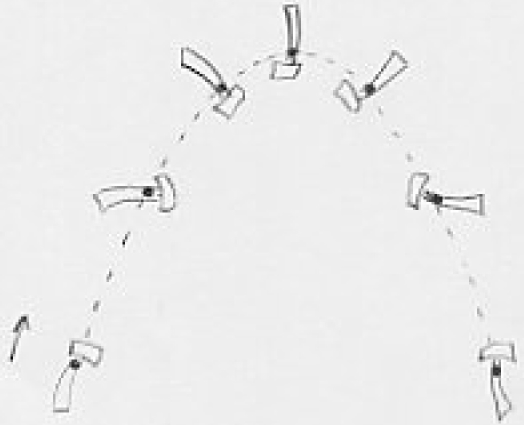
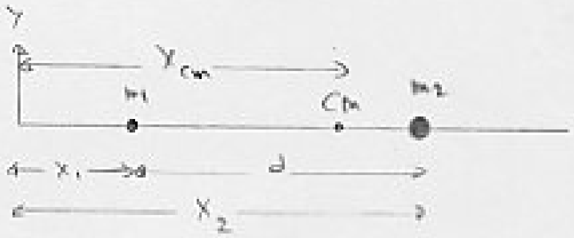
Chapter 9
Systems of Particles

9-1 A special point



The axe contains a system of particles.
Every part of axe moves in a <u>different</u> way (complicated).
But its center of mass moves in a <u>simple</u> parabolic path.

9.2 The center of Mass



Def.: $X_{cm} = \dfrac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$

$$\begin{cases} X_{cm} = \dfrac{1}{M} \sum\limits_{i=1}^{n} m_i x_i \\ Y_{cm} = \dfrac{1}{M} \sum\limits_{i=1}^{n} m_i y_i \\ Z_{cm} = \dfrac{1}{M} \sum\limits_{i=1}^{n} m_i z_i \end{cases}$$

$$r_{cm} = x_{cm}\hat{i} + y_{cm}\hat{j} + z_{cm}\hat{k}$$

$$\rightarrow \bar{r}_{cm} = \frac{1}{M} \sum_{i=1}^{n} m_i \bar{r}_i$$

Rigid Bodies:

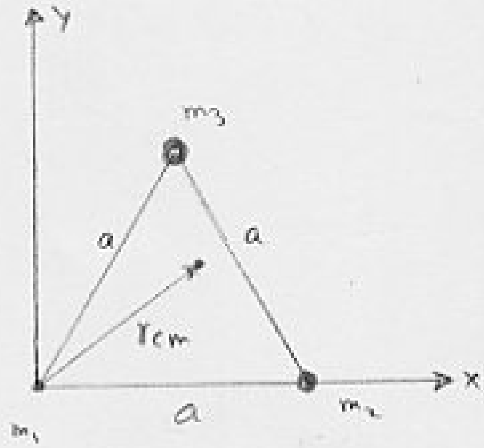
$$\begin{cases} X_{cm} = \dfrac{1}{M} \int x\, dm \\ Y_{cm} = \dfrac{1}{M} \int y\, dm \\ Z_{cm} = \dfrac{1}{M} \int z\, dm \end{cases}$$

$dm = \rho\, dv$

$$\begin{cases} X_{cm} = \dfrac{1}{M} \int x \rho\, dv \\ Y_{cm} = \dfrac{1}{M} \int y \rho\, dv \\ Z_{cm} = \dfrac{1}{M} \int z \rho\, dv \end{cases}$$

For uniform density:

$$\rho = \frac{M}{V} = \text{const.} \rightarrow \begin{cases} x_{cm} = \frac{1}{V}\int x\,dv \\ y_{cm} = \frac{1}{V}\int y\,dv \\ z_{cm} = \frac{1}{V}\int z\,dv \end{cases}$$



Sample prob. 9-1

$m_1 = 1.2$ kg, $m_2 = 2.5$ kg, $m_3 = 3.4$ kg

at the corners of an equilateral triangle of edge $a = 140$ cm

$r_{cm} = ?$

Sol.

