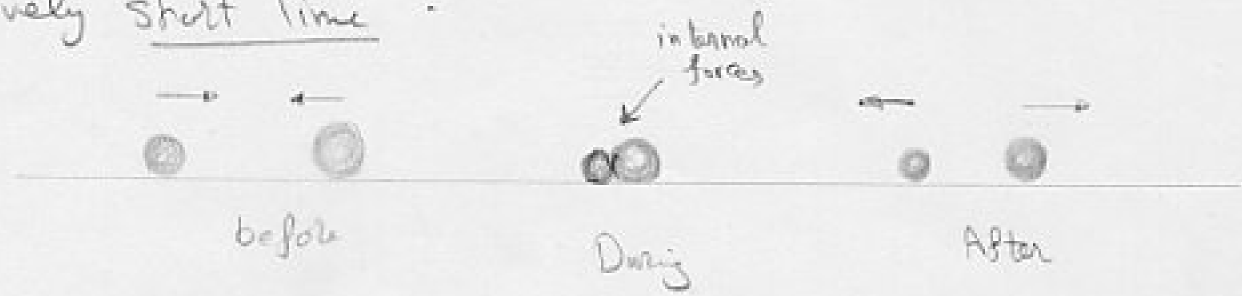
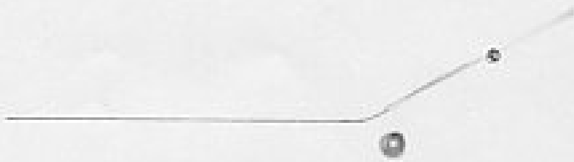
Chapter 10

Collisions

10-1 What is a Collision?

Def.: A collision is an isolated event in which a relatively <u>strong</u> force acts on each of two or more colliding bodies for a relatively <u>short time</u>.

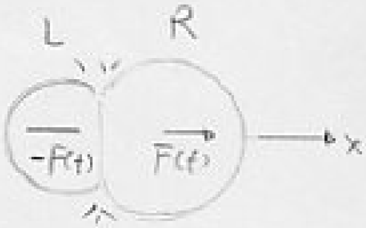




Collision

10-2 Impulse and Linear Mom.

Single Collision:



head-on collision

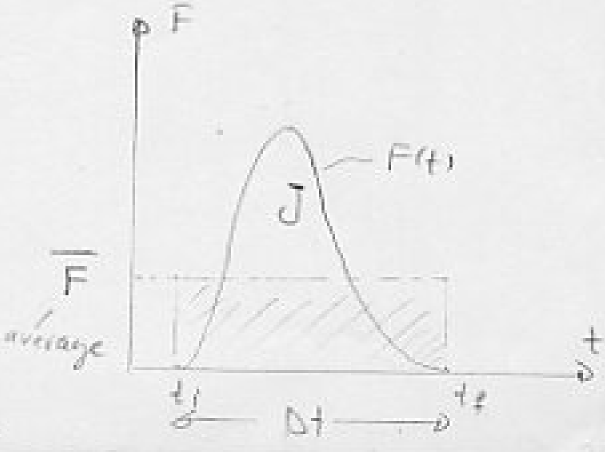
Acc. to Newton's Second law;

$$\bar{F} = \frac{d\bar{P}}{dt} \rightarrow d\bar{P} = \bar{F}(t)\,dt$$

$$\int_{P_i}^{P_f} d\bar{P} = \int_{t_1}^{t_2} \bar{F}(t)\,dt$$

$$\Delta P = P_f - P_i$$

$$J = \int_{t_i}^{t_f} F(t)\,dt$$ the collision <u>impulse</u>



$\rightarrow P_f - P_i = \Delta P = J$ impulse-linear momentum theorem

From the conservation of linear mom.;

$\Delta \bar{P}$ for body R $\longrightarrow$ $-\Delta \bar{P}$ for body L

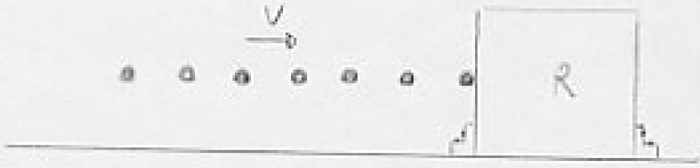
$$\begin{cases} P_{f_x} - P_{i_x} = \Delta P_x = J_x \\ P_{f_y} - P_{i_y} = \Delta P_y = J_y \\ P_{f_z} - P_{i_z} = \Delta P_z = J_z \end{cases}$$

Impulse-linear momentum theorem like work-kinetic energy theorem is a direct consequence of Newton's second law.

Also $J = \bar{F} \Delta t$ ($\bar{F}$: average)

Series of Collisions

A steady stream of bodies, with identical linear momenta mv collides with body R, which is fixed in place.



If n bodies collide in time interval Δt, then the total impulse J acting on body R during Δt is

$J = -n\Delta P$

acting on body R

of colliding body

Substituting in $J = \bar{F}\Delta t \Rightarrow \bar{F} = -\frac{n}{\Delta t}\Delta P = -\frac{n}{\Delta t} m\Delta V$

$\frac{n}{\Delta t}$: the rate at which the bodies collide with body R.

