Chap. 24

24-1 Charges and Forces: A Closer look

How does a charge Know the presence of another charge?

Answer: The charge sets up an electric field in the space surrounding it.

The electric field has (at any point in the space) $\begin{cases} \text{1- Magnitude} \\ \text{2- Direction} \end{cases}$

If the electric charge is moved, the information about the movement (the change in the field) travels with the speed of light c.

24-2 The Electric Field:

Scalar field: These fields are specified just by their magnitudes (like temperature field, pressure field)

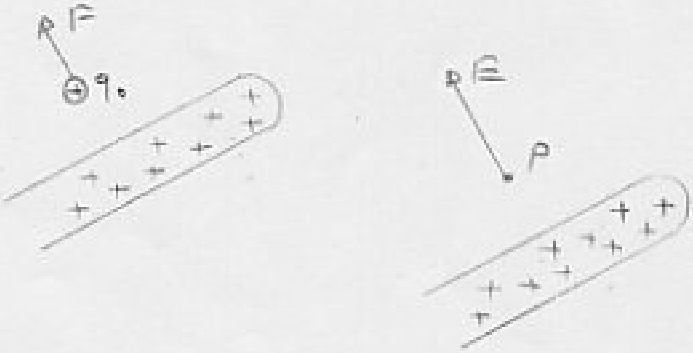
Vector field: These fields are specified by $\begin{cases} \text{Magnitude} \\ \text{Director} \end{cases}$

Therefore these are a distribution of vectors.

By def.:

$$E = \frac{F}{q_0}$$ electric field at any point (N/C)

q_0: small positive charge (test charge). We assume that its presence does not affect the configuration.



24-3 Electric Field Lines:

Michael Faraday (19th Century); introduced the idea of electric field (and line of forces).

Electric field lines provide a nice way to visualize patterns in electric field.

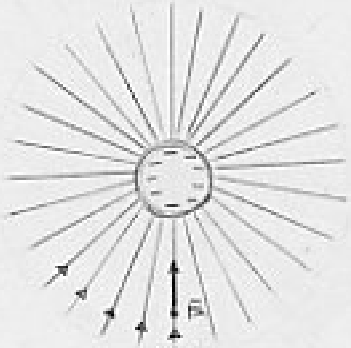
Rule:

i) The direction of a straight field line or the direction of the tangent to the curved field gives the direction of E at any point.

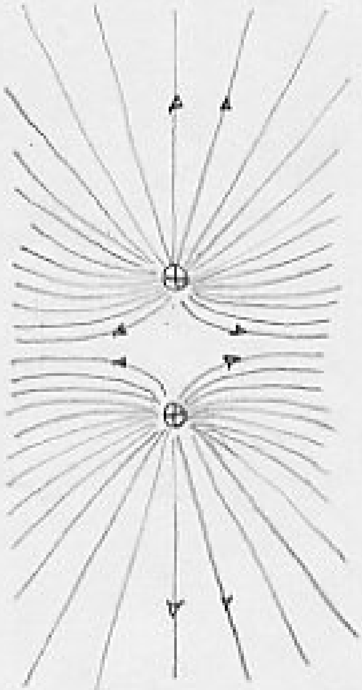
ii) The number of field lines per unit area is proportional to the magnitude of E

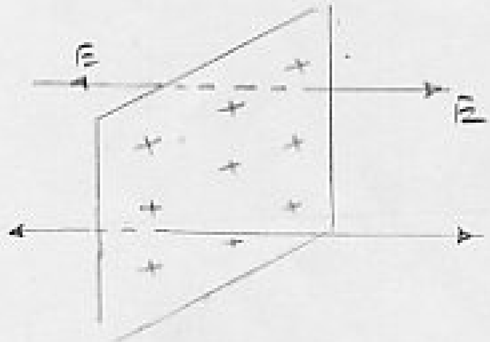
Ex.-

Positive
Test Charge

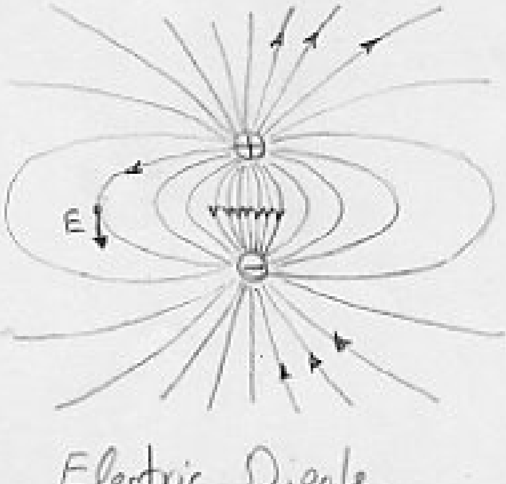


Electric field lines extend away from positive charge and toward negative charge.





Uniformaly charged nonconducting sheet



Electric Dipole

24-1.) The electric field due to a point charge:

$(q<0)$

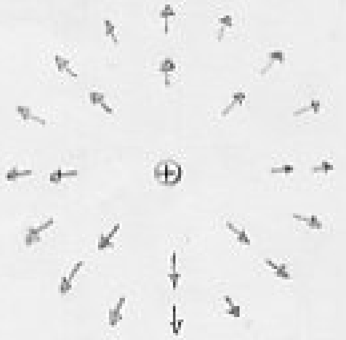
$(q>0)$

$$F = \frac{1}{4\pi\epsilon_0}\frac{q q_0}{r^2}$$

$$E = \frac{F}{q_0} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2} \quad \text{(magnitude)}$$

q_0: positive test charge

Direction: The direction of $\bar{E}$ is the same as $\bar{F}$ on the positive test charge.