$x_1 = 0$ $\quad y_1 = 0$

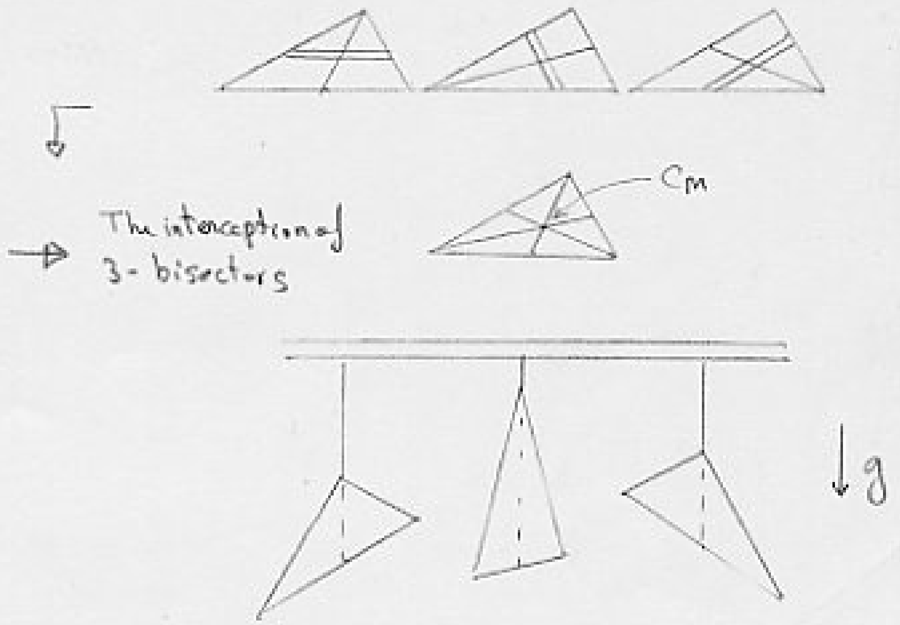
$x_2 = 140$ $\quad y_2 = 0$

$x_3 = 70$ $\quad y_3 = 121$

$$x_{cm} = \frac{1}{M}\sum_{i=1}^{3} m_i x_i = 83\text{ cm}$$

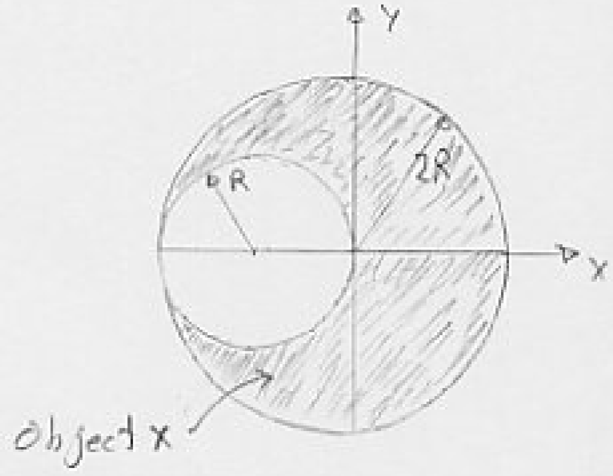
$$y_{cm} = \frac{1}{M}\sum_{i=1}^{3} m_i y_i = 58\text{ cm}$$

Sample prob. 9-2



Sample prob. 9-3

$X_{cm(x)} = ?$

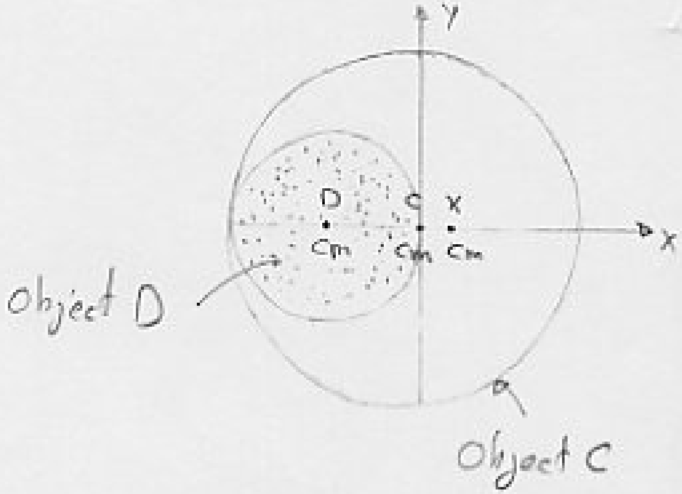


Sol.

$X_{cm(c)} = \frac{m_D X_{cm(D)} + m_x X_{cm(x)}}{m_D + m_x}$

$X_{cm(c)} = 0$

$\longrightarrow X_{cm(x)} = -\frac{m_D}{m_x} X_{cm(D)}$



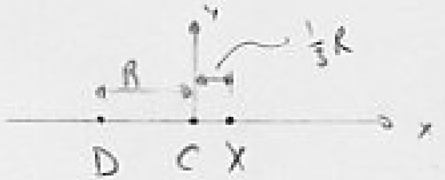
$m_D = \pi R^2 t \rho$

$m_x = \pi (2R)^2 t \rho - \pi R^2 t \rho$

t: the thickness of the plate

Also $X_D = -R$

$X_{cm(x)} = -\frac{\pi R^2 t \rho}{\pi (2R)^2 t \rho - \pi R^2 t \rho} (-R) = \frac{1}{3} R$



Sample prob 9-4

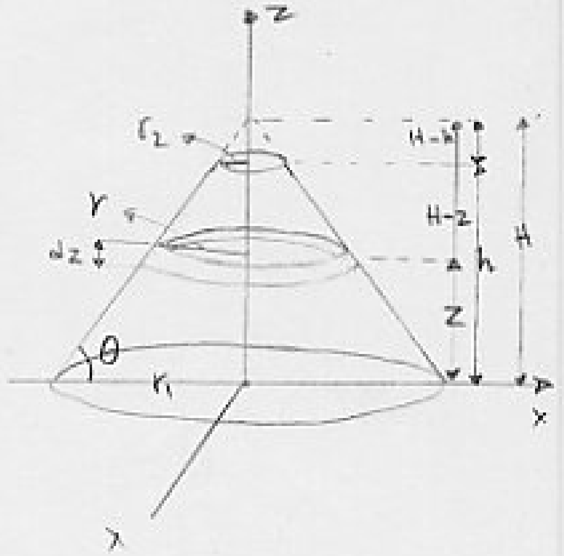
Incomplete right circular conical hill,
$r_1 = 88\,m$, $r_2 = 16\,m$, $h = 60\,m$, $\theta = 30^\circ$

a) $z_{cm} = ?$

Sol.

From the symmetry

$x_{cm} = y_{cm} = 0$



$Z_{cm} = \frac{1}{V}\int z\,dv$

$dv = \pi r^2 dz \qquad \tan\theta = \frac{H}{r_1} = \frac{H-z}{r} \quad \rightarrow r = (H-z)\frac{r_1}{H}$

$$Z_{cm} = \frac{1}{V}\int z\,dv = \frac{1}{V}\frac{\pi r_1^2}{H^2}\int_0^h z(H-z)^2 dz = \frac{\pi r_1^2}{VH^2}\int_0^h (z^3 - 2z^2H + zH^2)dz$$

$$= \frac{\pi r_1^2}{VH^2}\left[\frac{z^4}{4} - \frac{2z^3H}{3} + \frac{z^2H^2}{2}\right]_0^h = \frac{\pi r_1^2 h^4}{VH^2}\left[\frac{1}{4} - \frac{2H}{3h} + \frac{H^2}{2h^2}\right]$$

$V_{\text{right circular cone}} = \frac{1}{3}\pi R^2 Z$ R: base radius
Z: height

$V_{\text{incomplete cone}} = \frac{1}{3}\pi r_1^2 H - \frac{1}{3}\pi r_2^2 (H-h)$

where $H = r_1 \tan 30^\circ = 50.8\,m$

$\rightarrow V = 4.091\times 10^5\,m^3 \qquad \rightarrow Z_{cm} = 12.37\,m$

b) If this hill has a density $\rho = 1.5\times10^3\,kg/m^3$, then how much work is required to build it (from the level of the base)

sol.
We may assume all the mass is concentrated at its CM.

