Dr. M.H. Shahnas

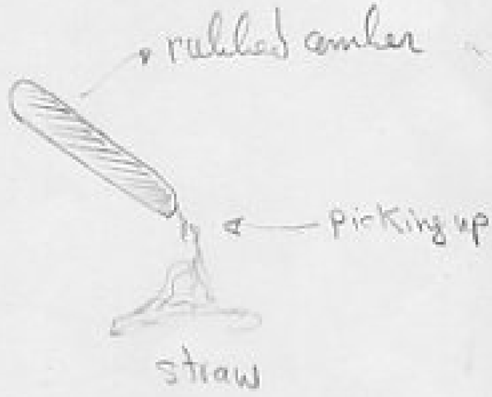
Physics-II

Useful Book : Fundamentals of Physics
David Halliday
& Robert Resnik

Chap 23

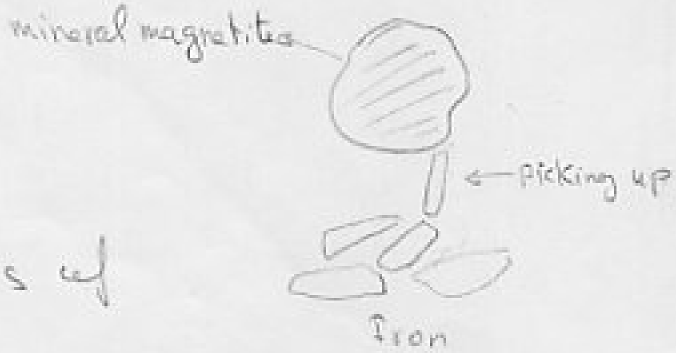
23-1 Electromagnetism:

electron = amber (Greek word)



i) Amber, picks up bits of straw.

ii) Mineral magnetite " " iron.



These are the origin of the sciences of electricity and magnetism.

1820: Hans Christian Oersted found a connection between them: An electric current in a wire can deflect a magnetic compass needle.

Michael Faraday, developed the science of electromagnetism

James Clerk Maxwell (mid of 19th century):

put Faraday's idea into mathmatical form.

23-2 Electric Charge:

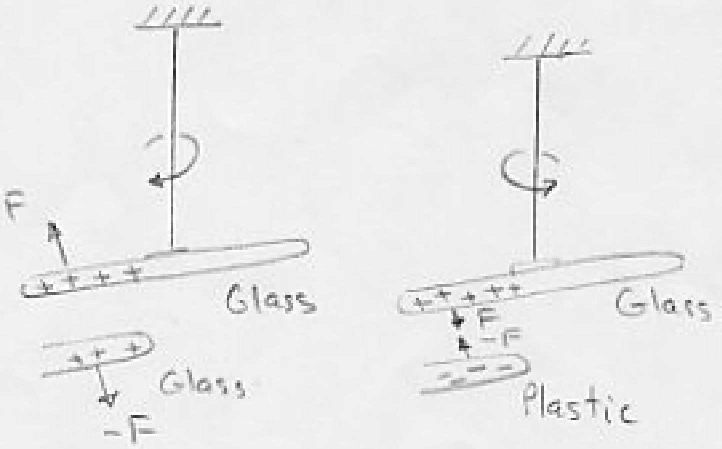
Every object in our life contains electric charge.

Two kinds of charge exist $\begin{cases} \text{negative charge} \\ \text{positive} \quad " \end{cases}$

Generally they are in balance (equal amount of each other). Such an object is called to be electrically neutral, and doesn't interact with the other objects.

If the two types of charges are not in balance, then there is a net charge, that can interact with other objects.

Interaction of charged objects:



Positive object: Glass rubbed with silk

Negative " : Plastic " " fur

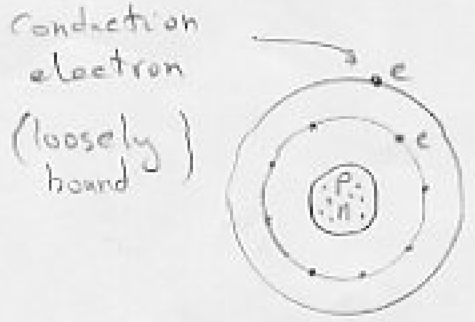
The negative and positive lables and signs for electric charge were chosen arbitrarily by Benjamin Franklin.

Result: Like charges repel each other, and unlike charges attract each other.

