Chapter 8

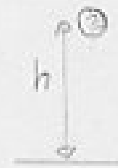
Conservation of Energy

8-1 Work and Potential Energy

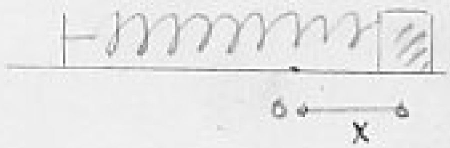
Energy is a measure that is associated with a <u>state</u> or condition of one or more bodies.

i) Kinetic energy K is associated with the state of motion of a body
ii) Potential " U " " " " configration " "

Potential energy is often said to be <u>stored</u> in a system, in the sense that it could later result in motion.



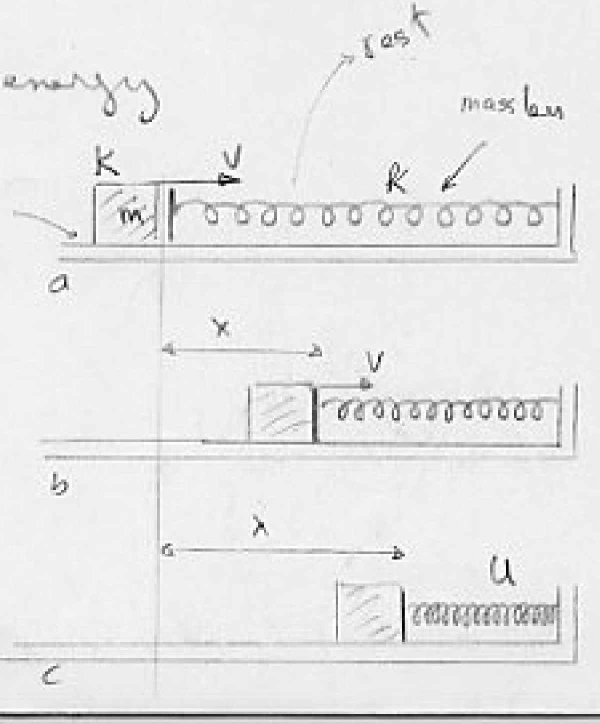
<u>Ex.</u> i) Gravitational pot. energy
ii) Elastic pot. energy (spring)

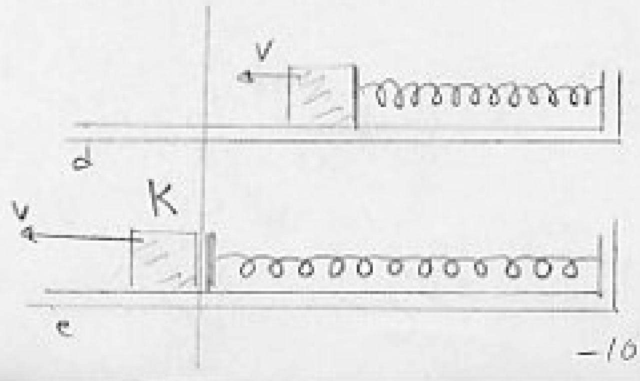


8-2 Mechanical Energy

$E = K + U$ mechanical energy

The spring Force:





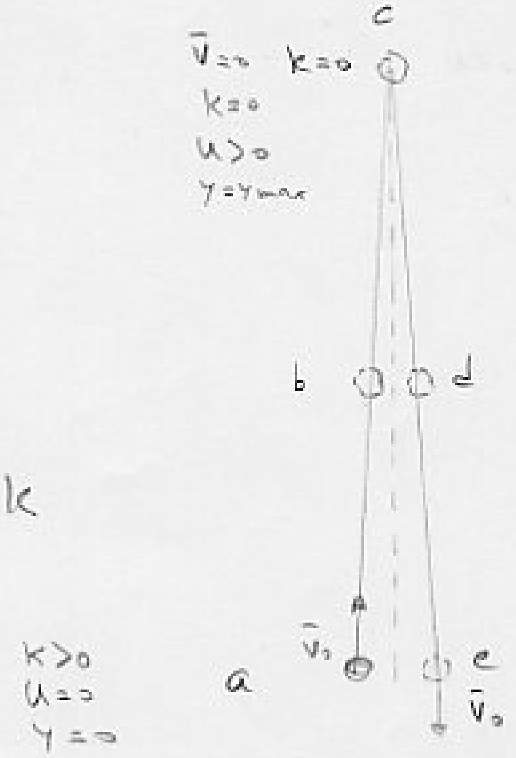
$F(x) = -kx$ spring force

$E = U_a + K_a = U_b + K_b = \dots = U_e + K_e$ (because $K_a = K_e$)

$E = U_1 + K_1 = \dots = U_n + K_n$

Or $\Delta K + \Delta U = 0$

The Gravitational Force



compare with the spring case.