If the colliding bodies stop upon impact, then;

$\Delta V = V_f - V_i = 0 - V = -V$

But if instead the colliding bodies bounce directly backward from body R with <u>no change</u> in speed, then $V_f = -V$;

$\Delta V = V_f - V_i = -V - V = -2V$

In time interval Δt, an amount of mass $\Delta m = nm$ collides with body R, then;

$$\bar{F} = -\frac{\Delta m}{\Delta t}\Delta V$$

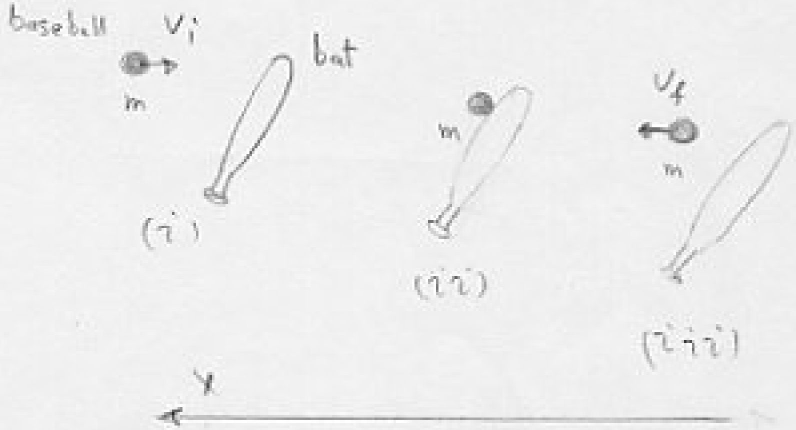
$\frac{\Delta m}{\Delta t}$: the rate at which mass collides with body R.

Sample prob. 10-1

$m = 140g$

$V_i = 39 \, m/s$

$V_f = 39 \, m/s$ (in opposite dir.)



a) $J = ?$ acting on ball while it is in contact with the bat.

$J = P_f - P_i = mv_f - mv_i = (0.14 kg)(39 m/s) - (0.14 kg)(-39 m/s)$
$= 10.9 kg m/s$ (in the dir. of V_f)

b) If $\Delta t = 1.2$ ms (impact time) $\bar{F} = ?$ average force acting on the ball

$\bar{F} = \frac{J}{\Delta t} = \frac{10.9 kgm/s}{0.0012 s} = 9100 N$ (J and $\bar{F}$ in the same dir.) in the dir. of V_f.

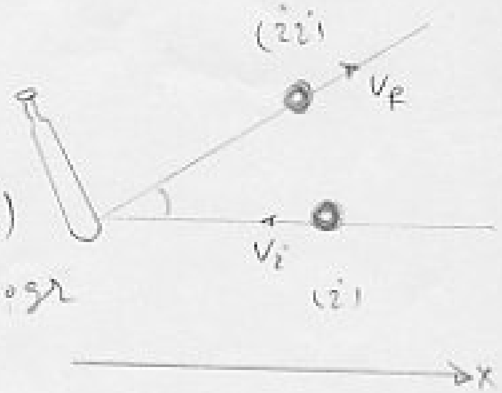
The max. force will be larger than this.

c) $\bar{a} = ?$ average acc.

$\bar{a} = \frac{\bar{F}}{m} = \frac{9100 N}{0.14 kg} = 6.5 \times 10^4 m/s^2$ $\sim 6600 g$

Remark: we neglected the mg force.

Sample Prob. 10-2



$v_i = 39 m/s$ $V_f = 45 m/s$ $\theta = 30°$ (not head on)
$\Delta t = 1.2$ ms $\bar{F} = ?$ on ball $m = 140 gr$

Sol.

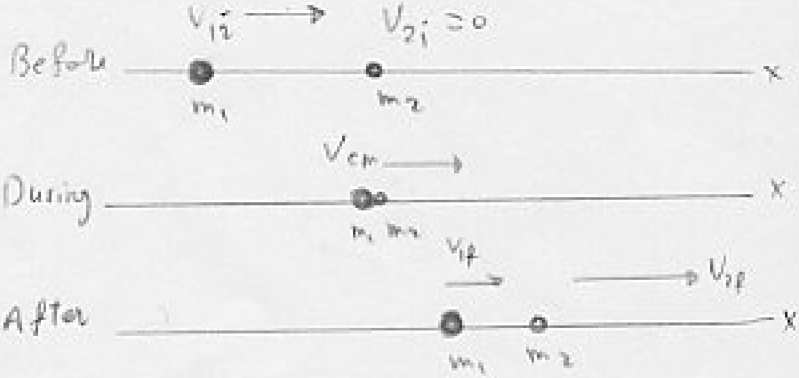
$J_x = P_{fx} - P_{ix} = mV_{fx} - mV_{ix} = (0.14)[(45)\cos 30 - (-39)] = 10.92 kg m/s$
$J_y = P_{fy} - P_{iy} = mV_{fy} - mV_{iy} = (0.14)[(45)\sin 30 - 0] = 3.150 kg m/s$
$J = \sqrt{J_x^2 + J_y^2} = 11.37 kg m/s$ $\bar{F} = \frac{J}{\Delta t} = \frac{11.37}{0.0012} = 9475 N$
$\tan\theta = \frac{J_y}{J_x} = 0.288 \rightarrow \theta = 16°$ (dir. of J or $\bar{F}$) (not in the dir. of V_f)
(We have neglected the mg force)