$\bar{E}$

The electric field of more than one point charge:

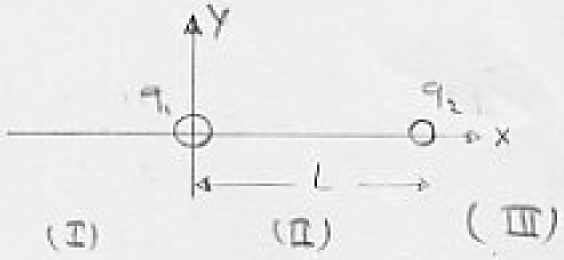
$$\bar{F}_0 = \bar{F}_{01} + \bar{F}_{02} + \cdots + \bar{F}_{0n}$$

$$\bar{E} = \frac{\bar{F}_0}{q_0} = \frac{\bar{F}_{01}}{q_0} + \frac{\bar{F}_{02}}{q_0} + \cdots + \frac{\bar{F}_{0n}}{q_0}$$

$$= \bar{E}_1 + \bar{E}_2 + \cdots + \bar{E}_n$$

Ex. 24.2) $\begin{cases} q_1 = +8q \\ q_2 = -2q \end{cases}$

$\begin{cases} x = ? \\ y = ? \end{cases}$ such that $\bar{E} = 0$



Sol.

$\bar{E} = \bar{E}_1 + \bar{E}_2 = 0 \longrightarrow \bar{E}_1 = -\bar{E}_2 \longrightarrow |E_1| = |E_2|$

The only possibility $\longrightarrow$ Such a point should be on x-axis ($y = 0$)

Region I) Impossible, because $r_1 < r_2$, $q_1 > q_2$

$\longrightarrow |E_1| > |E_2|$

Region II) Impossible, because direction of $\bar{E}_1$ = direction of $\bar{E}_2$

Region III) Possible;

$$\frac{1}{4\pi\epsilon_0}\frac{8q}{x^2} = \frac{1}{4\pi\epsilon_0}\frac{2q}{(x-L)^2} \longrightarrow \left(\frac{x-L}{x}\right)^2 = \frac{1}{4}$$

$$\frac{x-L}{x} = \frac{1}{2} \longrightarrow x = 2L$$

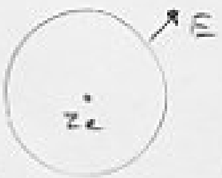
Ex. 24-3) The nucleus of uranium atom has a radius R of 6.8 fm. Assuming that the positive charge of the nucleus is distributed uniformaly, determine the electric field at a point of the nucleus due to that charge.

Sol.

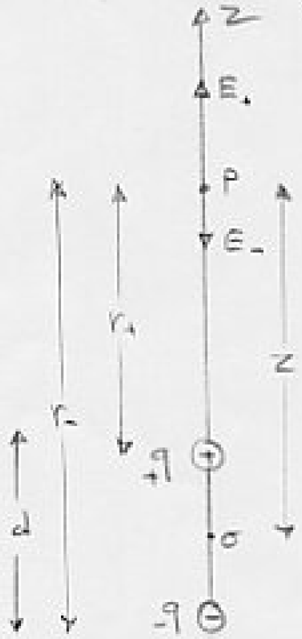
$$Q = Ze \qquad Z = 92 \qquad e = 1.60\times10^{-19}\,C$$

Acc. to first shell theorem; the nuclear charge can be assumed consentrated at the center of nucleus.

$$E = \frac{1}{4\pi\epsilon_0}\frac{Ze}{R^2} = 8.99\times10^{9}\ N\cdot m^2/C^2\ \frac{92\,(1.60\times10^{-19}\,C)}{(6.8\times10^{-15}\,m)^2} = 2.9\times10^{21}\ N/C$$



24-5) The electric field due to an Electric Dipole;



$$E = E_+ - E_- \quad \text{(magnitude)}$$

$$E = \frac{1}{4\pi\epsilon_0}\left(\frac{q}{r_+^2} - \frac{q}{r_-^2}\right)$$

$$E = \frac{q}{4\pi\epsilon_0}\left(\frac{1}{(z-\frac{d}{2})^2} - \frac{1}{(z+\frac{d}{2})^2}\right)$$

$$E = \frac{q}{4\pi\epsilon_0 z^2}\left[\left(1-\frac{d}{2z}\right)^{-2} - \left(1+\frac{d}{2z}\right)^{-2}\right]$$

$$E \simeq \frac{q}{4\pi\epsilon_0 z^2}\left[\left(1+\frac{d}{z}+\cdots\right) - \left(1-\frac{d}{z}+\cdots\right)\right]$$

for $z \gg d$

$\rightarrow \frac{d}{z} \ll 1$

where we have used;

$$(1 \pm x)^n = 1 \pm \frac{nx}{1!} + \frac{n(n-1)x^2}{2!} + \cdots \qquad |x|<1$$

$$E = \frac{q}{4\pi\epsilon_0 z^2}\,\frac{2d}{z} = \frac{1}{2\pi\epsilon_0}\,\frac{qd}{z^3}$$

Def.: $\quad p \equiv qd \quad$ electric dipole moment

$$E = \frac{1}{2\pi\epsilon_0}\,\frac{p}{z^3} \qquad \bar{E} = \frac{1}{2\pi\epsilon_0}\,\frac{\bar{p}}{z^3}$$

For distant points; $\quad E \sim \frac{1}{r^3}$

(whether or not the point lies on the dipole axis)

With increasing distance;

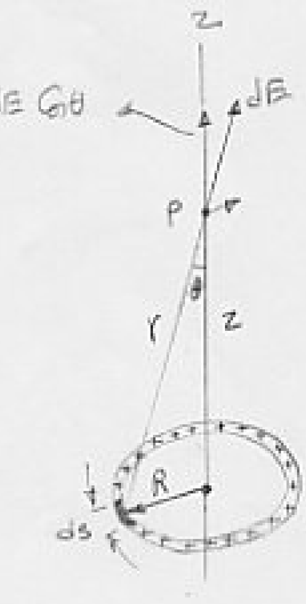
$$\underset{\sim 1/r^3}{E_{\text{dipole}} \xrightarrow{\text{more rapidly}} 0} \qquad \text{than} \qquad \underset{\sim \frac{1}{r^2}}{E_{\text{point charge}} \longrightarrow 0}$$

Because the presence of both <u>positive</u> and <u>negative</u> charges cancels out <u>almost</u> but not <u>quite</u> the effect of each other for <u>long</u> distances.

24-6) The electric field due to a <u>line</u> of Charge;

Discrete charge ⟶ continuous charge

Charge	q	C
Linear charge density	λ	C/m
Surface " "	σ	C/m^2
Volume " "	ρ	C/m^3



Ex. - Find the electric field of a ring of uniform positive charge at a point P, a distance z from the plane of the ring along its central axis.