23-3 Conductors and Insulators:

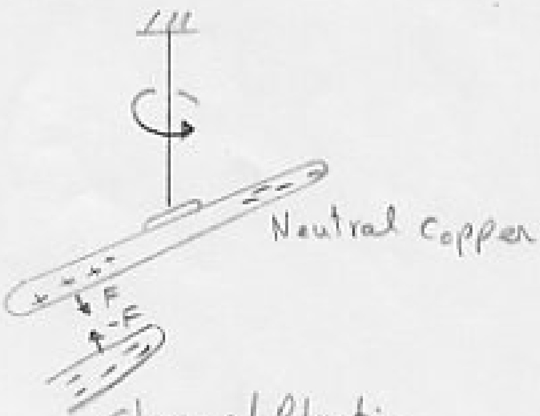
Conductor: Negative charge can move rather freely in this materials.

Insulators: None of charge can move freely in this materials.



Conductor

Experiment (the mobility of charge):



Charged Plastic

Semiconductors:

conductivity →

Insulators < Semiconductors < conductors

Superconductors:

They present no resistance to the movement of electric charge through them. $R = 0$ (Precisely)

Superconductivity was discovered in 1911 by Dutch Phisicist Kammerlingh Onnes

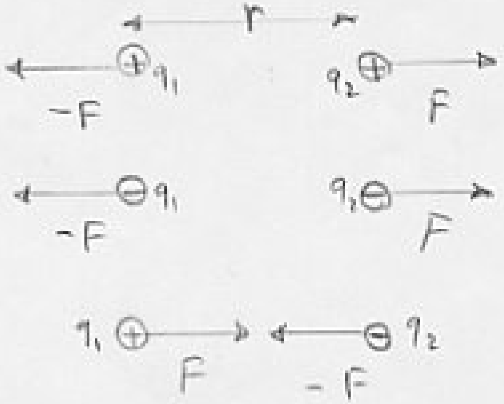
23-4 Coulomb's Law (Charles Augustus Coulomb) 1785

$$F = k\,\frac{q_1 q_2}{r^2}$$

q_1, q_2 : Point charges

F : Electrostatic Force

K : " const.



Def. - One Coulomb is the amount of charge that is transfered through the cross-section of a wire in 1 second when there is a current of 1 ampere in the wire.

In SI unit: charge → Coulomb

Current → Ampere

We will see later: $dq = i\,dt$

$$F = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2} \qquad \frac{1}{4\pi\epsilon_0} = 8.99 \times 10^{9}\ N \cdot m^2/C^2$$

$\epsilon_0 = 8.85 \times 10^{-12}\ C^2/(N \cdot m^2)$ permitivity Const.

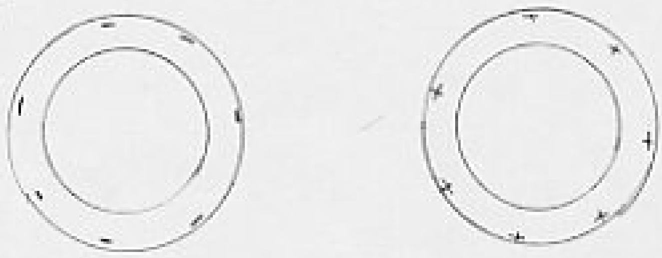
Superposition: $F_1 = F_{12} + F_{13} + F_{14} + \cdots + F_{1n}$

Shell theorem 1: A shell of uniform charge attracts or repels a charged particle that is outside the shell as if all the shell's charge were concentrated at its center.

Shell theorem 2: A shell of uniform charge exerts no electrostatic force on a charged particle that is located inside it.

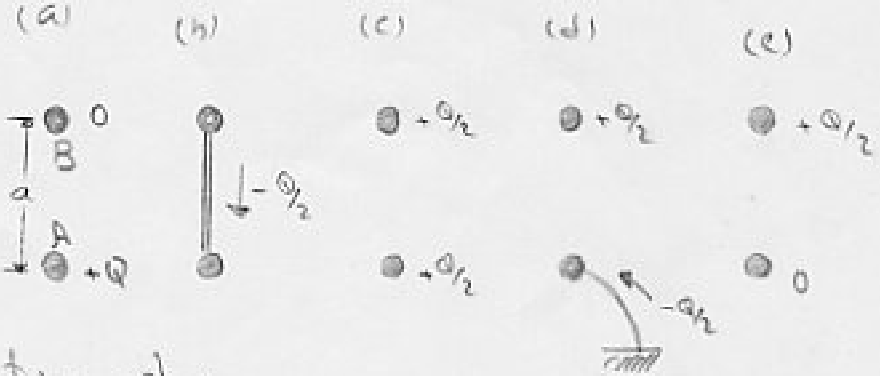
Spherical Conductors:

Exess negative (or positive) charge spreads uniformly over the external surface of shell.



The arrangement maximizes the distances between all pairs of the exess charges.

Ex. 23-1)



A, B: Two small conducting spheres.

(a) Charge of A = +Q   Charge of B = 0

i) A and B are connected by a thin conducting wire (Fig.b). What is the electrostatic force between the conductors after the wire is removed (Fig. C)?

