=-\frac{q}{8\pi\epsilon_0 R}$$

This result is the same as the result obtained in Prob. 11P (because ΔV is indep. of the choice of ref. point)

16P — Assume $V=0$ at $r=\infty$

$V=?$ for $r>r_2$, $r_2>r>r_1$, $r<r_1$

Charge q uniformly distributed throughout the spherical shell.

Sol.

For $r>r_2$ $\quad V=\dfrac{1}{4\pi\epsilon_0}\dfrac{q}{r}$

$$q=\rho\,\frac{4\pi}{3}(r_2^3-r_1^3)$$

$$\to V=\frac{1}{4\pi\epsilon_0}\frac{4\pi}{3}\frac{\rho(r_2^3-r_1^3)}{r} \qquad r>r_2$$

For $r_1 < r < r_2$

$$V(r) - V_\infty = -\int_\infty^r E.dl = -\int_\infty^{r_2} E.dl - \int_{r_2}^r E.dl$$

$dl = -dr$

$$V(r) = -\int_\infty^{r_2} E dl \cos\pi - \int_{r_2}^r E dl \cos\pi = \int_\infty^{r_2} E dr \cos\pi + \int_{r_2}^r E dr \cos\pi$$

$$V(r) = -\int_\infty^{r_2} \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} dr - \int_{r_2}^r \frac{1}{4\pi\epsilon_0} \frac{4\pi}{3} \rho \, (r^3 - r_1^3) \frac{dr}{r^2} \quad \begin{pmatrix} \cos\pi = -1 \\ q > 0 \end{pmatrix}$$

$$V(r) = -\frac{q}{4\pi\epsilon_0} \left[-\frac{1}{r}\right]_\infty^{r_2} - \frac{1}{4\pi\epsilon_0} \frac{4\pi}{3} \rho \left\{ \int_{r_2}^r r\,dr - r_1^3 \int_{r_2}^r \frac{dr}{r^2} \right\}$$

$$V(r) = \frac{q}{4\pi\epsilon_0} \frac{1}{r_2} - \frac{\rho}{3\epsilon_0} \left\{ \left(\frac{r^2}{2} - \frac{r_2^2}{2}\right) + \left(+\frac{r_1^3}{r} - \frac{r_1^3}{r_2}\right) \right\}$$

$$V(r) = \frac{\rho}{3\epsilon_0} (r_2^3 - r_1^3) \frac{1}{r_2} - \frac{\rho}{3\epsilon_0} \left[\frac{r^2}{2} - \frac{r_2^2}{2} + \frac{r_1^3}{r} - \frac{r_1^3}{r_2} \right]$$

$$V(r) = \frac{\rho}{3\epsilon_0} \left[-\frac{1}{2} r^2 + \frac{3}{2} r_2^2 - \frac{r_1^3}{r} \right]$$

For $r < r_1$ there is no charge inside and $V = $ const.

Since V is continuous; $V_{inside} = V(r_1)$

$$V_{inside} = \frac{\rho}{3\epsilon_0} \left[-\frac{1}{2} r_1^2 + \frac{3}{2} r_2^2 - \frac{r_1^3}{r_1} \right] = \frac{\rho}{2\epsilon_0} (r_2^2 - r_1^2)$$

<u>Ex</u> Show that $1\frac{N}{C}$ and $1\frac{V}{m}$ the unit of E are equivalent.

Sol.

$$\frac{N}{C} = \frac{N}{C} \underbrace{\left(\overset{*}{\frac{V.C}{J}}\right)}_{\text{dim.-less}} \underbrace{\left(\overset{**}{\frac{J}{N.m}}\right)}_{\text{dim.-less}} = \frac{V}{m}$$

Remark:

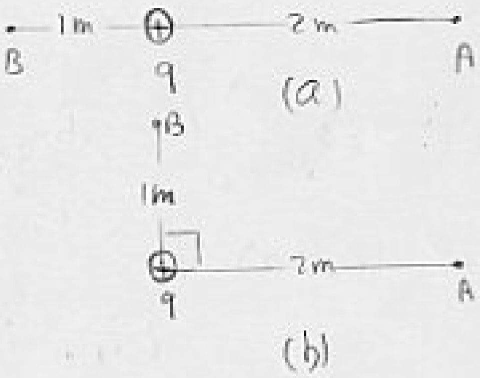
$**$ $W = \int F.ds$

$*$ $V = \frac{-W_{\infty}}{q_0}$

15E − $q = 1.0\,\mu C$

a) $V_A - V_B = ?$ for Fig(a)

b) 〃 〃 〃 (b)