<u>Sol.</u>

$dq = \lambda \, ds$

$$dE = \frac{1}{4\pi\epsilon_0}\frac{dq}{r^2} = \frac{1}{4\pi\epsilon_0}\frac{\lambda \, ds}{r^2} = \frac{1}{4\pi\epsilon_0}\frac{\lambda \, ds}{(z^2+R^2)}$$

The components of all dE (due all ds) perpendicular to z-axis cancels out each other.

The components of dE along z-axis are all in the same direction and we have to integrate them.

$$\cos\theta = \frac{z}{r} = \frac{z}{(z^2+R^2)^{1/2}}$$

$$dE\cos\theta = \frac{z\lambda}{4\pi\epsilon_0 (z^2+R^2)^{3/2}}\, ds$$

$$E = \int dE\cos\theta = \frac{z\lambda}{4\pi\epsilon_0 (z^2+R^2)^{3/2}} \int ds$$

$$\int ds = \int_0^{2\pi} R\,d\theta = R\int_0^{2\pi} d\theta = 2\pi R \longrightarrow E = \frac{z\lambda(2\pi R)}{4\pi\epsilon_0 (z^2+R^2)^{3/2}}$$

$$\lambda(2\pi R) = q \longrightarrow E = \frac{qz}{4\pi\epsilon_0 (z^2+R^2)^{3/2}}$$

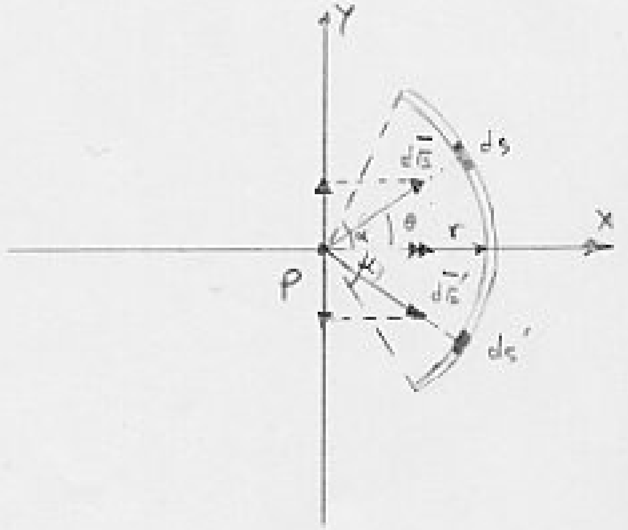
If $z \gg R \longrightarrow z^2+R^2 \simeq z^2 \longrightarrow E = \frac{1}{4\pi\epsilon_0}\frac{q}{z^2}$

$\longrightarrow$ Reasonable result (since at distant points the ring looks like a point.)

At $z=0 \longrightarrow E=0$ Reasonable

Ex. 24-5)

$-Q$: the total charge uniformaly distributed over the bent rod.



$\alpha = 60^\circ$

$\bar{E}_p = ?$

Sol.

$dq = \lambda ds \qquad dE = \frac{1}{4\pi\epsilon_0}\frac{dq}{r^2} = \frac{1}{4\pi\epsilon_0}\frac{\lambda ds}{r^2}$

$E_y = 0 \qquad$ (from the symmetry)

$dE_x = dE\cos\theta = \frac{1}{4\pi\epsilon_0}\frac{\lambda}{r^2}\cos\theta\, ds$

$ds = r\,d\theta \qquad E_x = \int dE_x = \int_{-60^\circ}^{60^\circ}\frac{1}{4\pi\epsilon_0}\frac{\lambda}{r^2}\cos\theta\, r\,d\theta$

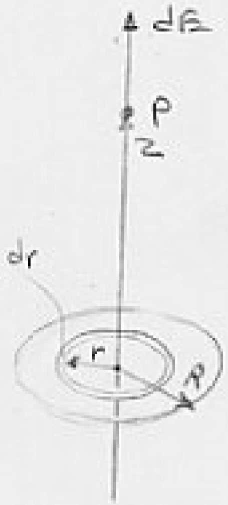
$E_x = \frac{\lambda}{4\pi\epsilon_0 r}\int_{-60^\circ}^{60^\circ}\cos\theta\, d\theta = \frac{\lambda}{4\pi\epsilon_0 r}\left[\sin\theta\right]_{-60^\circ}^{60^\circ}$

$E_x = \frac{\sqrt{3}\lambda}{4\pi\epsilon_0 r}$

$\lambda = \frac{Q}{\frac{2\pi r}{3}} = \frac{3Q}{2\pi r} \qquad E_x = \frac{3\sqrt{3}\,Q}{8\pi^2\epsilon_0 r^2} = \frac{3\sqrt{3}}{2\pi}\frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}$

$\bar{E} = E_x\hat{i} + 0\hat{j} + 0\hat{k}$

Ex. 24-7) The electric field due to a charged disk;



Sol.

$$dq = \sigma\, dA = \sigma(2\pi r)dr$$

$$dE = \frac{z(\sigma 2\pi r dr)}{4\pi\epsilon_0 (z^2+r^2)^{3/2}} = \frac{\sigma z}{4\epsilon_0} \frac{2r dr}{(z^2+r^2)^{3/2}}$$

$$E = \int dE = \frac{\sigma z}{4\epsilon_0} \int_0^R (z^2+r^2)^{-3/2} (2r)dr$$

$$\int u^m du = \frac{1}{m+1} u^{m+1} \qquad \begin{cases} u = (z^2+r^2) \\ du = 2r\,dr \\ m = -\frac{3}{2} \end{cases}$$

$$E = \frac{\sigma z}{4\epsilon_0} \left[\frac{(z^2+r^2)^{-1/2}}{-\frac{1}{2}} \right]_0^R = \frac{\sigma}{2\epsilon_0}\left(1 - \frac{z}{\sqrt{z^2+R^2}}\right)$$

$(z>0)$

If $R \to \infty$ (with z finite)

$$E \to \frac{\sigma}{2\epsilon_0}$$

Also, if $z \to 0$ (with R finite)