Sol. The configration becomes as like as (Fig.C). Since a >> dimensions of sphere $\longrightarrow$ The charge on either sphere does not disturb the uniformity of distribution on the other sphere.

Thus $\longrightarrow$ We can apply first shell theorem.

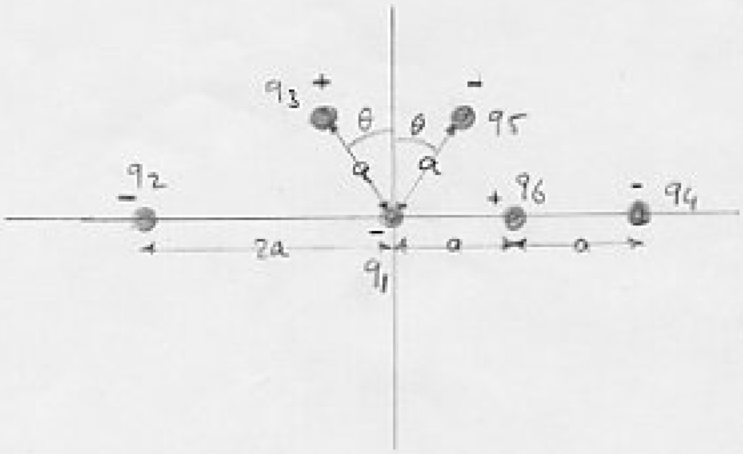
$$F = \frac{1}{4\pi\epsilon_0}\frac{(+Q/2)(+Q/2)}{a^2} = \frac{1}{16\pi\epsilon_0}\left(\frac{Q}{a}\right)^2 \quad \text{repulsive force}$$

ii) Next, suppose sphere A is grounded (Fig. d) and then the ground connection is removed (Fig. e). What now is the electostatic force?

Sol.

$$F = \frac{1}{4\pi\epsilon_0}\frac{(0)(+Q/2)}{a^2} = 0$$

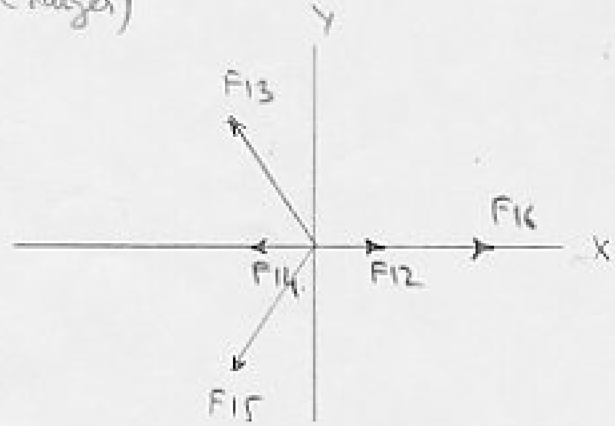
Ex. 23-2)



$a = 2.0\,\text{cm} \qquad \theta = 30°$

$|q| = 3.0\times10^{-6}\,\text{C}$

$F_1 = ?$ (acting on q_1 due to the other charges)



$$|F_{12}| = \frac{1}{4\pi\epsilon_0}\frac{|q_1||q_2|}{(2a)^2} = \frac{1}{4\pi\epsilon_0}\frac{q^2}{(2a)^2} \equiv A$$

$$|F_{14}| = \frac{1}{4\pi\epsilon_0}\frac{|q_1||q_4|}{(2a)^2} = \frac{1}{4\pi\epsilon_0}\frac{q^2}{(2a)^2} \equiv A$$

$$|F_{13}| = \frac{1}{4\pi\epsilon_0}\frac{|q_1||q_3|}{a^2} = \frac{1}{4\pi\epsilon_0}\frac{q^2}{a^2} = B$$

$$|F_{15}| = \frac{1}{4\pi\epsilon_0}\frac{|q_1||q_5|}{a^2} = \frac{1}{4\pi\epsilon_0}\frac{q^2}{a^2} = B$$

$$|F_{16}| = \frac{1}{4\pi\epsilon_0}\frac{|q_1||q_6|}{a^2} = \frac{1}{4\pi\epsilon_0}\frac{q^2}{a^2} = B$$

$$\begin{cases} F_{12x} = A \\ F_{12y} = 0 \end{cases} \quad \begin{cases} F_{14x} = -A \\ F_{14y} = 0 \end{cases} \quad \begin{cases} F_{13x} = -B\cos 60^\circ \\ F_{13y} = B\cos 30^\circ \end{cases}$$

$$\begin{cases} F_{15x} = -B\cos 60^\circ \\ F_{15y} = -B\cos 30^\circ \end{cases} \quad \begin{cases} F_{16x} = B \\ F_{16y} = 0 \end{cases}$$

$$F_{1x} = F_{12x} + F_{14x} + F_{13x} + F_{15x} + F_{16x} = 0$$

$$F_{1y} = F_{12y} + F_{14y} + F_{13y} + F_{15y} + F_{16y} = 0$$

23-5 Charge is quantized

Experiment shows that charge is quantized.

$$q = ne \qquad n = \pm 1, \pm 2, \cdots \qquad e = 1.60 \times 10^{-19}\,C$$

elementary charge

Ex. 23-3)

What is the magnitude q of the total positive (or negative) charge in a copper penny of mass $m = 3.11\,gr$?