$m = \rho V$

$W = U = mgZ_{cm} = \rho V g Z_{cm} = (1.5\times10^3\,kg/m^3)(4.091\times10^5\,m^3)$
$\cdot(9.8\,m/s^2)(12.37\,m)$
$= 7.4\times10^{10}\,J$

9-3 Newton's Second Law for a system of particles:

$$\sum \bar{F}_{ext} = M \bar{a}_{cm} \quad \rightarrow \quad \begin{cases} \sum F_{ext,x} = M a_{cm,x} \\ \sum F_{ext,y} = M a_{cm,y} \\ \sum F_{ext,z} = M a_{cm,z} \end{cases}$$

(1)

Some Care:

1- $\sum F_{ext}$ are the vector some of all the external forces. Forces exerted by one part of the system on the other are called <u>internal</u> forces, and must be excluded.

2- M is the total mass of the system. We assume that <u>no mass</u> enters or leaves the system. The system is said to be <u>closed</u>.

3- a_{cm} is the acc. of the center of mass of the system. Equ. (1) gives <u>no</u> information about the acc. of any other point.

<u>Ex.</u>

(i) before collision — (ii) — (iii) after collision

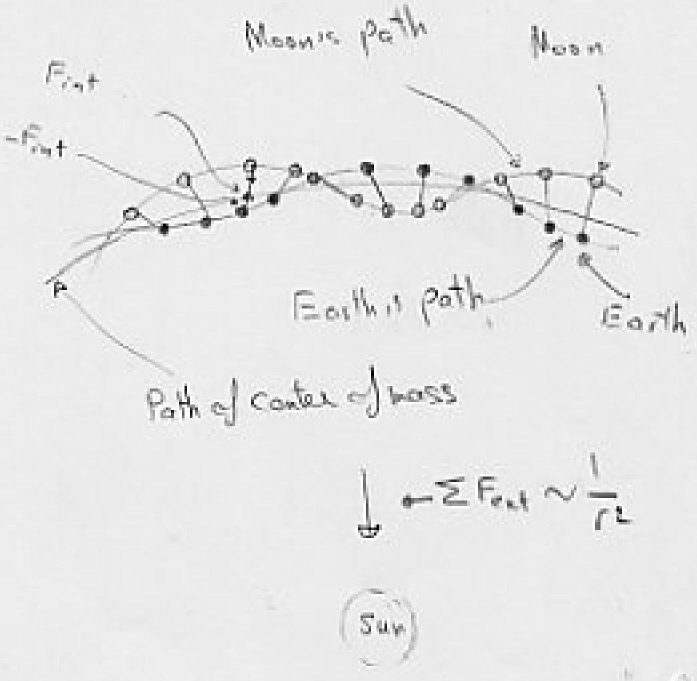
$\sum F_{ext} = 0$ — $\sum F_{ext} = 0$ — $\sum F_{ext} = 0$

$\xrightarrow{V}$ CM, $a_{cm} = 0$

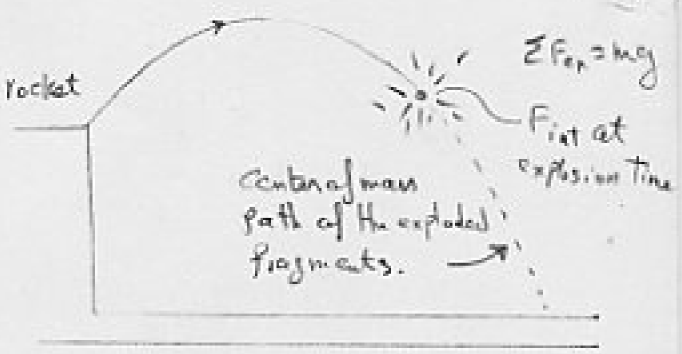
$\xrightarrow{V}$ CM, $a_{cm} = 0$ (the same as before)

Ex.

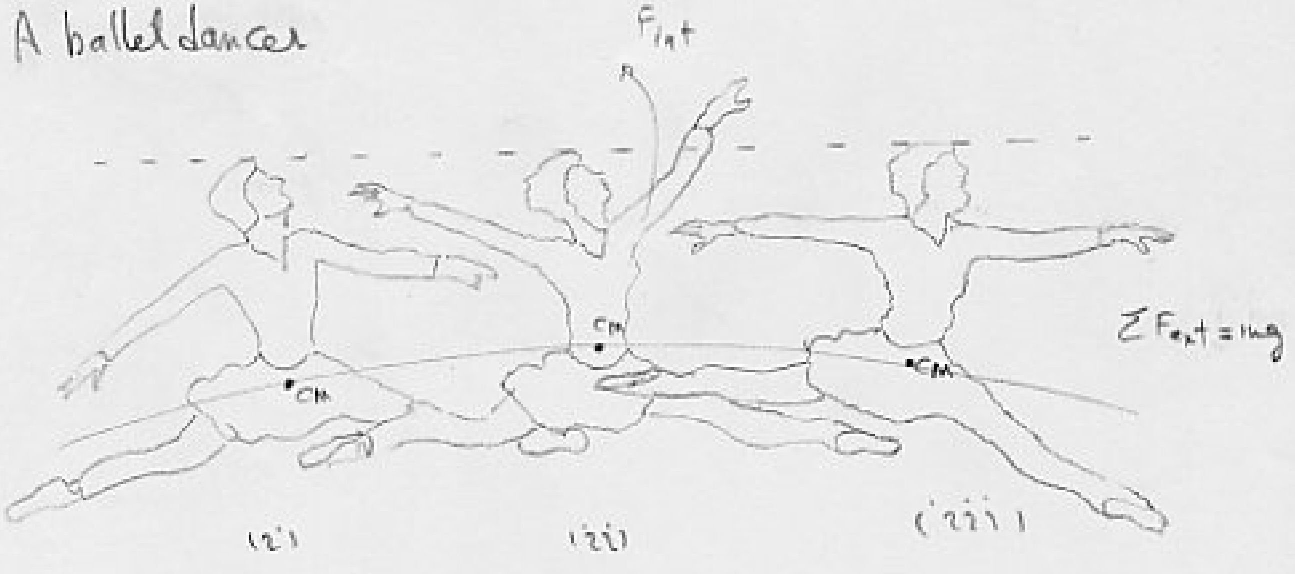
Moon and Earth interact with each other.