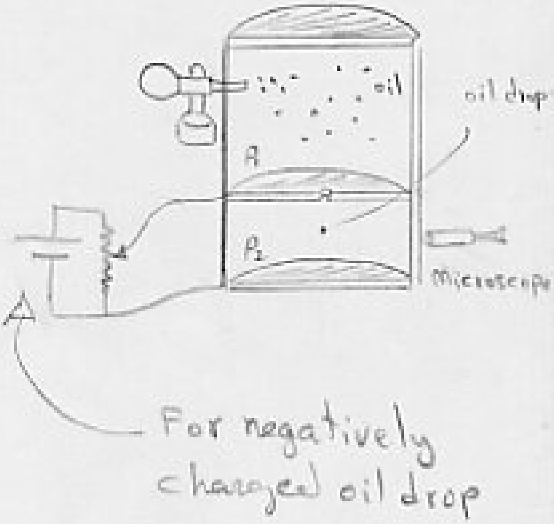
$$E \to \frac{\sigma}{2\epsilon_0}$$

24-8) A point charge in an electric field;

$$\bar{F} = q\bar{E}$$ $\bar{E}$: external field (not due to q)

Measuring the elementary charge;

1910-1913) Robert A. Millikan



By adjusting the electric field he made the oil drop stationary, so;

$$F_{grav} = F_{elec.}$$

$$mg = qE$$

$$\frac{4}{3}\pi R^3 \rho g = qE$$

$$\longrightarrow q = ne \quad , n = 0, \pm 1, \pm 2 \ldots$$

Ex. In the Millikan experiment;

$R = 2.76\ \mu m$ oil drop radius

Excess charge: $3e$ $\rho = 920\ kg/m^3$

$a = 0$ (stationary) $E = ?$

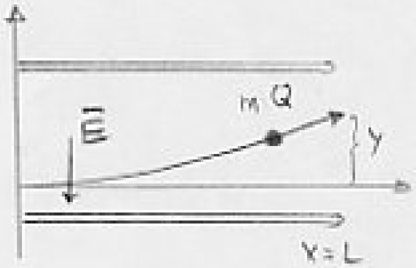
Sol.

$F_{grav.} = F_{coul.}$ $mg = qE$ $(\frac{4}{3}\pi R^3)\rho g = (3e)E$

$$E = \frac{4\pi R^3 \rho g}{9e} = \frac{(4\pi)(2.76\times10^{-6}\ m)^3(920\ kg/m^3)(9.8\ m/s^2)}{(9)(1.6\times10^{-19}\ C)} = 1.65\times10^{6}\ N/C$$

$$\overline{F} = -3e\overline{E}$$

Ex. Ink-jet printer;



$m = 1.3\times10^{-10}\ kg$ mass of ink drop

$Q = -1.5\times10^{-13}\ C$

$mg \approx 0$ negligable

$V_x = 18\ m/s$ $L = 1.6\ cm$ $E = 1.4\times10^{6}\ N/C$

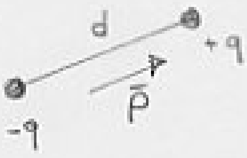
$Y = ?$

Sol.

$$\begin{cases} a_y = \frac{F}{m} = \frac{|Q|E}{m} \\ Y = \frac{1}{2}a_y t^2 \\ L = V_x t \end{cases} \rightarrow Y = \frac{|Q|EL^2}{2mV_x^2}$$

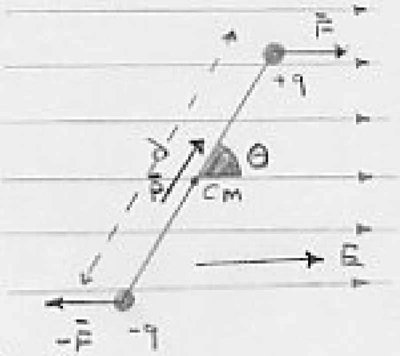
$$Y = \frac{(1.5\times10^{-13}\ C)(1.4\times10^{6}\ N/C)(1.6\times10^{-2}\ m)^2}{(2)(1.3\times10^{-10}\ kg)(18\ m/s)^2} = 6.4\times10^{-4}\ m = 0.64\ mm$$

24-9) A Dipole in an Electric Field;



$\Sigma F = 0$ on dipole

$\bar{\tau} = \bar{r} \times \bar{F}$ $|\bar{\tau}| = \frac{d}{2} F \sin\theta + \frac{d}{2} F \sin\theta$



$|\bar{\tau}| = F d \sin\theta$

$|\bar{\tau}| = (qE) d \sin\theta = E P \sin\theta$

$|\bar{\tau}| = PE \sin\theta \rightarrow \bar{\tau} = \bar{P} \times \bar{E}$

$\bar{\tau} \xrightarrow{\text{tends}}$ to rotate $\bar{P}$ in $\bar{E}$-dir (reducing θ)

Potential Energy:

Potential energy can be associated with the orientation of an electric dipole in the electric field.

Lowest potential energy; occurs in equilibrium orientation.

$\xrightarrow{\text{i.e.}}$ $\bar{\tau} = \bar{P} \times \bar{E} = 0$

Convention:

$\tau = rF\sin\alpha$ counterclockwise rot. (increasing θ)

$\tau = -rF\sin\alpha$ clockwise rot. (decreasing θ)

Zero-level of the pot. is arbitrary -

The difference in the potential energy has physical meaning -

For simplicity we choose;

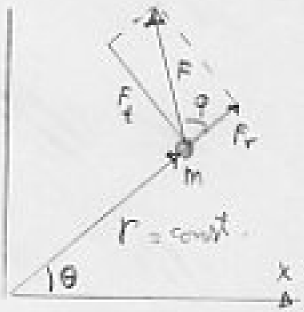
At $\theta = 90^\circ$ $\quad U = 0$

Since; $\Delta K + \Delta U = 0$ (cons. of energy)

$\Delta K = W$ work done by F or τ

$\Delta U = -W \qquad U - U_0 = -W$ $\quad (U_0 = 0$ at $\theta = 90)$

$U = -W \qquad W = \int F \cdot ds = \int F_t \underbrace{(r\,d\theta)}_{ds}$

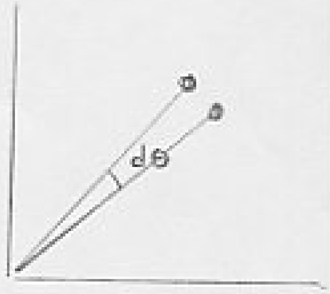


$\bar{\tau} = \bar{r} \times \bar{F} \qquad \tau = r F_t$

$W = \int \tau\, d\theta$

The field E tries to rotate P in such a way to decrease θ

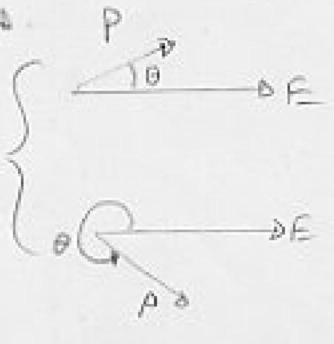
$$U = -W = -\int_{\pi/2}^{\theta} \tau\, d\theta = -\int_{\pi/2}^{\theta} -PE \sin\theta\, d\theta$$