Sol.

$$\begin{cases} q_+ = Z(+e) \\ q_- = Z(-e) \end{cases} \qquad Z_{Cu} = 29$$

$N_A = 6.02 \times 10^{23}$ Atoms/mol The number of atoms in a mole

$\frac{m}{M}$ The number of moles of copper in the penny

$$N = N_A \frac{m}{M} = 6.02\times10^{23} \text{ Atoms/mol} \; \frac{3.11 \text{ gr}}{63.5 \text{ gr/mol}}$$

$M = 63.5$ gr/mol molar mass of copper

$N = 2.95 \times 10^{22}$ Atoms

$$q = NZe = (2.95\times10^{22})(29)(1.60\times10^{-19} C) = 1.37\times10^{5} C$$

Ex. 23-4) In prob. 23-3, if each kind of charges could be concentrated into two separate bundles, 100 m apart, what attractive force would act on each bundle?

Sol.

$$F = \frac{1}{4\pi\epsilon_0}\frac{(q)(-q)}{r^2} = \frac{(8.99\times10^{9}\, N.m^2/C^2)(-1)(1.37\times10^{5}C)^2}{(100m)^2}$$

$$= -1.69\times10^{16} N \quad \text{(huge)}$$

There is also another point:
Such a separation of charges and collecting each kind in a small bundle is a nonsense assumption, because of very strong repulsive forces.

Ex. 23-5)

$r = 5.3\times10^{-11}$ m The average distance between electron and proton in Hydrogen atom.

a) $F_{elec} = ?$ b) $F_{grav} = ?$

Sol.

$$F_{elec} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2} = 8.99\times10^{9}\ N.m^2/C^2\ \frac{(-1)(1.6\times10^{-19}C)^2}{(5.3\times10^{-11})^2}$$

$$= -8.2\times10^{-8}\ N$$

$$F_{grav} = G\frac{m_1 m_2}{r^2} = 6.67\times10^{-11}\ \underbrace{m^3/(kg.s^2)}_{N.m^2/kg^2}\ \frac{(9.11\times10^{-31}kg)(1.67\times10^{-27}kg)}{(5.3\times10^{-11}m)^2}$$

$$= 3.6\times10^{-47}\ N$$

Ex. 23-6)

$r_{Fe} = 4.0\times10^{-15}$ m radius of iron nucleus ($z=26$)

$F_{elect\,(p\text{-}p)} = ?$ the force btween two protons

(if $r_{p\text{-}p} = 4.0\times10^{-15}$ m)

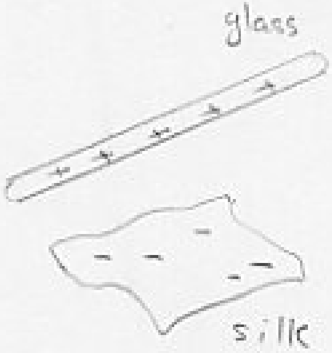
$$F_{elec} = \frac{1}{4\pi\epsilon_0}\frac{q_p q_p}{r^2} = 8.99\times10^{9}\ N.m^2/C^2\ \frac{(1.60\times10^{-19}C)^2}{(4.0\times10^{-15}m)^2}$$

$= 14$ N (an enormous repulsive force on P)

But there is another stronger attractive force present namely <u>nuclear force</u>.

23-6 Charge is Conserved

Benjamin Franklin: Charge conservation



Ex. - Rubbed glass with silk

After rubbing a glass rod with silk equal amount of charges with opposite signs appear on glass and silk.

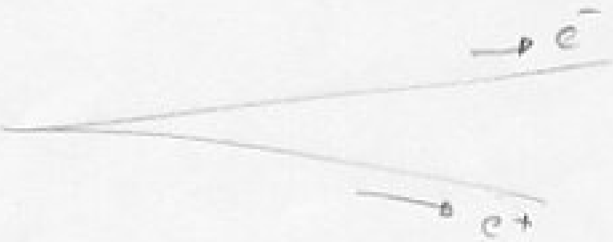
Ex. -

$$^{238}_{92}U \longrightarrow {}^{234}_{90}Th + {}^{4}_{2}He$$

Ex. -

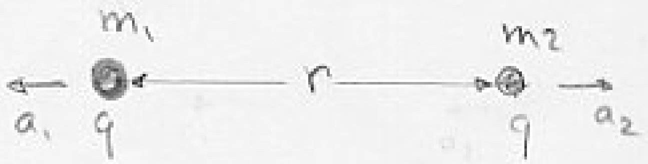
$$e^- + e^+ \longrightarrow \gamma + \gamma \quad \text{annihilation}$$