Ex. A fire works display;



Ex. A ballet dancer



CM follows a parabolic path

She rises her arms and stretches her legs $\longrightarrow$ The CM shifts upward through her body $\longrightarrow$ her head follows a straight path.

Proof of Equ. (1):

We have $\bar{r}_{cm} = \frac{1}{M}\sum_{i=1}^{n} m_i \bar{r}_i$ $\qquad \frac{d}{dt}\bar{r}_{cm} = \frac{1}{M}\frac{d}{dt}\sum_{i=1}^{n} m_i \bar{r}_i$

$$\rightarrow \quad M\bar{V}_{cm} = \sum_{i=1}^{n} m_i \bar{V}_i \qquad \frac{d}{dt}(M\bar{V}_{cm}) = \frac{d}{dt}\sum_{i=1}^{n} m_i \bar{V}_i$$

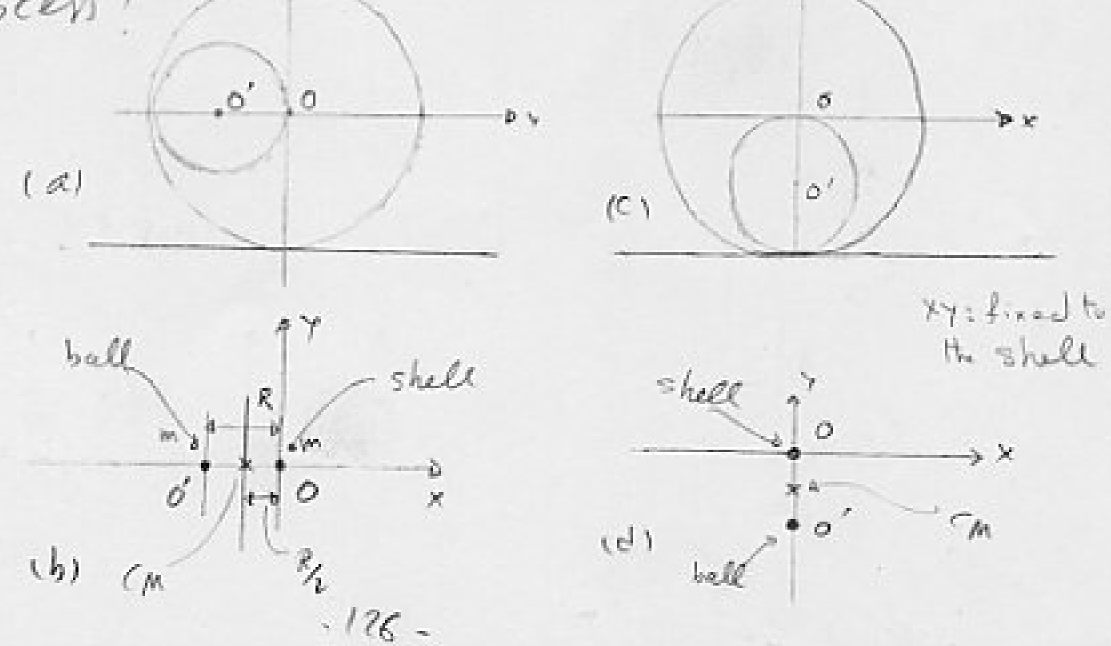
$$\rightarrow \quad M\bar{a}_{cm} = \sum_{i=1}^{n} m_i \bar{a}_i \quad \rightarrow \quad M\bar{a}_{cm} = \sum_{i=1}^{n} F_i$$

$$M\bar{a}_{cm} = \sum_{i=1}^{n_1} F_{i(ext)} + \underbrace{\sum_{i=n_1}^{n} F_{i(int)}}_{=0} = \sum_{i=1}^{n_1} F_{i(ext)}$$

$$\rightarrow \quad \sum \bar{F}_{ext} = M\bar{a}_{cm}$$

Sample prob 9-5

A ball of mass m and radius R is placed inside a spherical shell of the same mass m and inner radius 2R. The combination is at rest on a tabletop in the position shown in Fig. a. The ball is released, rolls back and forth inside, and finally comes to rest at the bottom of the shell, as in Fig. c. What is the displacement d of the shell during this process?



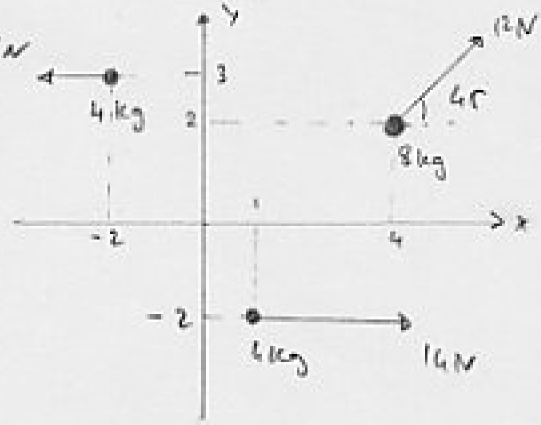
sol.

$\sum F_{ext(x)} = 0 \quad \rightarrow a_{cm(x)} = 0$

Since there is no initial motion $\rightarrow x_{cm}$: stationary

$\rightarrow$ At the final position the shell must move to the left through a displacement $d = \frac{1}{2}R$

Sample prob 9-6

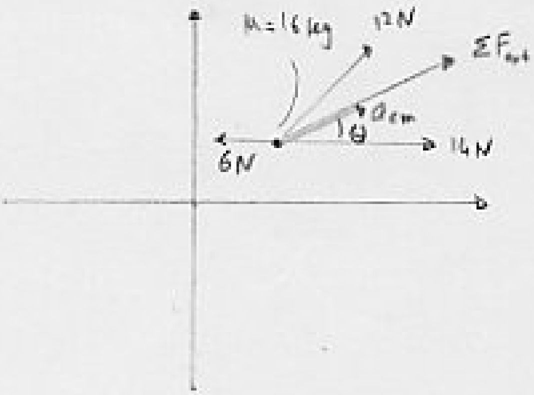


$a_{cm} = ?$

Sol.