10-3 Elastic Collision in one dim.:

Stationary Target:

m_2: at rest (target)

m_1: Projectile



The system is assumed to be
$\begin{cases} \text{i- Closed} \\ \text{ii- Isolated } (\Sigma F_{ext}=0) \end{cases}$

Furthermore: $T_{1i}+T_{2i}=T_{1f}+T_{2f}$ (Elastic collision)

Due to (i)(ii) $\longrightarrow$ $P_{1i}+P_{2i}=P_{1f}+P_{2f}$ (even if the collision is inelastic, because the forces involved in collision are internal)

Then;

$$\begin{cases} m_1\bar{V}_{1i} = m_1\bar{V}_{1f} + m_2\bar{V}_{2f} & (1) \\ \frac{1}{2}m_1V_{1i}^2 = \frac{1}{2}m_1V_{1f}^2 + \frac{1}{2}m_2V_{2f}^2 & (2) \end{cases}$$

$(1) \rightarrow m_1(V_{1i}-V_{1f}) = m_2V_{2f}$

$(2) \rightarrow m_1(V_{1i}-V_{1f})(V_{1i}+V_{1f}) = m_2V_{2f}^2$

$$\rightarrow \begin{cases} V_{1f} = \dfrac{m_1-m_2}{m_1+m_2}V_{1i} & (3) \\ V_{2f} = \dfrac{2m_1}{m_1+m_2}V_{1i} > 0 \text{ always} & (4) \end{cases}$$

Equal Masses:

If $m_1=m_2 \longrightarrow V_{1f}=0$, $V_{2f}=V_{1i}$

In head on collisions, bodies of equal mass simply exchange velocities. This is true even if the target particle is not initially at rest.

A Massive Target:

i.e. $m_2 >> m_1$, (3)(4) $\rightarrow$ $V_{1f} \simeq -V_{1i}$, $V_{2f} \simeq \left(\frac{2m_1}{m_2}\right) V_{1i}$

$(\Delta V_1 = 2V_{1i})$

A Massive Projectile:

i.e. $m_1 >> m_2$ (3)(4) $\longrightarrow$ $V_{1f} \simeq V_{1i}$, $V_{2f} \simeq 2V_{1i}$

$(\Delta V_2 = 2V_{1i})$

Motion of the CM:

The CM of two colliding bodies continues to move, totally uninfluenced by the collision.

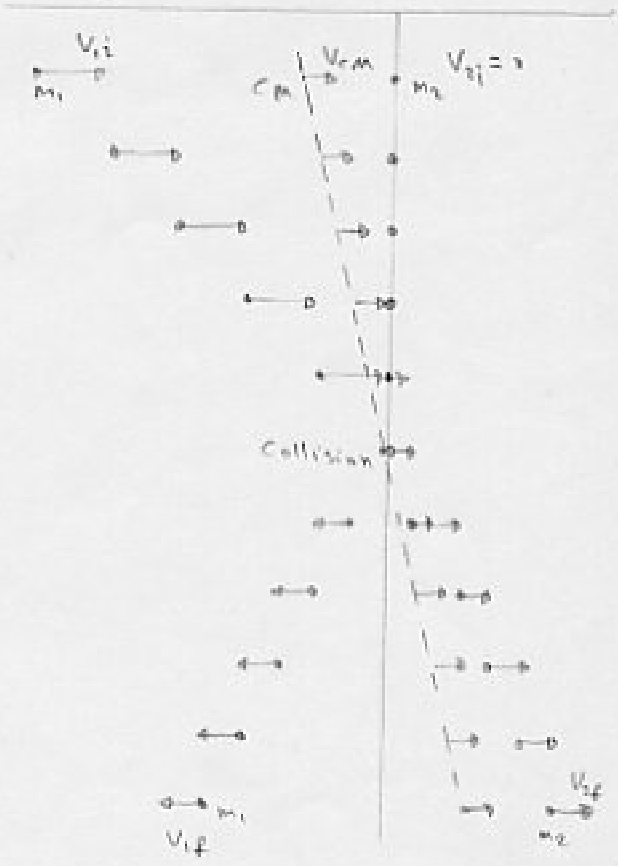
This follows from i) conservation of linear mom., ii) and

$P = M V_{cm} = (m_1+m_2) V_{cm}$

Since $P = P'$ $\longrightarrow$ V_{cm} unchanged

$$V_{cm} = \frac{P}{m_1+m_2} = \frac{m_1}{m_1+m_2} V_{1i}$$

(for target at rest)