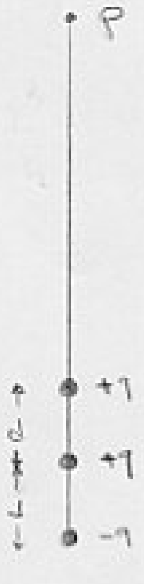
Sol.

$$V_A = \frac{1}{4\pi\epsilon_0}\frac{q}{r_A}$$
$$V_B = \frac{1}{4\pi\epsilon_0}\frac{q}{r_B}$$
$$\longrightarrow V_A - V_B = \frac{q}{4\pi\epsilon_0}\left(\frac{1}{r_A} - \frac{1}{r_B}\right)$$

$$V_A - V_B = (8.99\times10^{9})\times10^{-6}\left(\frac{1}{2} - \frac{1}{1}\right) = -4.495\times10^{3}\ \text{V}$$

for both cases.

33P − show that $V_P = \frac{1}{4\pi\epsilon_0}\frac{q}{r}\left(1 + \frac{2d}{r}\right)$



Sol.

$$V_P = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{r-d} + \frac{1}{r} - \frac{1}{r+d}\right]$$

$$V_P = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{r} + \left(\frac{1}{r-d} - \frac{1}{r+d}\right)\right]$$

$$V_P = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{r} + \left(\frac{r+d-(r-d)}{r^2-d^2}\right)\right]$$

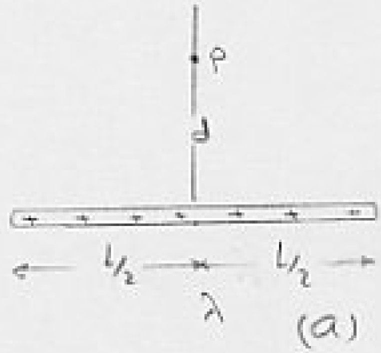
$$V_P = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{r} + \left(\frac{2d}{r^2-d^2}\right)\right]$$

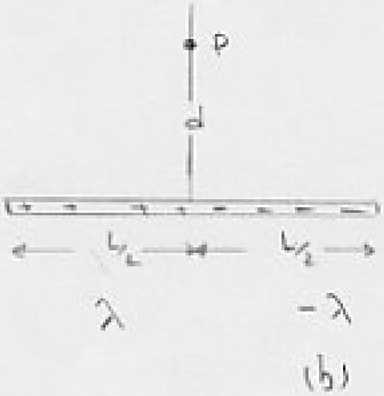
$$V_P = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{r} + \left(\frac{r\,2\frac{d}{r}}{r^2\left(1-\frac{d^2}{r^2}\right)}\right)\right]$$

if $r \gg d$ $\quad \frac{d^2}{r^2} < \frac{d}{r} < 1$

$$V_P = \frac{q}{4\pi\epsilon_0}\frac{1}{r}\left(1 + \frac{2d}{r}\right)$$

36E – $V_p = ?$





Sol.

For (a)

We have obtained before:

$$V = \frac{\lambda}{4\pi\epsilon_0} \ln\left[\frac{L + (L^2 + d^2)^{1/2}}{d}\right]$$

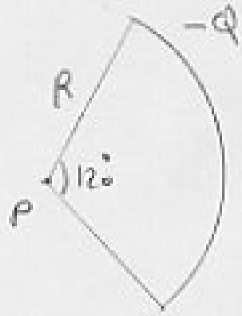
In our prob. $L \longrightarrow L/2$

$$\Rightarrow V_p = \frac{\lambda}{4\pi\epsilon_0} \ln\left[\frac{L/2 + \left(\frac{L^2}{4} + d^2\right)^{1/2}}{d}\right] \times 2$$

For (b)

$$V_p = \frac{\lambda}{4\pi\epsilon_0}\left\{\ln[\cdots] - \ln[\cdots]\right\} = 0$$

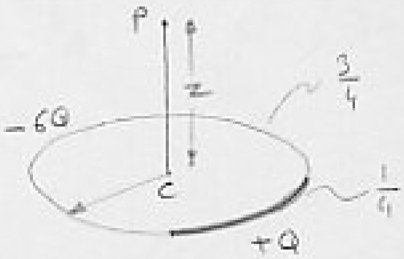
37E – $V_p = ?$



Sol.