$\sum F_{ext(x)} = 14N - 6N + (12N)\cos 45 = 16.5\,N$

$\sum F_{ext(y)} = (12N)\sin 45° = 8.49\,N$



$\sum F_{ext} = \sqrt{(16.5)^2 + (8.49)^2} = 18.6\,N$

$\theta = \tan^{-1} \frac{8.49}{16.5} = 27°$

$a_{cm} = \frac{\sum F_{ext}}{M} = \frac{18.6\,N}{16\,kg} = 1.16\,m/s^2$

But the acc. of each particle is different in mag. and dir.

9.4 Linear Momentum

Def.: $\bar{P} = m\bar{v}$ linear Mom.

Since $m > 0$ and scalar $\rightarrow \bar{P} \parallel \bar{V}$

Newton actually expressed his <u>second</u> law of motion in terms of momentum:

$$\sum \bar{F} = \frac{d\bar{P}}{dt}$$

$$\sum \bar{F} = \frac{d\bar{P}}{dt} = \frac{d}{dt}(m\bar{v}) = m\frac{dv}{dt} = ma \qquad (m = \text{const. incl. M.})$$

Momentum at Very High Speeds

$\bar{P} = m(v)\,\bar{V}$ $\qquad \bar{P} = \dfrac{m\bar{v}}{\sqrt{1-(\frac{v}{c})^2}}$

$\bar{F} = \dfrac{d\bar{P}}{dt}$ holds good with the mentioned change.

9-5 The Linear Mom. of a system of particles

$$\bar{P} = \sum_{i=1}^{n} \bar{P}_i = \sum_{i=1}^{n} m_i \bar{V}_i$$

Comparing with $M\bar{V}_{cm} = \sum_{i=1}^{n} m_i \bar{V}_i$

$\bar{P} = M\bar{V}_{cm}$ linear mom. of system of particles

$$\rightarrow \frac{d\bar{P}}{dt} = M\frac{d\bar{V}_{cm}}{dt} = M\bar{a}_{cm}$$

Comparing with $\sum F_{ext} = M\bar{a}_{cm} \rightarrow \sum F_{ext} = \frac{dP}{dt}$

system of particles

9-6 Conservation of Linear Mom.

Suppose; $\sum \bar{F}_{ext} = 0$ (isolated system)

and <u>no</u> particles <u>leave</u> or <u>enter</u> the system (system is closed)

Then acc. to $\sum \bar{F}_{ext} = \frac{d\bar{P}}{dt} \rightarrow \frac{d\bar{P}}{dt} = 0$

$\rightarrow \bar{P} =$ const. Conservation of linear mom.

$\rightarrow \bar{P}_i = \bar{P}_f$

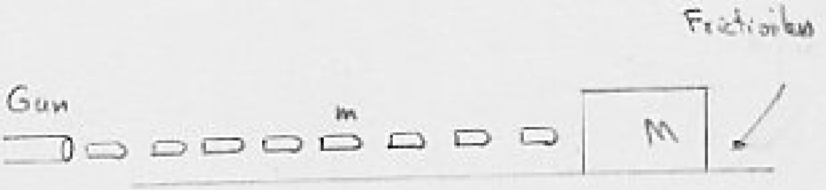
Acc. to $\bar{P} = M\bar{V}_{cm}$, if $\bar{P} =$ const. $\longrightarrow V_{cm} =$ const.

$\rightarrow \bar{a}_{cm} = 0$ which is consistant with $\sum \bar{F}_{ext} = Ma_{cm}$

when $\sum \bar{F}_{ext} = 0$

Sample prob. 9-7

Eight bullets are absorbed by the wooden block.



$m = 3.8\,g$ $v = 1100\,m/s$

$M = 12\,kg$ $V_i = 0$

$V_{final} = ?$

Sol.

Eight bullets and the block form a closed system.
But the system is not isolated, because the block and the bullets all have weight and the surface exerts a normal force on the block. However these forces are vertical and can not change the horizontal mom. of the block and bullets.

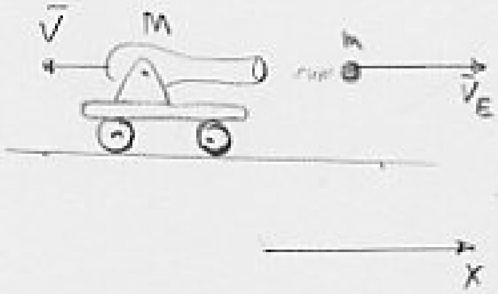
Horizontally, there is no force acting on the system. The forces involved in the collisions are internal forces (within the system) and can not change the mom. of the system.

$\rightarrow P_i = P_f$ horizontally

$P_i = n(mv) + M(0)$ $P_f = (M + nm)V$

$\rightarrow n(mv) = (M+nm)V$ $V = \frac{nm}{M+nm}v = 2.8\ m/s$

Sample prob. 9-8



$M = 1300\,kg \qquad m = 72\,kg$

$V_{relative} = 55\,m/s$

a) $\bar{V} = V_x = ?$

Sol.

Cannon + ball : our system → the forces involved in the firing are internal.

The external forces acting on the system have no components in the horizontal dir.

→ Thus the horizontal component of the total linear mom. of the system must remain unchanged.

$V_{rel\,x} = V_{Ex} - V_x$ the ball's velocity relative to the cannon

$\rightarrow V_{Ex} = V_{rel\,x} + V_x$

$P_{fx} = MV_x + mV_{Ex} = MV_x + m(V_{rel\,x} + \bar{V}_x)$

$P_{ix} = 0$

$P_{ix} = P_{fx} \rightarrow 0 = MV_x + m(V_{rel\,x} + \bar{V}_x) \rightarrow \bar{V}_x = -\dfrac{m V_{rel\,x}}{M+m} = -2.9\,m/s$