Ex. - $\gamma \longrightarrow e^+ + e^-$ Pair Production



5E – Two equally charged particles, held 3.2×10^{-3} m apart, are released from rest. The initial acceleration of the first particle is observed to be $7.0\,m/s^2$ and that of the second to be $9.0\,m/s^2$. If the mass of the first particle is 6.3×10^{-7} kg, what are (a) the mass of the second particle and (b) the magnitude of the common charge?

Sol.



$r = 3.2\times10^{-3}$ m

$m_1 = 6.3\times10^{-7}$ kg

$m_2 = ?$

$a_1 = 7\,m/s^2$

$a_2 = 9\,m/s^2$

$q = ?$

$$\begin{cases} F = \dfrac{1}{4\pi\epsilon_0}\dfrac{q_1 q_2}{r^2} = 8.99\times10^{9}\,N.m^2/C^2\,\dfrac{q^2}{(3.2\times10^{-3})^2} \\ F = m_1 a_1 = 6.3\times10^{-7}\,kg \times 7\,m/s^2 = 4.41\times10^{-6}\,N \\ F = m_2 a_2 = m_2 \times 9\,m/s^2 \end{cases}$$

$q = 7.09\times10^{-11}$ C

$m_2 = 4.9\times10^{-7}$ kg

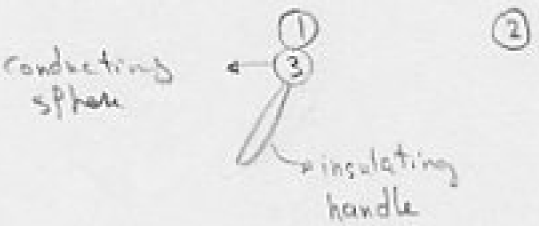
7E –

a) $q_1 = q_2 = q$

1,2: conducting spheres

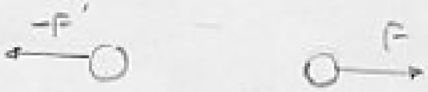
b) $q_3 = 0 \quad q_1' = ?$

$q_3' = ?$



c) $q_2 = ?$

d) $F' = ?$



Sol.

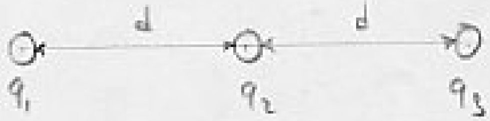
$$F = \frac{1}{4\pi\epsilon_0}\frac{q^2}{r^2}$$

$$q_1' = \frac{q}{2} \quad , q_3 = 0 \qquad q_1' = q_3' = \frac{q}{2} \qquad q_2' = q_3'' = \frac{1}{2}\left(q + \frac{q}{2}\right) = \frac{3}{4}q$$

$$F' = \frac{1}{4\pi\epsilon_0}\frac{q_1' q_2'}{r^2} = \frac{1}{4\pi\epsilon_0}\frac{\frac{3}{8}q^2}{r^2} \qquad F' = \frac{3}{8}F$$

8P –

q_1, q_2 are held fixed.



If $F_3 = 0$ (i.e. to be stationary) then find $q_1 = f(q_2)$

Sol. $\quad F_{31} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_3}{r_{31}^2} \qquad F_{32} = \frac{1}{4\pi\epsilon_0}\frac{q_2 q_3}{r_{32}^2}$

$$|F_{31}| = |F_{32}| \longrightarrow \frac{|q_1|}{r_{31}^2} = \frac{|q_2|}{r_{32}^2}$$

$$|q_1| = |q_2| \frac{r_{31}^2}{r_{32}^2} = |q_2| \frac{(2d)^2}{d^2} = 4|q_2|$$

$$q_1 = -4q_2$$

9P —

$F_Q = 0 \quad q_1 = f(q_2)$?

$F_{Q'} = 0 \quad q_1 = f(q_2)$?