$$V_p = \frac{1}{4\pi\epsilon_0}\,\frac{-Q}{R}$$

39P – $V_c = ?$ $V_p = ?$



Sol.

$$V_c = \frac{1}{4\pi\epsilon_0}\left(\frac{Q}{R} - \frac{6Q}{R}\right) = -\frac{1}{4\pi\epsilon_0}\,\frac{5Q}{R}$$

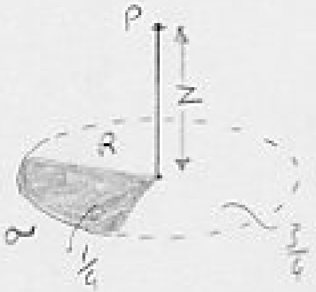
$$V_p = \frac{1}{4\pi\epsilon_0}\left(\frac{1}{\sqrt{R^2+z^2}}\right)(Q - 6Q)$$

$$= \frac{-1}{4\pi\epsilon_0}\,\frac{5Q}{\sqrt{R^2+z^2}}$$

40P — $V_P = ?$

<ins>Sol.</ins>

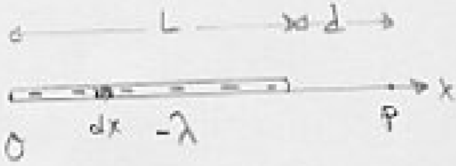
For charged disk we obtained;



$$V = \frac{\sigma}{2\epsilon_0}\left(\sqrt{z^2+R^2} - z\right)$$

$$\rightarrow V_P = \frac{1}{4}\frac{\sigma}{2\epsilon_0}\left(\sqrt{z^2+R^2} - z\right)$$

41P — $V_P = ?$ charge $= -Q$



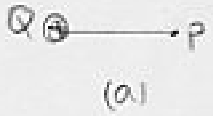
<ins>Sol.</ins>

$$dV = \frac{1}{4\pi\epsilon_0}\frac{dq}{r_P} \qquad dq = -\lambda\, dx \qquad r_P = d + L - x$$

$$V_P = \frac{1}{4\pi\epsilon_0}\int_0^L \frac{-\lambda\, dx}{d+L-x} = \frac{-\lambda}{4\pi\epsilon_0}\int_{d+L}^{d}\frac{-du}{u} \qquad d+L-x = u,\ -dx = du$$

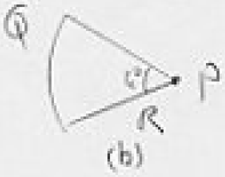
$$= \frac{\lambda}{4\pi\epsilon_0}\left[\ln u\right]_{d+L}^{d} = \frac{\lambda}{4\pi\epsilon_0}\ln\frac{d}{d+L} \qquad -Q = -\lambda L$$

$$V_P = \frac{Q/L}{4\pi\epsilon_0}\ln\frac{d}{d+L}$$



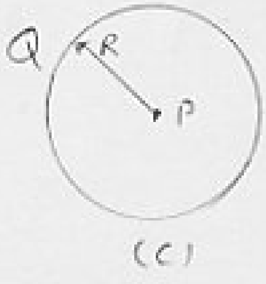
38P — $V_P = ?$ in (a), (b), (c)

Rank the three situations acc. to the magnitude of E at P from largest to least.



<ins>Sol.</ins>

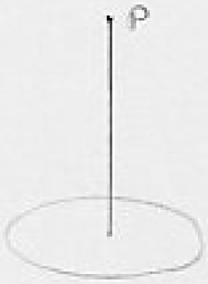
$$V_P = \frac{1}{4\pi\epsilon_0}\frac{Q}{R} \text{ in all three cases.}$$



$$|E_{Pa}| > |E_{Pb}| > |E_{Pc}| = 0$$

45E - $V_P = \frac{\sigma}{2\epsilon_0}\left(\sqrt{z^2+R^2} - z\right)$

Show; $E_P = \frac{\sigma}{2\epsilon_0}\left(1 - \frac{z}{\sqrt{R^2+z^2}}\right)$



charged disk

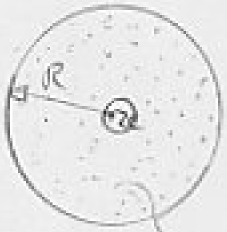
Sol.

$$E_z = -\frac{\partial V}{\partial z} = \frac{\sigma}{2\epsilon_0}\left(1 - \frac{z}{\sqrt{R^2+z^2}}\right)$$

47E -

$$E = \frac{Ze}{4\pi\epsilon_0}\left(\frac{1}{r^2} - \frac{r}{R^3}\right)$$ for Rutherford Atom

for $r < R$



a) Show that $V = \frac{Ze}{4\pi\epsilon_0}\left(\frac{1}{r} - \frac{3}{2R} + \frac{r^2}{2R^3}\right)$ gives the above electric field.