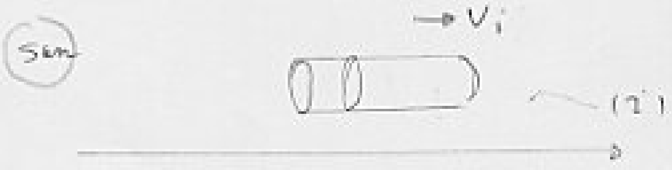
(recoil to the left)

b) $V_E = ?$

$V_{Ex} = V_{rel\,x} + \bar{V}_x = 55\,m/s + (-2.9\,m/s) = 52.1\,m/s$

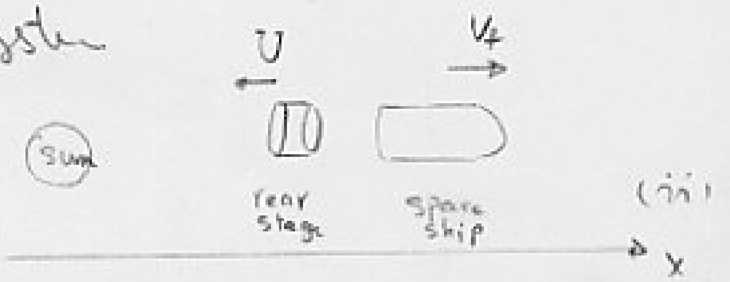
Sample prob. 9-9

A space ship with mass M is traveling in deep space ($g=0$), with velocity $v_i = 2100\ km/h$ rel. to the sun. It ejects a rear stage of mass $0.2M$ with a rel. speed $u = 500\ km/h$. What then is the velocity of the ship?



<u>Sol.</u>

ship + rear stage : our system



Because the system is <u>closed</u>, and isolated ($\Sigma F_{ext} = 0$) $\rightarrow P_i = P_f$

$P_i = Mv_i$, $P_f = 0.2M\,U + 0.8M\,v_f$

$u = v_f - U \rightarrow U = v_f - u$ w.r.t. the sun

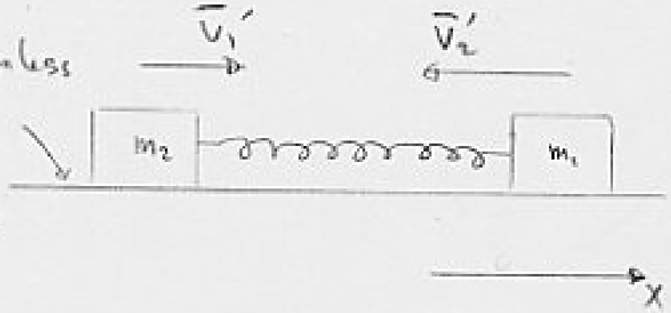
$P_i = P_f \rightarrow Mv_i = 0.2M(v_f - u) + 0.8Mv_f$

$\rightarrow v_f = v_i + 0.2u$ $v_f = 2100\ km/h + 0.2(500\ km/h) = 2200\ km/h$

Sample prob. 9-10

Two blocks are connected by a spring and free to slide on a frictionless horizontal surface. The blocks whose masses are m_1 and m_2, are <u>pulled apart</u> and then released from rest. What fraction of the total kinetic energy of the system will each block have at any later time?

Sol.



Two blocks and spring: our system

$P_i = m_1 V_1 + m_2 V_2 = 0 \quad (V_1 = V_2 = 0)$

$P_f = m_1 V_1' + m_2 V_2'$

$P_i = P_f \longrightarrow 0 = m_1 V_1' + m_2 V_2' \longrightarrow \dfrac{V_1'}{V_2'} = -\dfrac{m_2}{m_1}$ (1)

at any instant

$K_1' = \frac{1}{2} m_1 V_1'^2 \quad K_2' = \frac{1}{2} m_2 V_2'^2$

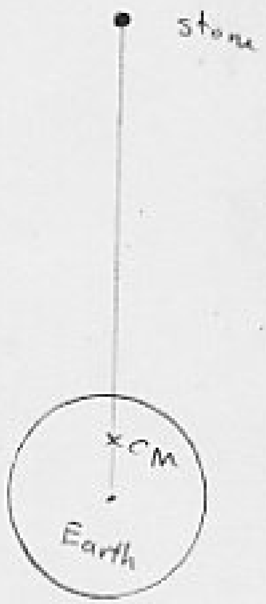
$\text{fraction } 1 = \dfrac{K_1'}{K_1' + K_2'} = \dfrac{\frac{1}{2} m_1 V_1'^2}{\frac{1}{2} m V_1'^2 + \frac{1}{2} m_2 V_2'^2}$

$V_2' = -V_1' \left(\frac{m_1}{m_2}\right) \longrightarrow \text{frac1} = \dfrac{m_2}{m_1 + m_2}$ (indep. of time) (2)

Similarly; $\text{frac 2} = \dfrac{m_1}{m_1 + m_2}$ (3) (")

For example if $m_2 = 10 m_1 \longrightarrow \begin{cases} \text{frac1} = 0.91 \\ \text{frac2} = 0.09 \end{cases}$

If $m_2 >> m_1 \longrightarrow \begin{cases} \text{frac1} \simeq 100\% \\ \text{frac2} \simeq 0\% \end{cases}$



Equs. (2) and (3) apply equally well to a falling stone (if we choose the origin to be CM)

The spring is replaced by the gravitational force.

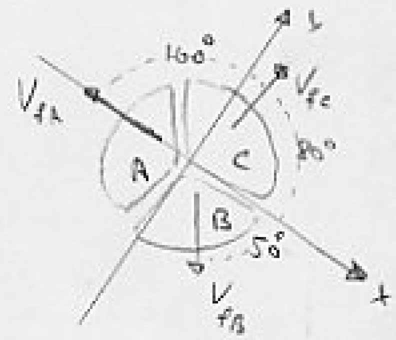
m_1: stone, m_2: Earth

frac1 $\simeq 1$, frac2 $\simeq 0$ but (1) still holds.

Sample prob. 9-11

A firecracker placed inside a coconut of mass M, initially at rest on a frictionless floor, blows the fruit into 3 pieces and sends them sliding across the floor. Piece C, with mass 0.3M, has speed $V_{fC} = 5\ m/s$.

a) What is the speed of piece B with mass 0.2M?



Sol.

Explosion forces are internal, and $\Sigma F_{ext} = 0$.