Sol.

$$F_Q = 0 \longrightarrow |F_{Qq_1}| = F_{Qq_2}| \longrightarrow \frac{1}{4\pi\epsilon_0} \frac{|Q||q_1|}{r_{Qq_1}^2} = \frac{1}{4\pi\epsilon_0} \frac{|Q||q_2|}{r_{Qq_2}^2}$$

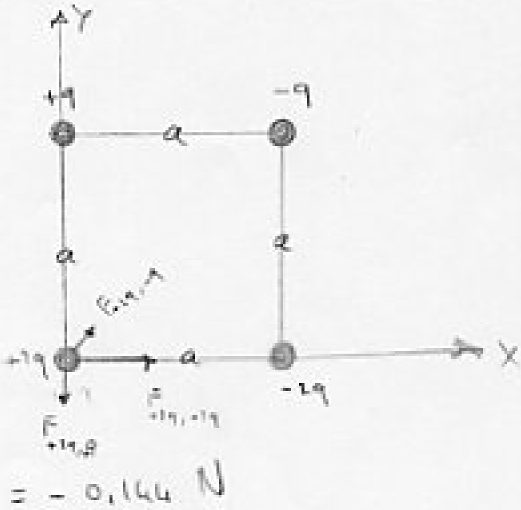
$$|q_1| = |q_2| \frac{r_{Qq_1}^2}{r_{Qq_2}^2} = |q_2| \frac{(\frac{3}{2}a)^2}{(\frac{a}{2})^2} \qquad |q_1| = 9|q_2| \Rightarrow q_1 = 9q_2$$

$$F_{Q'} = 0 \longrightarrow |F_{Q'q_1}| = |F_{Q'q_2}| \longrightarrow \frac{1}{4\pi\epsilon_0} \frac{|Q'||q_1|}{r_{Q'q_1}^2} = \frac{1}{4\pi\epsilon_0} \frac{|Q'||q_2|}{r_{Q'q_1}^2}$$

$$|q_1| = |q_2| \frac{r_{Q'q_1}^2}{r_{Q'q_2}^2} = |q_2| \frac{(\frac{5}{2}a)^2}{(\frac{1}{2}a)^2} \qquad |q_1| = 25|q_2| \Rightarrow q_1 = -25q_2$$

10P — $q = 1.0 \times 10^{-7}\,C \quad a = 5.\,cm$

$F_{+2qx} = ? \quad F_{+2qy} = ?$



Sol.

$$F_{+2q,-2q} = \frac{1}{4\pi\epsilon_0} \frac{(+2q)(-2q)}{a^2} = 8.99 \times 10^9 \frac{(-1)(2 \times 10^{-7})^2}{(5 \times 10^{-2})^2} = -0.144\ N$$

$$F_{+2q,+q} = \frac{1}{4\pi\epsilon_0}\frac{(+2q)(+q)}{a^2} = 8.99\times10^{9}\,\frac{(2\times10^{-7})(10^{-7})}{(5\times10^{-2})^2} = 0.072 \text{ N}$$

$$F_{+2q,-q} = \frac{1}{4\pi\epsilon_0}\frac{(+2q)(-q)}{(\sqrt{2}a)^2} = 8.99\times10^{9}\,\frac{(2\times10^{-7})(-10^{-7})}{(\sqrt{2}\times5\times10^{-2})^2} = -0.036$$

$$F_{+2qx} = |F_{2q,-2q}|\cos 0 + |F_{2q,q}|\cos 270^\circ + |F_{2q,-q}|\cos 45^\circ$$

$$F_{+2qy} = |F_{2q,-2q}|\sin 0 + |F_{2q,q}|\sin 270^\circ + |F_{2q,-q}|\sin 45^\circ$$

$$F_{2qx} = 0.144 + 0 + 0.036\times\frac{\sqrt{2}}{2} = 0.169 \text{ N}$$

$$F_{2qy} = 0 + (-1)(0.072) + 0.036\,\frac{\sqrt{2}}{2} = -0.047 \text{ N}$$

11P —

$r = 2.0\text{ m}$

$F = 1.0\text{ N}$

$q + q' = 5.0\times10^{-5}\text{ C}$

$q = ?\quad q' = ?$

Sol.

$$F = \frac{1}{4\pi\epsilon_0}\frac{qq'}{r^2} = 8.99\times10^{9}\,\frac{qq'}{(2)^2} = 2.2475\times10^{9}\,qq'$$

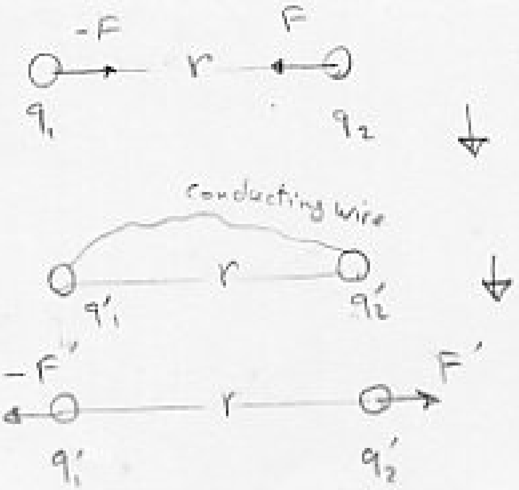
$$1 = 2.2475\times10^{9}\,qq' \longrightarrow qq' = 4.45\times10^{-10}$$