Moving Target:

$$m_1 V_{1i} + m_2 V_{2i} = m_1 V_{1f} + m_2 V_{2f} \qquad (1)$$

$$\frac{1}{2} m_1 V_{1i}^2 + \frac{1}{2} m_2 V_{2i}^2 = \frac{1}{2} m_1 V_{1f}^2 + \frac{1}{2} m_2 V_{2f}^2 \qquad (2)$$

$$(1) \rightarrow m_1 (V_{1i} - V_{1f}) = - m_2 (V_{2i} - V_{2f})$$

$$(2) \rightarrow m_1 (V_{1i} - V_{1f})(V_{1i} + V_{1f}) = - m_2 (V_{2i} - V_{2f})(V_{2i} + V_{2f})$$

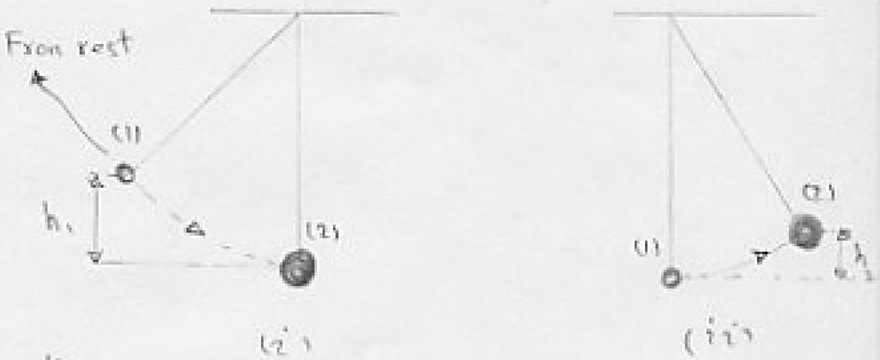
$$\rightarrow \begin{cases} V_{1f} = \dfrac{m_1 - m_2}{m_1 + m_2} V_{1i} + \dfrac{2 m_2}{m_1 + m_2} V_{2i} \\[2ex] V_{2f} = \dfrac{2 m_1}{m_1 + m_2} V_{1i} + \dfrac{m_2 - m_1}{m_1 + m_2} V_{2i} \end{cases}$$

Sample prob. 10-3

$m_1 = 30\,gr \quad m_2 = 75\,gr$

$h_1 = 8\,cm$

An elastic collision;



a) $V_{1f} = ?$ just after collision

Let V_{1i} for ball 1 just before collision

$$\overset{\circ}{T_i} + \overset{\circ}{U_i} = T_1 + U_1 \qquad \frac{1}{2} m_1 V_{1i}^2 = m_1 g h_1 \rightarrow V_{1i} = \sqrt{2 g h_1} = \sqrt{2(9.8)(0.08)} = 1.252\ m/s$$

$$V_{1f} = \frac{m_1 - m_2}{m_1 + m_2} V_{1i} = \frac{0.03 - 0.075}{0.03 + 0.075}(1.252) = -0.537\ m/s \qquad (P146)$$

ball (1) moves to the left just after collision.

b) To what height h_1' does ball 1 swing to the left after the collision?

$$m_1 g h_1' = \frac{1}{2} m_1 V_{1f}^2 \qquad h_1' = \frac{V_{1f}^2}{2g} = \frac{(-0.537)^2}{(2)(9.8)} = 0.0147\ m$$

c) What is the velocity V_{2f} of ball (2) just after the collision?

$$V_{2f} = \frac{2m_1}{m_1+m_2} V_{1i} = \frac{(2)(0.030)}{0.030+0.075}(1.252) = 0.715 \, m/s$$

d) To what height h_2 does ball (2) swing after the collision?

$$m_2 g h_2 = \frac{1}{2} m_2 V_{2f}^2 \quad \rightarrow h_2 = \frac{V_{2f}^2}{2g} = \frac{(0.715)^2}{(2)(9.8)} = 0.0261 \, m$$

Sample prob. 10-4

In nuclear reactor, newly produced fast neutrons must be slowed down, before they can participate effectively in the chain reaction process. This is done by alowing the to collide with the nuclei of atoms in a moderator.

a) By what fraction is the kinetic energy of a neutron (of mass m_1) reduced in a head on elastic collision with a nucleus of mass m_2 initially at rest?

$$K_i = \frac{1}{2} m_1 V_{1i}^2 \,, \quad K_f = \frac{1}{2} m_1 V_{1f}^2 \qquad frac = \frac{K_i - K_f}{K_i} = \frac{V_{1i}^2 - V_{1f}^2}{V_{1i}^2} = 1 - \frac{V_{1f}^2}{V_{1i}^2} \quad (1)$$

But $V_{1f} = \frac{m_1 - m_2}{m_1 + m_2} V_{1i}$ (P146) (2)

(2) in (1) $\rightarrow$ $frac = \frac{4 m_1 m_2}{(m_1+m_2)^2}$

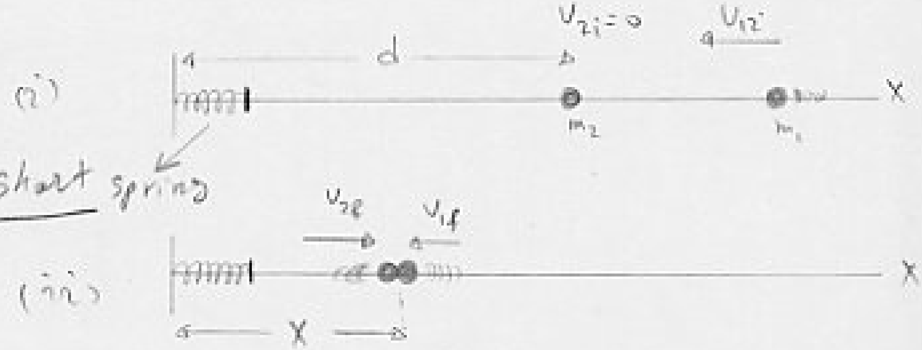
b) Evaluate the fraction for lead, carbon and hydrogen.