$U = -PE\cos\theta \longrightarrow U = -\bar{P} \cdot \bar{E}$

At $\theta = 0$ $\quad U = -PE$ (least)

" $\theta = \pi$ $\quad U = PE$ (most)



<u>Ex.</u> H_2O-molecule;

$P = 6.2\times10^{-30}$ C.m

a) $d = ?$

$p = qd = (10e)d \quad d = \frac{P}{10e} = \frac{6.2\times10^{-30}\ C.m}{(10)(1.60\times10^{-19}\ C)} = 3.9\times10^{-12}$ m

b) $E = 1.5\times10^4$ N/C $\quad \tau_{max} = ?$

$\tau = PE\sin\theta = (6.2\times10^{-30}\ C.m)(1.5\times10^4\ N/C)(\sin\pi/2) = 9.3\times10^{-26}$ N.m

c) $W = ?$ by external agent to turn the molecule from $\theta = 0$ to $\theta = \pi$

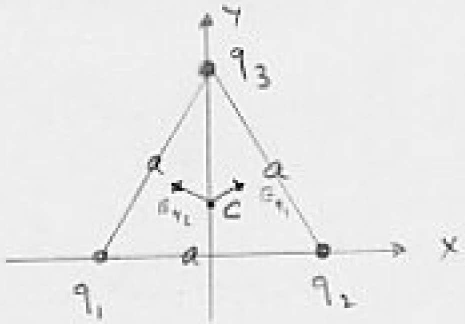
$W = U(\pi) - U(0) = (-PE\cos\pi) - (-PE\cos0) = 2PE$

$= (2)(6.2\times10^{-30}\ C.m)(1.5\times10^4\ N/C) = 1.9\times10^{-25}$ J

8E – $q_1 = q_2 = +1.0\,\mu C$

$q_3 = ?$ in such a way $E_c = 0$

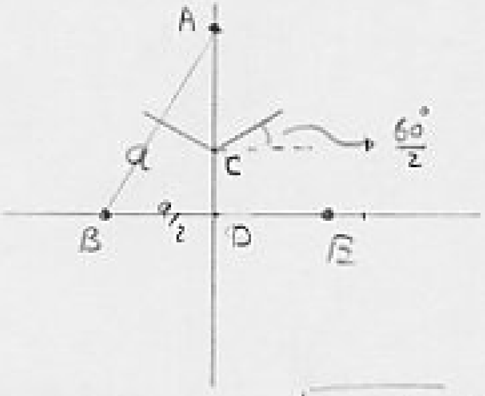
C: Center



Sol.

$$E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$$

$$E_{q_1} = \frac{1}{4\pi\epsilon_0}\frac{q_1}{(\frac{\sqrt{3}}{3}a)^2} \qquad E_{q_1 y} = E_{q_1}\sin 30^\circ = \frac{1}{4\pi\epsilon_0}\frac{3}{2}\frac{q_1}{a^2}$$



Similarly $E_{q_2 y} = \frac{1}{4\pi\epsilon_0}\frac{3}{2}\frac{q_2}{a^2}$

$$E_y = E_{q_1 y} + E_{q_2 y} = \frac{1}{4\pi\epsilon_0}\frac{3}{a^2}(1\times10^{-6})$$

$E_x = 0$

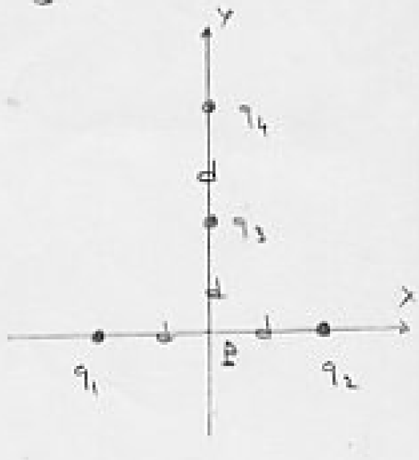
In $\overset{\triangle}{ABD}$: $AD = \sqrt{a^2 - (\frac{a^2}{4})} = \frac{\sqrt{3}}{2}a$

$AC = BC = EC = \frac{2}{3}AD$

$= \frac{\sqrt{3}}{3}a$

$$E_{q_3} = E_y \qquad \frac{1}{4\pi\epsilon_0}\frac{q_3}{(\frac{\sqrt{3}}{3}a)^2} = \frac{1}{4\pi\epsilon_0}\frac{3}{a^2}(1\times10^{-6})$$

$q_3 = 1\times10^{-6}\,C$



18P – $q_1 = q_2 = +5.0q$,

$q_3 = +3.0q$, $q_4 = -12q$ $\quad E_p = ?$

Sol.

$\bar{E}_{q_1} = -\bar{E}_{q_2} \rightarrow \bar{E}_{q_1} + \bar{E}_{q_2} = 0$

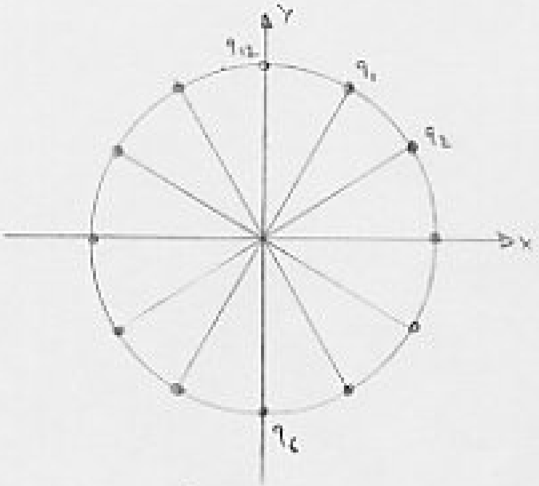
$$|E_{q_3}| = \frac{1}{4\pi\epsilon_0}\frac{3q}{d^2} \qquad |E_{q_4}| = \frac{1}{4\pi\epsilon_0}\frac{12q}{(2d)^2} = \frac{1}{4\pi\epsilon_0}\frac{3q}{d^2}$$

But since they are in opposite-dir, they cancel out eachother.