$$\begin{cases} q + q' = 5\times10^{-5} \\ qq' = 4.45\times10^{-10} \end{cases}$$

$$ax^2 + bx + c = 0 \qquad x^2 + \frac{b}{a}x + \frac{c}{a} = 0 \qquad x^2 - 5\times10^{-5}x + 4.45\times10^{-10} = 0$$

$$x = \frac{-b\pm\sqrt{b^2-4ac}}{2a} = \frac{+5\times10^{-5}\pm\sqrt{(5\times10^{-5})^2-4(4.45\times10^{-10})}}{2} = +3.84\times10^{-5},\ +1.16\times10^{-5}$$

12P —



a) $r = 50\text{cm}$, $F = 0.108\text{N}$

b) $r = 50\text{cm}$ $F = 0.036\text{ N}$

$q_1' = q_2' \equiv q'$

$q_1 = ?$ $q_2 = ?$

Sol.

$$|F| = \frac{1}{4\pi\epsilon_0}\frac{|q_1||q_2|}{r^2} \qquad 0.108 = 8.99\times10^{9}\frac{|q_1||q_2|}{(0.5)^2} \qquad |q_1||q_2| = 3\times10^{-12}$$

$$|F'| = \frac{1}{4\pi\epsilon_0}\frac{q'^2}{r^2} \qquad 0.036 = 8.99\times10^{9}\frac{q'^2}{(0.5)^2} \qquad |q'| = 1\times10^{-6}\text{ C}$$

$$|q_1 + q_2| = |q' + q'| = 2\times10^{-6}$$

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0 \qquad -\frac{b}{a} = 2\times10^{-6} \qquad \frac{c}{a} = -|q_1||q_2| = -3\times10^{-12}$$

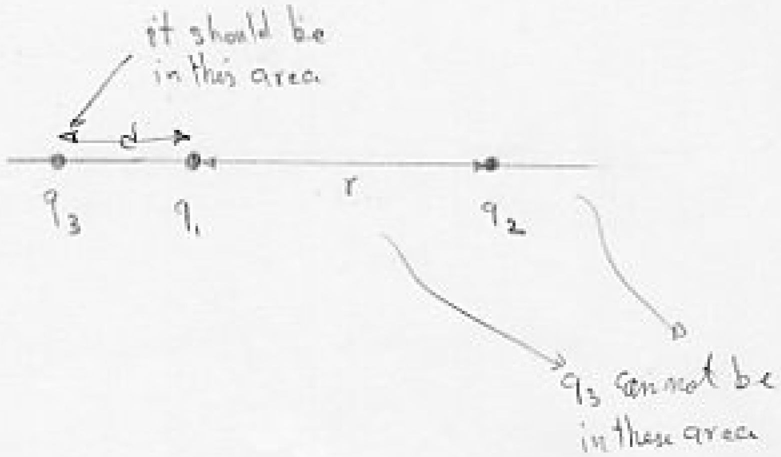
$$x^2 - 2\times10^{-6}x - 3\times10^{-12} = 0 \qquad x = \frac{-b' \pm \sqrt{b'^2 - ac}}{a}$$

$$x = \frac{10^{-6} \pm \sqrt{10^{-12} + 3\times10^{-12}}}{1} = 3\times10^{-6},\ -10^{-6}\text{ C}$$

$$\begin{cases} q_1 = 3\times10^{-6}\text{ C} \\ q_2 = -10^{-6}\text{ C} \end{cases} \quad \text{or} \quad \begin{cases} q_1 = -3\times10^{-6}\text{ C} \\ q_2 = 10^{-6}\text{ C} \end{cases}$$

13P —

$$\begin{cases} q_1 = 1\,\mu C \\ q_2 = -3\,\mu C \\ r = 10\text{ cm} \end{cases}$$



$x_{q_3} = ?$ $\quad F_{q_3} = 0$

Sol.

$$|F_{31}| = \frac{1}{4\pi\epsilon_0}\frac{|q_3||q_1|}{d^2} \qquad |F_{32}| = \frac{1}{4\pi\epsilon_0}\frac{|q_3||q_2|}{(r+d)^2}$$

$$|F_{31}| = |F_{32}| \rightarrow \frac{|q_1|}{d^2} = \frac{|q_2|}{(r+d)^2} \qquad (r^2+d^2+2rd)|q_1| = |q_2|d^2$$

$$2d^2 - 2(0.1)d - (0.1)^2 = 0 \qquad d^2 - 0.1d - 0.005 = 0$$

$$d = \frac{+0.1 \pm \sqrt{0.01+0.02}}{2} = 0.1366,\ -0.0366$$

(not acceptable)

18P — A certain charge Q is divided into two parts, q and Q-q, which are then separated by a certain distance. What must q be in terms of Q to maximize the electrostatic repulsion between the two charges?