($\frac{m_2}{m_1} = 206$ for lead, $\frac{m_2}{m_1} = 12$ for carbon, $\frac{m_2}{m_1} = 1$ for hydrogen, m_1: neutron mass)

$$frac_{Lead} = \frac{4(206)}{(1+206)^2} = 0.019 \; (1.9\%) \qquad frac_{Carbon} = \frac{4(12)}{(1+12)^2} = 0.28 \; (28\%)$$

$$frac_{Hyd.} = \frac{4(1)}{(1+1)^2} = 1 \; (100\%)$$

Prob. 10-5



$d = 53\ cm$

$m_1 = 590\ gr$

$m_2 = 350\ gr$

$v_{1i} = -75\ cm/s$

$X = ?$

Sol.

$$V_{1f} = v_{1i}\frac{m_1 - m_2}{m_1 + m_2} = (-75\ cm/s)\frac{(590-350)gr}{(590+350)gr} = -19\ cm/s$$

$$V_{2f} = V_{1i}\frac{2m_1}{m_1+m_2} = -94\ cm/s \qquad t = \frac{d-x}{v_{1f}} = \frac{d+x}{v_{2f}}$$

$$\frac{53-X}{-19} = \frac{53+X}{-94} \rightarrow X = 35\ cm, \quad \text{But indeed } X = d\,\frac{m_1+m_2}{3m_1-m_2}$$

or $X = d\,\frac{r+1}{3r-1}$ $\left(r = \frac{m_1}{m_2}\right)$ indep. of V_{1i}! -

10-4 Inelastic Collisions in One Dim.:

$T_{1i} + T_{2i} \neq T_{1f} + T_{2f}$ in inelastic collision

$\Delta T = (T_{1i} + T_{2i}) - (T_{1f} - T_{2f})$ changes to other kind of energies

Still if the system is $\begin{cases}\text{(i) isolated} \\ \text{(ii) and closed}\end{cases} \longrightarrow P_{1i} + P_{2i} = P_{1f} + P_{2f}$

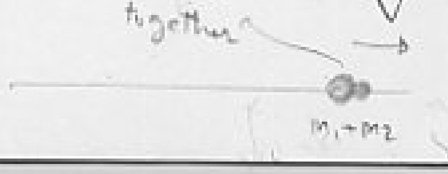
Completely inelastic collision: The bodies stick together.

Ex. Completely inelastic collision; Before (a)

$m_1 v = (m_1 + m_2)V$

$V = v\,\frac{m_1}{m_1+m_2} < v$ always

After (b)

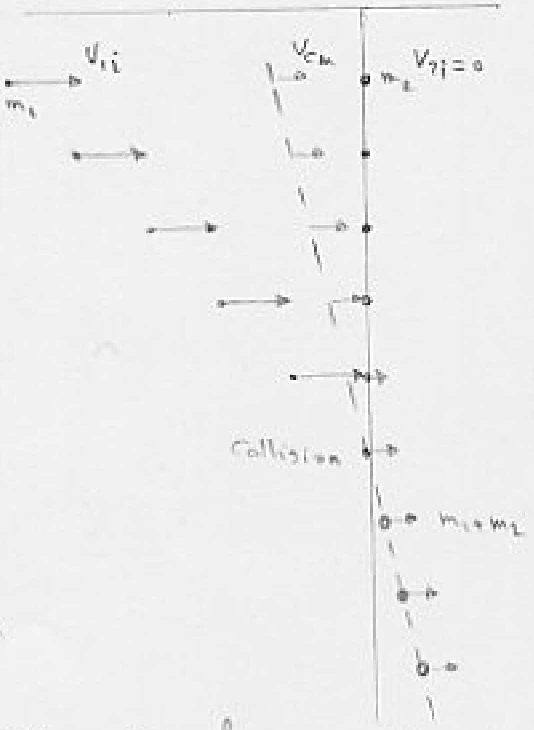


If both bodies are moving prior to a collision in which they stick together, we have,

$$m_1 v_1 + m_2 v_2 = (m_1 + m_2)\bar{V}$$

If $P_{1i} + P_{2i} = P_{1f} + P_{2f} = 0$

$\rightarrow$ All the kinetic energy in a completely inelastic collision is dissipated.