$\bar{P}_i = \bar{P}_f \longrightarrow \qquad P_{iy} = P_{fy}$

$P_{iy} = P_{Ay} + P_{By} + P_{cy} = 0 \qquad P_{fy} = P'_{Ay} + P'_{By} + P'_{cy}$

$P'_{Ay} = 0$, $P'_{By} = -0.2M\,V'_{By} = -0.2M\,V'_B \sin 50°$

$P'_{cy} = +0.3M\,V'_{cy} = 0.3M\,V'_c \sin 80$

with $V'_c = 5\ m/s \longrightarrow 0 = 0 - 0.2M\,V'_B \sin 50° + 0.3M(5\ m/s)\sin 80°$

$V'_B \simeq 9.6\ m/s$

b) What is the speed of piece A?

$P_{ix} = P_{fx} \qquad P_{ix} = P_{Ax} + P_{Bx} + P_{cx} = 0 \qquad P_{fx} = P'_{Ax} + P'_{By} + P'_{cx}$

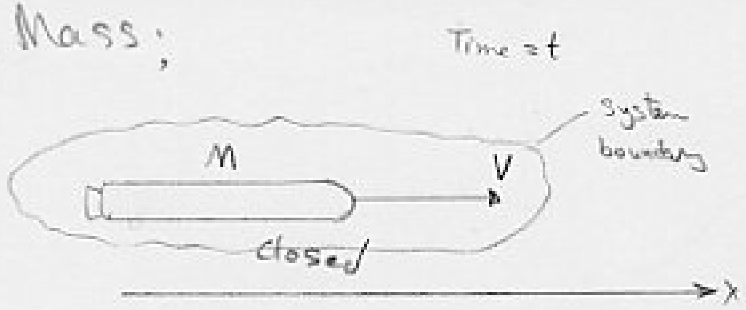
$P'_{Ax} = -0.5M\,V'_A$, $P'_{Bx} = 0.2M\,V'_{Bx} = 0.2M\,V'_B \cos 50°$

$P'_{cx} = 0.3M\,V'_{cx} = 0.3M\,V'_c \cos 80°$

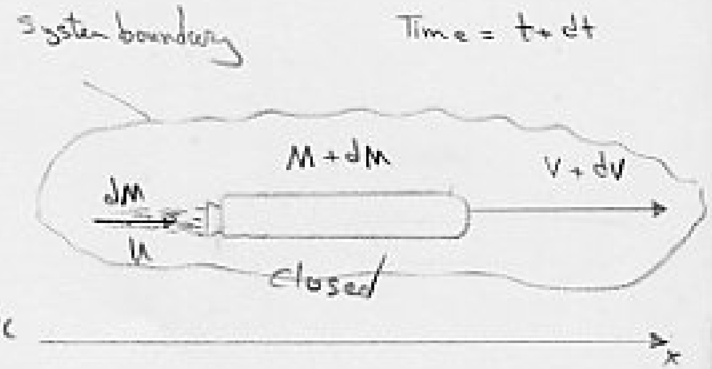
with $V'_c = 5\ m/s$, $V'_B = 9.6\ m/s \longrightarrow 0 = -0.5M\,V'_A + 0.2M(9.6\ m/s)\cos 50° + 0.3M(5\ m/s)\cos 80°$

$\longrightarrow V'_A = 3\ m/s$

9-7 Systems with Varying Mass; A Rocket



Finding the Acceleration



Assume you are in an <u>inertial</u> ref. frame, watching a rocket accelerate through deep space with <u>no</u> <u>gravitional</u> or <u>atmospheric</u> <u>drag</u> forces.

For the rocket;

At t: M, V

At $t+dt$: $M+dM$ $\quad V+dv$ $\qquad$ (dM: negative)

For the exhaust product;

$-dM$, $\quad U$, $\quad$ (during dt)

$\sum F_{ext} = 0$ and the system is closed $\rightarrow \bar{P}_i = \bar{P}_f$

$$Mv = -dM\,U + (M+dM)(v+dv) \qquad (1)$$

$u = (v+dv) - U$ $\qquad$ speed of exhaust products relative to the rocket

$$U = v + dv - u \qquad (2)$$

(2) in (1) $\rightarrow$ $\quad -dM\,u = M\,dv \quad \rightarrow \quad -\frac{dM}{dt}u = M\frac{dv}{dt}$

Let $R \equiv -\frac{dM}{dt}$ rate of fuel consumption $(R>0)$

$\rightarrow$ $Ru = Ma$ at any instant

Here $M = M(t)$, $R = R(t)$, $a = a(t)$ are evaluvated at that instant.

Ru has the dim. of a force ($kg \cdot m/s^2 = N$) and depends only on design characteristics of the rocket engine, namely, the rate R at which it ejects mass and the speed u.

We call this term Ru the thrust of the rocket engine;

$$T \equiv Ru \longrightarrow T = M(t)\,a(t)$$

Finding the Velocity:

$$dv = -u\frac{dM}{M} \rightarrow \int_{v_i}^{v_f} dv = -u\int_{M_i}^{M_f}\frac{dM}{M}$$

$$v_f - v_i = u \ln\frac{M_i}{M_f}$$

Sample prob. 9-12

A rocket whose initial mass M_i is 850 kg consumes fuel at the rate $R = 2.3$ kg/s. The speed u of the exhust gases relative to the rocket engine is 2800 m/s.

a) What thrust does the rocket engine provide?

$$T = Ru = (2.3 \text{ kg/s})(2800 \text{ m/s}) = 6440 \text{ N}$$

b) What is the initial acc. of the rocket?

From Newton's second law;

$$a = \frac{T}{M_i} = \frac{6440}{850} = 7.6 \text{ m/s}^2$$

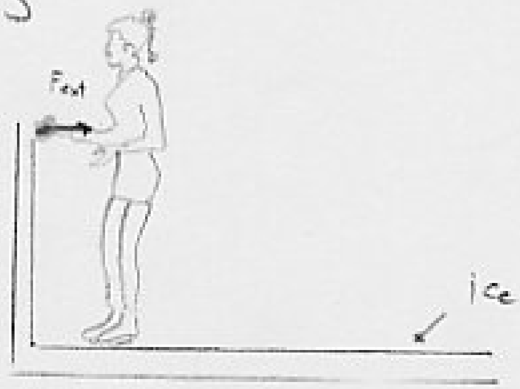
Remark: To be lunched from the Earth's surface, a rocket <u>must</u> have an initial acc. <u>greater</u> than $g = 9.8 \text{ m/s}^2$

c) $v_f = ?$ when $M(t) = 180$ kg

$$v_f = u \ln \frac{M_i}{M_f} = (2800 \text{ m/s}) \ln \frac{850}{180} \simeq 4300 \text{ m/s}$$