$E_p = 0$

14P- $\quad q_n = -nq$

At what time hour hand and the electric field are in the same dir?

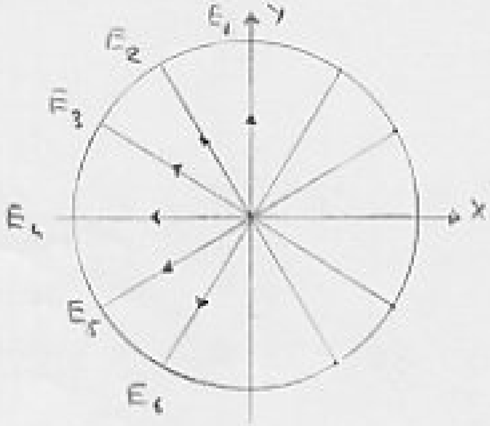


Clock

Sol.

$|E_1| = |E_2| = |E_3| = |E_4| = |E_5| = |E_6| = E$

$E = \frac{1}{4\pi\epsilon_0}\frac{(6q)}{r^2}$



$E_x = E\left(\cos\frac{\pi}{2} + \cos\left(\frac{\pi}{2}+\frac{\pi}{6}\right) + \cos\left(\frac{\pi}{2}+\frac{2\pi}{6}\right) + \right.$

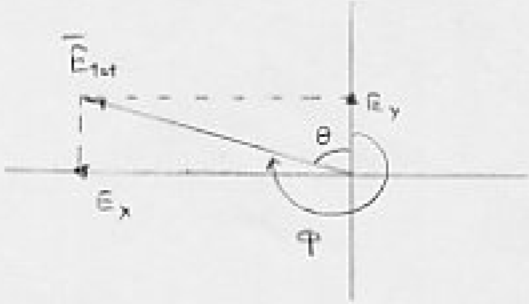
$\left. + \cos\left(\frac{\pi}{2}+\frac{3\pi}{6}\right) + \cos\left(\frac{\pi}{2}+\frac{4\pi}{6}\right) + \cos\left(\frac{\pi}{2}+\frac{5\pi}{6}\right)\right)$

$= -3.732\,E$

$E_y = E\left(\sin\frac{\pi}{2} + \sin\left(\frac{\pi}{2}+\frac{\pi}{6}\right) + \sin\left(\frac{\pi}{2}+\frac{2\pi}{6}\right) + \sin\left(\frac{\pi}{2}+\frac{3\pi}{6}\right) + \sin\left(\frac{\pi}{2}+\frac{4\pi}{6}\right)\right.$

$\left. + \sin\left(\frac{\pi}{2}+\frac{5\pi}{6}\right)\right) = +E$

$\overline{E}_{tot} = (-3.732\,\hat{i} + \hat{j})E$



$\tan\theta = \left|\frac{E_x}{E_y}\right| = 3.732 \rightarrow \theta = 75^\circ$

$\varphi = 360^\circ - 75^\circ = 285^\circ$

$\theta = 30^\circ \xrightarrow{\text{Corresponds}} 1$ o'clock

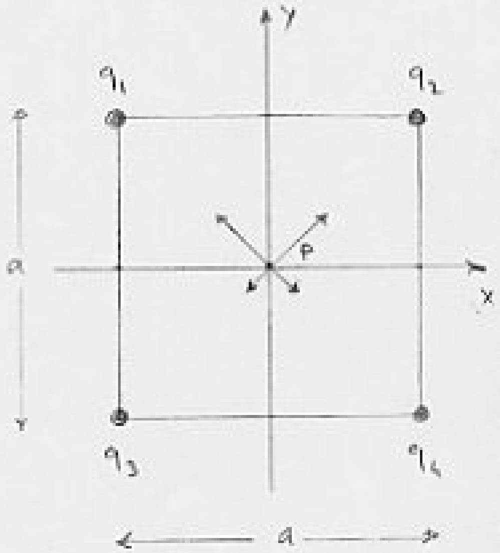
$\varphi = 285^\circ \qquad\qquad t = 9.5$

22P - $q_1 = +q$, $q_2 = -2q$

$q_3 = -q$, $q_4 = 2q$

$q = 1\times10^{-8}\,C$ $a = 5\,cm$

$E_p = ?$



sol.

From the symmetry; $E_x = 0$

$$|E_1| = |E_3| = \frac{1}{4\pi\epsilon_0}\frac{q}{(\frac{\sqrt{2}}{2}a)^2} = \frac{1}{4\pi\epsilon_0}\frac{2q}{a^2}$$

$$|E_2| = |E_4| = \frac{1}{4\pi\epsilon_0}\frac{2q}{(\frac{\sqrt{2}}{2}a)^2} = \frac{1}{4\pi\epsilon_0}\frac{4q}{a^2}$$

$$E_y = 2|E_2|\cos 45 - 2|E_1|\cos 45 = 2\,\frac{\sqrt{2}}{2}\,\frac{1}{4\pi\epsilon_0}\frac{2q}{a^2}(2-1)$$

$$E_y = 2\sqrt{2}\,(8.99\times10^{9})\,\frac{1\times10^{-8}}{(0.05)^2} = 101710\ N/C$$

25E - Show that $E = \frac{1}{4\pi\epsilon_0}\frac{2q}{z^2}$ $(z \gg d)$

sol.

$$E = E_1 + E_2 = \frac{q}{4\pi\epsilon_0}\left(\frac{1}{r_1^2} + \frac{1}{r_2^2}\right)$$

$$E = \frac{q}{4\pi\epsilon_0}\left(\frac{1}{(z-\frac{d}{2})^2} + \frac{1}{(z+\frac{d}{2})^2}\right)$$