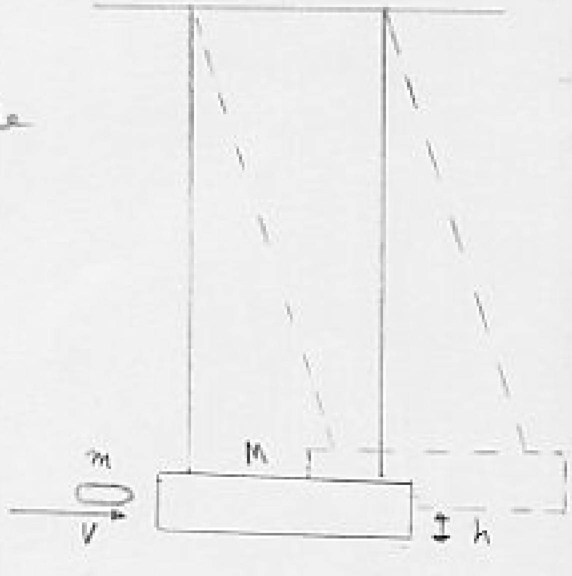


The center of mass motion is not affected by the inelastic collision

Sample prob. 10-6

A ballistic pendulum is a device that was used to measure the speeds of bullets before electronic devices were developed.

$M = 5.4\,kg \quad m = 9.5\,gr \quad h = 6.3\,cm$



a) $v = ?$

Sol.

$$mv = (m+M)\bar{V} \quad (1)$$

This is completely inelastic collision and the kinetic energy is not conserved. However after the collision the mechanical energy is conserved.

$\rightarrow \quad \frac{1}{2}(M+m)V^2 = (M+m)gh \qquad (2)$

$(1)(2) \rightarrow v = \frac{M+m}{m}\sqrt{2gh} = \left(\frac{5.4+0.0095}{0.0095}\right)\sqrt{2(9.8)(0.063)} = 630\,m/s$

b) Initial kinetic energy of the bullet ?

How much of this energy remains as mechanical energy of the swinging pendulum ?

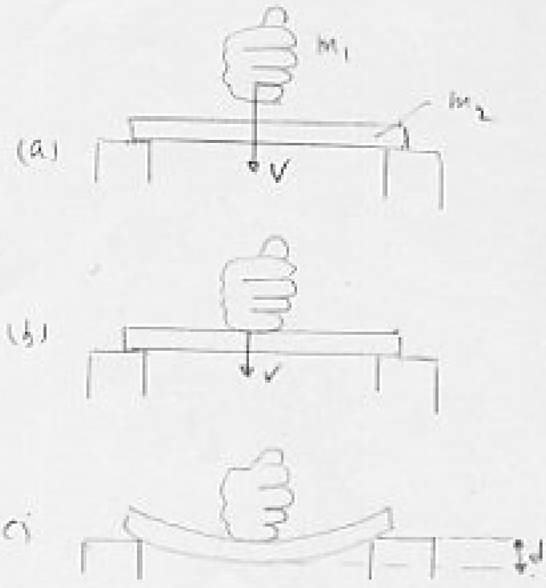
$K_{bullet} = \frac{1}{2}mv^2 = \frac{1}{2}(0.0095)(630)^2 = 1900\,J$

$E = (M+m)gh = (5.4+0.0095)(9.8)(0.063) = 3.3\,J$

$\frac{3.3}{1900} \sim \%2$

Sample prob. 10-7

A Karate expert strikes downward with his fist (of mass $m_1 = 0.7\,kg$) breaking a 0.14 kg board. He then does the same to a 3.2 kg concrete block. The spring consts. k for bending are $4.1\times10^4\,N/m$ for the board and $2.6\times10^6\,N/m$ for the block. Breaking occurs at a deflection $d = 16\,mm$ for the board and $d = 1.1\,mm$ for the block.



a) Just before the board and the block break, what is the energy stored in each?

$U = \frac{1}{2} k d^2 \qquad U = \frac{1}{2}\,(4.1\times 10^4\ N/m)\,(0.016\ m)^2 = 5.248\ J$ (board)

$U = \frac{1}{2}(2.6\times 10^6\ N/m)(0.0011\ m)^2 = 1.573\ J$ (block)

b) What fist speed v is required to break the board and the block? Assume that mechanical energy is conserved during the bending, that the fist and struck object stop just before the break, and that the fist-object collision at the onset of bending (Fig. b) is totally inelastic.

$T_{1i} + T_{2i} \neq T_{1f} + T_{2f}$

But $K \equiv T_{1f} + T_{2f} = \frac{1}{2}(m_1+m_2)\,V^2 = U$

$\rightarrow V = \sqrt{\dfrac{2U}{m_1+m_2}}$ the speed of fist + object at the onset of bending

$V = \sqrt{\dfrac{2(5.248)}{0.70+0.14}} = 3.534\ m/s$ (board)

$V = \sqrt{\dfrac{2(1.573)}{0.7+3.2}} = 0.8981\ m/s$ (block)