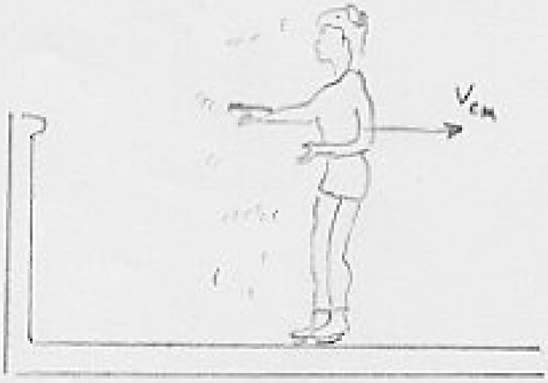
9.8 Systems of Particles: Changes in Kinetic Energy

A skater pushes herself from a railing, picking up some kinetic energy in the process (muscular energy).



$\Delta K = ?$

We cannot use $W = K_f - K_i = \Delta K$



(for a single particle, work-energy theorem), which tells us the increase in the kinetic energy of a particle is equal to the work done by the net force F_{ext} acting on the particle.

The reason:

The skater cannot be consider as a particle!

The criterion for representing a body as a single particle is that every part of it moves in the same way

In the essential act of extending her arm to push herself away from the railing, the skater fails to meet this requirement.

Thus she must be treated as a system of particles, to which $W = K_f - K_i = \Delta K$ simply does not apply.

We must use: $\Sigma F_{ext} = M a_{cm}$ (sys. of particles)

Skater: sys. of particles

F_{ext} : the force acting on her center of mass by the railing.

Suppose that:

dx_{cm} : the displacement of the center of mass due to F_{ext}.

$$F_{ext} = M a_{cm} \rightarrow F_{ext}\, dx_{cm} = M a_{cm}\, dx_{cm} = M \frac{dV_{cm}}{dt} dx_{cm}$$
$$= M \frac{dx_{cm}}{dt} dV_{cm} = M V_{cm}\, dV_{cm}$$

$$\rightarrow F_{ext}\, dx_{cm} = M V_{cm}\, dV_{cm}$$

$$\int_{x_i}^{x_f} F_{ext}\, dx_{cm} = \left(\tfrac{1}{2} M V_{cm}^2\right)_f - \left(\tfrac{1}{2} M V_{cm}^2\right)_i$$

$$\rightarrow \int_{x_i}^{x_f} F_{ext}\, dx_{cm} = K_{cm_f} - K_{cm_i} = \Delta K_{cm} \quad (1)$$

For $F_{ext} = \text{const.} \longrightarrow \int_{x_i}^{x_f} F_{ext}\, dx_{cm} = F_{ext} \int_{x_i}^{x_f} dx_{cm} = F_{ext}\,(x_{cm_f} - x_{cm_i})$
$$= F_{ext}\, d_{cm}$$

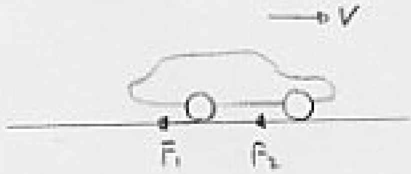
$$\rightarrow F_{ext}\, d_{cm} = K_{cm_f} - K_{cm_i} = \Delta K_{cm} \quad (2)$$

These equs. also apply to the breaking of a car.

In this situation, the frictional force exerted by the road on the tires points toward the rear of the car.

$$\longrightarrow \int_{x_i}^{x_f} F_{ext}\, dx_{cm} < 0 \rightarrow \Delta K_{cm} < 0$$

$\longrightarrow$ this energy is transferred to the thermal energy of the breaking surfaces.

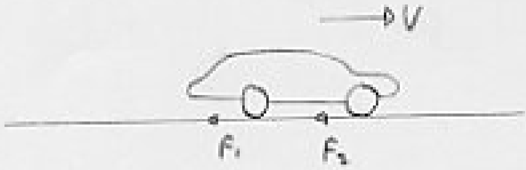


$F_{ext} = F_1 + F_2 + F_3 + F_4 < 0$

Sample prob. 9-13

A car of $M = 1400\,kg$ brakes to rest from a speed $V_{cm} = 24\,m/s$ covering a distance $d_{cm} = 180\,m$. $F_{ext} = ?$ (total frictional force)

Assume F_{ext} is const. and the tires do not skid.



$$F_{ext}\, d_{cm} = K_{cmf} - K_{cmi}$$

$$-F_{ext}\, d_{cm} = 0 - \frac{1}{2} M V_{cm}^2$$

$$F_{ext} = \frac{M V_{cm}^2}{2 d_{cm}} = \frac{(1400)(24)^2}{2(180)} = 2240\ N$$

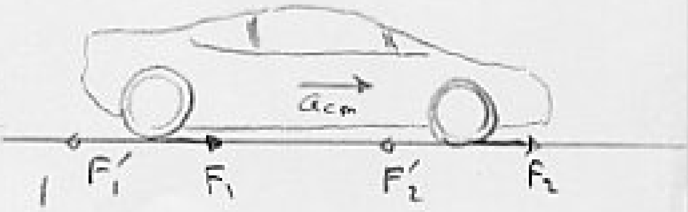
Note that ΔK_{cm}: change in kinetic energy associatied with only the translational motion (i.e. the motion of the system consentrated at on its CM).

Any changes in energy associated with rotation of the system or with internal motions or vibrations are not included in ΔK_{cm}.

Equ. (2) is equivalent to $-fd = \Delta K$ (P 114).

What makes a Car go?

F_1, F_2; frictional forces cause the car to move.



$F_{ext} = F_1 + F_2 + \underbrace{F_3 + F_4}_{\text{not shown}}$

Gasoline combustion $\longrightarrow$ Kinetic energy of the engine $\longrightarrow$ rotation of the tires $\longrightarrow$ F_1', F_2' exerted by the car to the road $\longrightarrow$ F_1, F_2 reation forces exerted by the road to the car

In this prob. we must use equ (1) and (2).