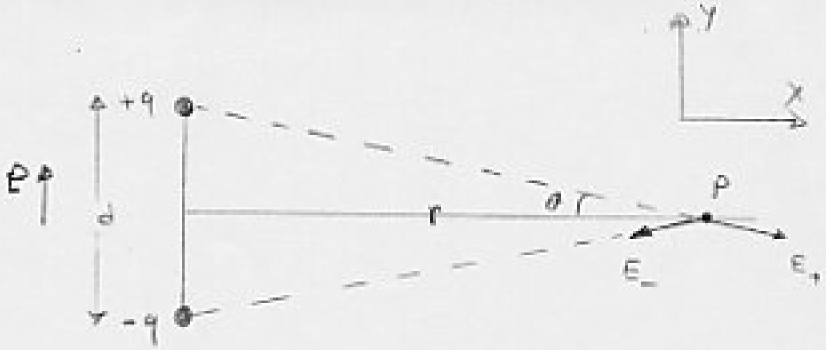
$$E = \frac{q}{4\pi\epsilon_0}\frac{1}{z^2}\left(\left(1-\frac{d}{2z}\right)^{-2} + \left(1+\frac{d}{2z}\right)^{-2}\right)$$

$$E = \frac{q}{4\pi\epsilon_0}\frac{1}{z^2}\left(\left(1+\frac{d}{2z}+\cdots\right) + \left(1-\frac{d}{2z}+\cdots\right)\right)$$

$$E = \frac{1}{4\pi\epsilon_0}\frac{2q}{z^2} \quad \text{for } z \gg d$$

26P — $\overline{E}_p = f(\overline{p}) = ?$

$r \gg d$



<u>Sol.</u>

From the symmetry; $E_x = 0$

$$|E_-| = |E_+| = E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$$

$$E_p = 2E\sin\theta = \frac{1}{4\pi\epsilon_0}\frac{2q}{r^2}\frac{\frac{d}{2}}{r} = \frac{1}{4\pi\epsilon_0}\frac{qd}{r^3} = \frac{1}{4\pi\epsilon_0}\frac{p}{r^3}$$

$$\overline{E}_p = \frac{-1}{4\pi\epsilon_0}\frac{\overline{p}}{r^3}$$

27P — Electric quadrupole;

show that $E = \dfrac{3Q}{4\pi\epsilon_0 z^4}$ $\quad (Q = 2qd^2)$

for $z \gg d$

<u>Sol.</u>

$$E = E_+ + E_+' - E_-$$

$$E = \frac{1}{4\pi\epsilon_0}\left[\frac{q}{(z-d)^2} + \frac{q}{(z+d)^2} - \frac{2q}{z^2}\right]$$

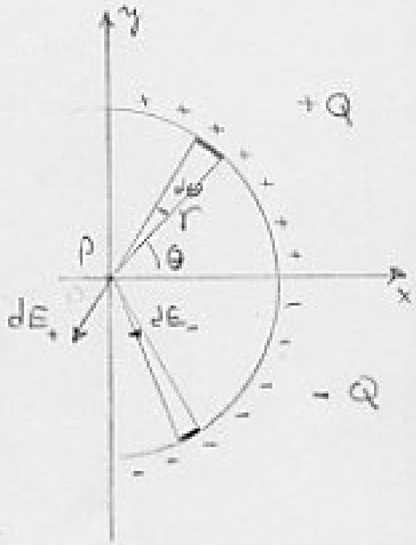
$$E = \frac{q}{4\pi\epsilon_0}\frac{1}{z^2}\left[\left(1-\frac{d}{z}\right)^{-2} + \left(1+\frac{d}{z}\right)^{-2} - 2\right]$$

$$E = \frac{q}{4\pi\epsilon_0}\frac{1}{z^2}\left[\left(1+\frac{2d}{z}+\frac{3d^2}{z^2}+\dots\right) + \left(1-\frac{2d}{z}+\frac{3d^2}{z^2}+\dots\right) - 2\right]$$

$$E = \frac{q}{4\pi\epsilon_0}\frac{1}{z^2}\left(2\,\frac{3d^2}{z^2}\right) = \frac{3Q}{4\pi\epsilon_0 z^4}$$

X

32P— $\bar{E}_p = ?$



Sol.

From the symmetry; $E_x = 0$

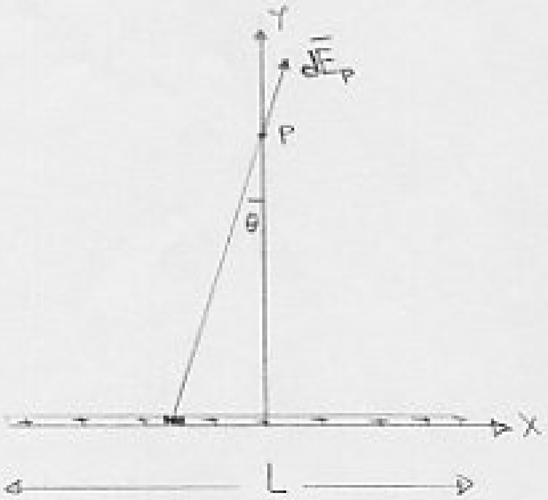
$|dE_+| = |dE_-|$ $\qquad dE_{+y} = \frac{1}{4\pi\epsilon_0}\frac{dq}{r^2} S\theta$

$$|E_y| = 2\int \frac{1}{4\pi\epsilon_0}\frac{dq}{r^2} S\theta = \frac{2}{4\pi\epsilon_0}\int \frac{\lambda(r d\theta)}{r^2} S\theta$$

$$|E_y| = \frac{2}{4\pi\epsilon_0}\frac{\lambda}{r}\int_0^{\pi/2} S\theta \, d\theta = \frac{2}{4\pi\epsilon_0}\frac{\lambda}{r}\left[-C\theta\right]_0^{\pi/2} = \frac{2}{4\pi\epsilon_0}\frac{\lambda}{r}$$

$$\lambda = \frac{Q}{\frac{2\pi r}{4}} = \frac{2Q}{\pi r} \qquad |E|_y = \frac{4}{4\pi\epsilon_0}\frac{Q}{\pi r^2}$$

$$\bar{E} = -\frac{4}{4\pi\epsilon_0}\frac{Q}{\pi r^2}\hat{j}$$



33P— Show that;

$$E = \frac{q}{2\pi\epsilon_0 Y}\frac{1}{(L^2+4Y^2)^{1/2}}$$

Sol.