Now, just before hitting the board or block,

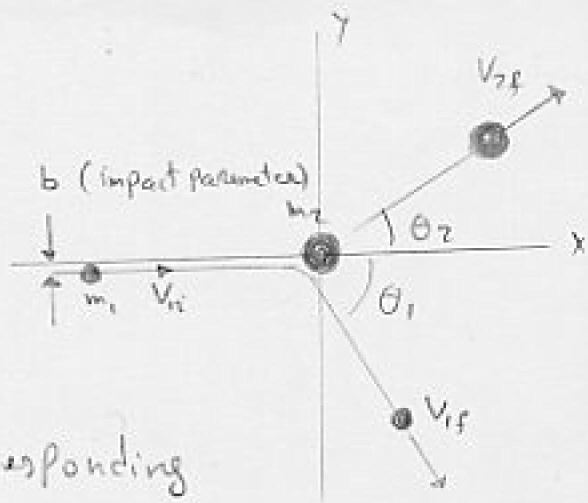
$v = \left(\dfrac{m_1+m_2}{m_1}\right) V$ speed of fist

$v = \left(\dfrac{0.7+0.14}{0.7}\right)(3.534) = 4.2\ m/s$ (for the board)

$v = \left(\dfrac{0.7+3.2}{0.7}\right)(0.8981) = 5\ m/s$ (" " block)

The fist speed must be about %20 faster for the fist to break the block, because the larger mass of the block makes the transfer of energy to the block more difficult.

10-5 Collisions in Two Dims.;



The distance b by which the collision fails to be head on is called the impact parameter. It is a measure of directness of the collision, b = 0 corresponding to head-on.

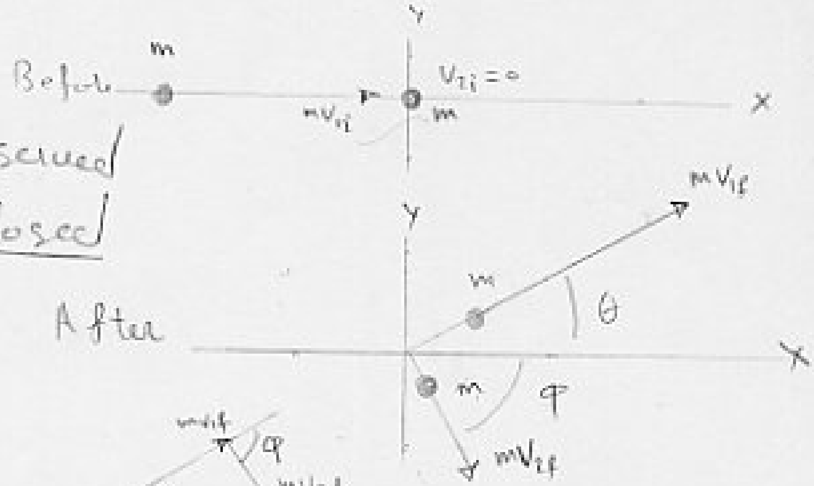
$$\begin{cases} m_1 V_{1i} = m_1 V_{1f} \cos\theta_1 + m_2 V_{2f} \cos\theta_2 \\ 0 = -m_1 V_{1f} \sin\theta_1 + m_2 V_{2f} \sin\theta_2 \end{cases}$$

$$\frac{1}{2} m_1 V_{1i}^2 = \frac{1}{2} m_1 V_{1f}^2 + \frac{1}{2} m_2 V_{2f}^2$$

Sample prob 10-8

Two particles of equal masses have an elastic collision, the target particle being initially at rest. Show that (unless the collision is head-on) the two particles will always move off perpendicular to each other after the collision.

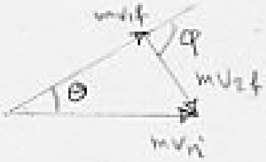
Sol.



Because the linear mom. is conserved these vectors must form a closed triangle.

Because $m_1 = m_2 = m$

→ We have also a closed velocity triangle.

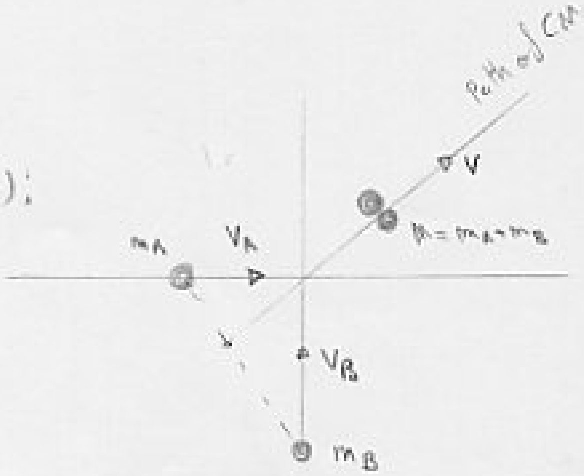


$$\bar{V}_{1i} = \bar{V}_{1f} + \bar{V}_{2f}$$

Also $V_{1i}^2 = V_{1f}^2 + V_{2f}^2$ (energy conservation)

$\rightarrow$ From Pythagorean theorem $\rightarrow$ The triangle must be a right triangle.