Sol.

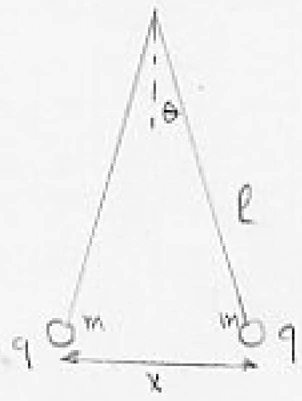
$$F = \frac{1}{4\pi\epsilon_0}\frac{q(Q-q)}{r^2} \qquad \frac{\partial F}{\partial q} = 0 \qquad \frac{1}{4\pi\epsilon_0}\frac{1}{r^2}(Q-2q) = 0$$

$$q = \frac{Q}{2}$$

19P –

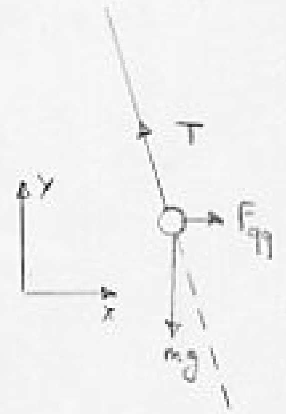
$\theta = \text{small} \rightarrow \tan\theta \simeq S\theta$

show that $x = \left(\dfrac{q^2 \ell}{2\pi\epsilon_0 mg}\right)^{\frac{1}{3}}$



<u>Sol.</u>

$\begin{cases} \sum F_x = 0 \\ \sum F_y = 0 \end{cases}$ $\begin{cases} F_{qq} C0^\circ + T C(90+\theta) + mg C270^\circ = 0 \\ F_{qq} S0^\circ + T S(90+\theta) + mg S270^\circ = 0 \end{cases}$

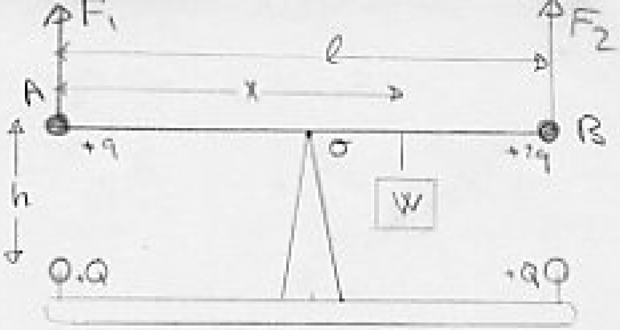


$\begin{cases} F_{qq} - T S\theta = 0 \\ T C\theta - mg = 0 \end{cases}$ $\tan\theta = \dfrac{F_{qq}}{mg}$

$\tan\theta = \dfrac{1}{4\pi\epsilon_0} \cdot \dfrac{q^2}{x^2}\left(\dfrac{1}{mg}\right)$ But $\tan\theta \simeq S\theta = \dfrac{x/2}{\ell}$

$\dfrac{x}{2\ell} = \dfrac{1}{4\pi\epsilon_0} \dfrac{q^2}{x^2}\left(\dfrac{1}{mg}\right)$ $x = \left(\dfrac{\ell q^2}{2\pi\epsilon_0 mg}\right)^{\frac{1}{3}}$

21P



a) $X = ?$ (in equilibrium)

b) $h = ?$ (in such a way no vertical force on O.)

Sol.

a)
$$\begin{cases} F_1 = \frac{1}{4\pi\epsilon_0}\frac{qQ}{h^2} \\ F_2 = \frac{1}{4\pi\epsilon_0}\frac{2qQ}{h^2} \end{cases} \qquad \begin{cases} N_A = Wx - F_2 l = 0 \\ N_B = F_1 l - W(l-x) = 0 \end{cases}$$

$$x = \frac{F_2 l}{W} \qquad l - x = \frac{F_1 l}{W} \rightarrow \frac{x}{l-x} = \frac{F_2}{F_1} = 2 \rightarrow x = \frac{2}{3} l$$

b) $$W = F_1 + F_2 \rightarrow W = \frac{3}{4\pi\epsilon_0}\frac{qQ}{h^2} \qquad h = \sqrt{\frac{3}{4\pi\epsilon_0}\frac{qQ}{W}}$$

Some Integral formulas:

$$\int \frac{dx}{(ax+b)(cx+d)} = \frac{1}{ad-bc} \ln\left|\frac{ax+b}{cx+d}\right| + c$$

$$\int \frac{x\,dx}{(ax+b)(cx+d)} = \frac{1}{ad-bc}\left[\frac{d}{c}\ln|cx+d| - \frac{b}{a}\ln|ax+b|\right] + c$$

$$\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c$$