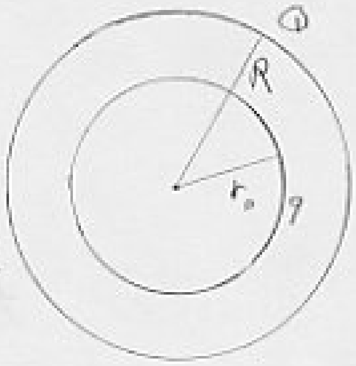
b) Why $V \not\to 0$ as $r \to \infty$

Sol.

a) $E = -\frac{\partial V}{\partial r} = \frac{Ze}{4\pi\epsilon_0}\left(\frac{1}{r^2} - \frac{r}{R^3}\right)$

b) Because this sol. is for $r < R$ region.

85P - $V_{r_0} - V_R = ?$



Sol.

For $r > R$ $\quad V_r - V_\infty = -\int_\infty^r E \cdot ds$

if $Q > 0$ $\quad V_r = -\int_\infty^r E\, ds \cos\pi = \int_\infty^r \frac{1}{4\pi\epsilon_0}\frac{Q+q}{r^2}(-dr) = \frac{1}{4\pi\epsilon_0}\frac{Q+q}{r}$

on $r = R$ $\quad V_R = \frac{1}{4\pi\epsilon_0}\frac{Q+q}{R}$

For $r_0 < r < R$ $\quad V_r - V_R = -\int_R^r \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\, dr = \frac{1}{4\pi\epsilon_0}\left(\frac{q}{r} - \frac{q}{R}\right) \to V_r = \frac{1}{4\pi\epsilon_0}\left(\frac{Q}{R} + \frac{q}{r}\right)$

$V_{r_0} - V_R = \frac{q}{4\pi\epsilon_0}\left(\frac{1}{r_0} - \frac{1}{R}\right)$ / If we connect two spheres by a fine wire $V_{r_0} - V_R = 0 \longrightarrow q = 0$

76E – $R_2 = 2R_1$

sphere (1) : initially has q

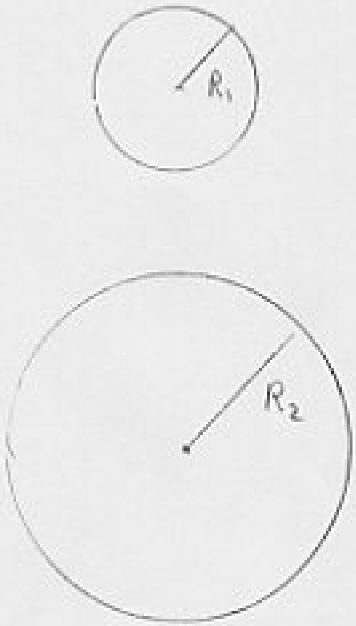
" (2) : " uncharged

Connect them by a long thin wire.

a) How V_1 and V_2 are related?

b) Final q_1, q_2 ?

c) " σ_1, σ_2 ?



Two widely separated conducting spheres

Sol.

a) $V_1 = V_2$

b) $V_1 = \frac{1}{4\pi\epsilon_0}\frac{q_1}{R_1}$ $\quad V_2 = \frac{1}{4\pi\epsilon_0}\frac{q_2}{R_2}$

$V_1 = V_2$ $\quad \frac{q_1}{R_1} = \frac{q_2}{R_2}$ $\quad \frac{q_1}{R_1} = \frac{q_2}{2R_1}$ $\quad \Rightarrow \quad q_2 = 2q_1$

$q_1 + q_2 = q$ $\quad 3q_1 = q$ $\quad q_1 = \frac{q}{3}$ $\quad q_2 = \frac{2}{3}q$

c) $\sigma_1 = \frac{q_1}{4\pi R_1^2}$ $\quad \sigma_2 = \frac{q_2}{4\pi R_2^2} = \frac{2q_1}{4\pi(2R_1)^2} = \frac{q_1}{8\pi R_1^2}$

$\frac{\sigma_1}{\sigma_2} = \frac{1}{2}$