From the symmetry; $E_x = 0$

$dE = \frac{1}{4\pi\epsilon_0}\frac{dq}{r^2}$ $\qquad dq = \lambda dx$

$dE_y = dE\, C\theta$ $\qquad E_y = \int dE\, C\theta = \frac{1}{4\pi\epsilon_0}\int \frac{\lambda dx}{(x^2+y^2)} C\theta$

$$E = \frac{\lambda}{4\pi\epsilon_0}\int \frac{dx}{(x^2+y^2)}\left(\frac{y}{\sqrt{x^2+y^2}}\right) = \frac{\lambda}{4\pi\epsilon_0} y \int_{-L/2}^{L/2} \frac{dx}{(x^2+y^2)^{3/2}}$$

But $\displaystyle\int \frac{dx}{(x^2 \pm a^2)^{3/2}} = \frac{\pm x}{a^2 (x^2 \pm a^2)^{1/2}}$

$$\longrightarrow E = \frac{\lambda}{4\pi\epsilon_0} y \left[\frac{x}{y^2 (x^2+y^2)^{1/2}} \right]_{-\frac{L}{2}}^{+\frac{L}{2}} = \frac{\lambda}{4\pi\epsilon_0} \frac{1}{y} \left[\frac{\frac{L}{2}}{\sqrt{\frac{L^2}{4}+y^2}} - \frac{-\frac{L}{2}}{\sqrt{\frac{L^2}{4}+y^2}} \right]$$

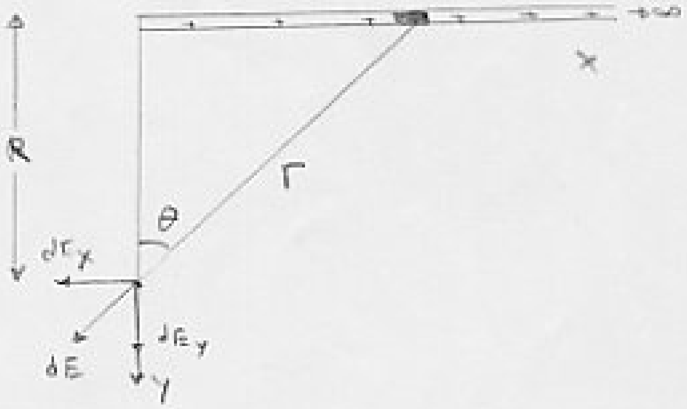
$$\lambda = \frac{q}{L} \rightarrow E = \frac{1}{4\pi\epsilon_0} \frac{q}{Ly} \left(\frac{L}{\sqrt{\frac{L^2}{4}+y^2}} \right) = \frac{1}{4\pi\epsilon_0} \frac{q}{Ly} \frac{2L}{\sqrt{L^2+4y^2}}$$

$$E = \frac{q}{4\pi\epsilon_0 y} \frac{1}{\sqrt{L^2+4y^2}}$$

x35P— Semi-infinite nonconducting rod, with charge density λ;

$\bar{E}_p = ?$

and show that $\bar{E}$ at P makes an angle 45° with the rod independent of R.



Sol.

$$dE = \frac{1}{4\pi\epsilon_0} \frac{dq}{r^2}$$

$r^2 = R^2 + x^2 \quad dq = \lambda dx$

$$|dE_y| = \frac{1}{4\pi\epsilon_0} \frac{dq}{r^2} C\theta$$

$$|dE_x| = \frac{1}{4\pi\epsilon_0} \frac{dq}{r^2} S\theta$$

Also $\begin{cases} C\theta = \dfrac{R}{(R^2+x^2)^{1/2}} \\ S\theta = \dfrac{x}{(R^2+x^2)^{1/2}} \end{cases}$

$$E_x = \frac{1}{4\pi\epsilon_0} \int_0^\infty \frac{\lambda x \, dx}{(R^2+x^2)^{3/2}} \qquad E_y = \frac{1}{4\pi\epsilon_0} \int_0^\infty \frac{\lambda R \, dx}{(R^2+x^2)^{3/2}}$$

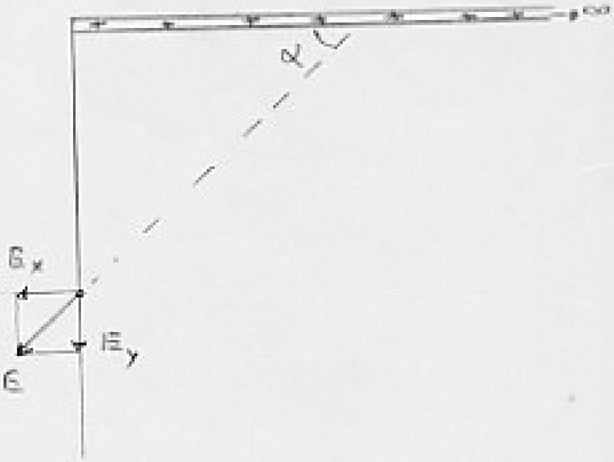
Since; $\int \frac{dx}{(x^2 \pm a^2)^{3/2}} = \frac{\pm x}{a^2\sqrt{x^2 \pm a^2}}$

and $\int \frac{x\,dx}{(x^2 \pm a^2)^{3/2}} = \frac{-1}{\sqrt{x^2 \pm a^2}}$

$$E_x = \frac{\lambda}{4\pi\epsilon_0}\left[\frac{-1}{\sqrt{x^2+R^2}}\right]_0^\infty = \frac{\lambda}{4\pi\epsilon_0}\left[0 + \frac{1}{R}\right] = \frac{1}{4\pi\epsilon_0}\frac{\lambda}{R}$$

$$E_y = \frac{\lambda R}{4\pi\epsilon_0}\left[\frac{x}{R^2\sqrt{x^2+R^2}}\right]_0^\infty = \frac{\lambda R}{4\pi\epsilon_0}\left[\frac{1}{R^2} - 0\right] = \frac{1}{4\pi\epsilon_0}\frac{\lambda}{R}$$

$$\bar{E} = \frac{1}{4\pi\epsilon_0}\frac{\lambda}{R}\left(-\hat{i} - \hat{j}\right)$$



$\tan\alpha = \frac{|E_y|}{|E_x|} = 1 \rightarrow \alpha = 45^\circ$

α indep of R