Sample prob. 10-9



They embrace at the origin (totally inelastic):

$m_A = 83$ kg $\quad m_B = 55$ kg

$V_A = 6.2$ km/h $\quad V_B = 7.8$ km/h

a) $V = ?$

$$\begin{cases} m_A V_A = MV\cos\theta \\ m_B V_B = MV\sin\theta \end{cases} \rightarrow \tan\theta = \frac{m_B V_B}{m_A V_A} = 0.834 \quad \theta = 39.8^\circ$$

$$V = \frac{m_B V_B}{M\sin\theta} = \frac{(55)(7.8)}{(83+55)(\sin 39.8)} = 4.86 \text{ km/h}$$

b) What is the velocity of the CM of the two skaters before and after the collision?

The velocity of the CM <u>is not</u> affected by the collision, then it is the <u>same</u> before and after the collision.

c) Fractional change in the kinetic energy = ?

$K_i = \frac{1}{2} m_A V_A^2 + \frac{1}{2} m_B V_B^2 = 3270 \text{ kg}\cdot\text{km}^2/\text{h}^2$

$K_f = \frac{1}{2} MV^2 = 1630 \text{ kg}\cdot\text{km}^2/\text{h}^2$

$\text{frac} = \dfrac{K_f - K_i}{K_i} = -0.50 \sim \%50$

10-6 Fractions and Decay Processes

Here we consider collisions (called reactions) in which the identity and even the number of interacting nuclear particles change because of the collision.

Sample prob. 10-10

$^{235}U \longrightarrow \alpha + {}^{231}Th$ Thorium

$m_\alpha = 4.00 u$ $\quad K_\alpha = 4.60 MeV$

$m_{Th} = 231 u$ $\quad K_{Th} = ?$

Sol.

^{235}U : initally at rest.

$0 = m_{Th} V_{Th} + m_\alpha V_\alpha \longrightarrow m_{Th} K_{Th} = m_\alpha K_\alpha \quad (k = \frac{1}{2} m v^2)$

$K_{Th} = K_\alpha \frac{m_\alpha}{m_{Th}} = (4.60 MeV) \frac{4.00 u}{231 u} = 7.97 \times 10^{-2} MeV$

$K_\alpha + K_{Th} = 4.60 + 0.0.0797 = 4.68 \qquad \frac{4.68}{0.0797} \sim 1.7\%$

Sample prob. 10-11

A nuclear reaction of great importance for the generating of the energy by nuclear fission is the so called d-d reaction, one form of which is $d + d \longrightarrow t + p$

p $\quad {}^1H \quad m_p = 1.00783\ u$

d $\quad {}^2H \quad m_d = 2.01410\ u$

t $\quad {}^3H \quad m_t = 3.01605\ u$ (triton)

a) How much kinetic energy appears because of the mass change Δm that occurs in this reaction?

$E = mc^2 \qquad Q = -\Delta m c^2 = -(m_p + m_t - 2m_d)c^2 = (2m_d - m_p - m_t)c^2$

$Q = (2\times 2.01410\ u - 1.00783\ u - 3.01605\ u)(932\ MeV/u) = 4.03\ MeV$

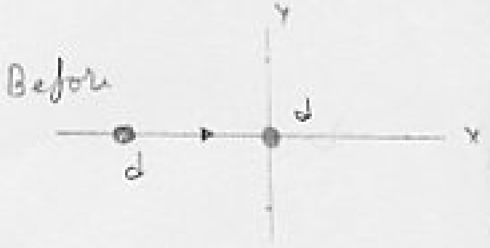
$Q > 0 \rightarrow$ the reaction is exothermic (i.e. $\Delta m \longrightarrow$ kinetic energy)

$\frac{\Delta m}{2m_d} = \frac{0.00432}{2\times 2.01410} \sim 0.1\%$

On the other hand if $Q < 0 \rightarrow$ endothermic reaction

(ie. kinetic energy $\rightarrow \Delta m$)

And $Q = 0 \rightarrow$ Elastic encounter



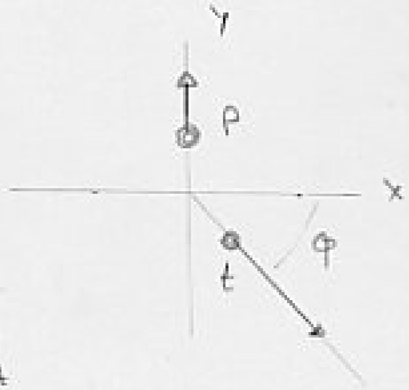
b)

$K_d = 1.50\ MeV \qquad K_d' = 0$

$K_p = 3.39\ MeV \qquad K_t = ?$

$Q = K_p + K_t - (K_d + K_{d'}) \qquad K_t = Q + K_d - K_p$

$K_t = 4.03 + 1.50 - 3.39 = 2.14\ MeV$



c) $\varphi = ?$

$\begin{cases} m_d V_d + 0 = m_t V_t \cos\varphi \\ 0 + 0 = m_p V_p + m_t V_t \sin\varphi \end{cases} \qquad \sin\varphi = -\frac{m_p V_p}{m_t V_t}$

$\varphi = \sin^{-1}\left(-\sqrt{\frac{m_p K_p}{m_t K_t}}\right) = \sin^{-1}\left(-\sqrt{\frac{(1.01u)(3.39MeV)}{(3.02u)(2.14MeV)}}\right) = -46.7$