In upward motion:

mg dose work $\longrightarrow$ decreasing $\bar{v}$ ad K
increasing U

In downward motion:

mg dose work $\longrightarrow$ increasing $\bar{v}$ ad K
decreasing U

The kinetic energy of the ball is trasfered to the ball-Earth system (as gravitational pot. energy) during the ball raise. As the ball falls this stored energy is tranfered to the ball (as kinetic energy).

During both processes: $E = K + U = \text{const}$.

The Kinetic Frictional Force

In this case there is no way the body can regain its original kinetic energy.

The kinetic energy in this case instead of being stored as the potential energy, is dissipated. (its transfer cannot be reversed).

8-3 Determining the Potential Energy

$$\begin{cases} \Delta K + \Delta U = 0 \\ W = \Delta K \quad \text{(Kinetic energy theorem)} \end{cases} \longrightarrow \Delta U = -W \quad \text{(definition of } U\text{)}$$

Thus: $F \longrightarrow$ changes the configuration of the system

$\xrightarrow{\text{causes}} \Delta U$

W: work done by F

For One-dim. motion:

$$\Delta U = -W = -\int_{x_1}^{x} F(x)\,dx$$

Only changes in pot. energy are physically important.

$\rightarrow \begin{cases} \text{i) We choose some arbitrary ref. configuration } x_0. \\ \text{ii) We assign it a pot. energy } U(x_0) \text{ (usually zero).} \end{cases}$

$$U(x) = U(x_0) + \Delta U = U(x_0) - \int_{x_0}^{x} F(x)\, dx$$

Elastic Potential Energy

For the spring, we choose, $x_0 = 0$ (its relaxed state)

and $U(x_0) = 0$

$$\rightarrow U(x) = 0 - \int_0^x (-kx)\, dx = \frac{1}{2} k x^2$$

$$U + K = E \quad \rightarrow \frac{1}{2} k x^2 + \frac{1}{2} m v^2 = 0$$

Gravitational Potential Energy

$$F(y) = -mg$$

Minus sign: the force points downward in the dir. of decreasing y.

Let $y_0 = 0$, $U(y_0) = 0$

$$U(y) = 0 - \int_0^y -(mg)\, dy = mgy$$

$mgy + \frac{1}{2}mv^2 = E$ particle-Earth system

In 3-dimensional motion

$$W = \int_{x_0}^{x} F_x \, dx + \int_{y_0}^{y} F_y \, dy + \int_{z_0}^{z} F_z \, dz$$

$$\vec{F} = -mg\hat{j} \rightarrow W = 0 + \int_0^y (-mg)\,dy + 0 = -mgy$$

the work done by $\vec{F}$

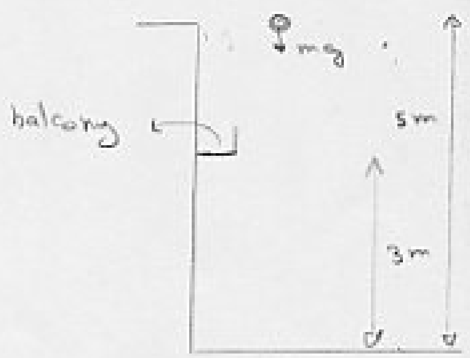
$$\Delta U = -W \longrightarrow U = mgy$$

$\rightarrow$ U depends on <u>y only</u> not <u>x or z</u>.

<u>Ex.</u> Pendulum

Sample Prob 8-1

A ball drops from the roof $(m = 2\,kg)$



a) $U = ?$ if $y_0 = 0$ 1) at the ground

$$U = mgy = 2(9.8)(5) = 98\,J$$

2) at the bottom of balcony

$$U = mgy = 2(9.8)(5-3) = 39\,J$$

3) at the roof

$$U = mgy = 2(9.8)(0) = 0\,J$$

4) 1m above the roof

$$U = mgy = 2(9.8)(-1) = -19.6\,J$$

b) For each choice of ref. configuration $\Delta U = ?$

$$\Delta U = mgy_f - mgy_i = mg\Delta y$$

$\Delta y = -5$ for all case

$$\Delta U = -98\ J$$

c) $V = ?$ just before landing

$$\Delta K = -\Delta U$$

$$\Delta K = K_f - K_i = \tfrac{1}{2} m V_f^2 - 0 = \tfrac{1}{2}(2) V_f^2$$

$$\Delta U = -98\ J \qquad V_f = \sqrt{\frac{98}{\frac{1}{2}(2)}} = 9.9\ m/s$$

d) If the mass of ball changes to $m + \Delta m$, how does the answer to (c) change!

$$\Delta k = -\Delta U = -mg\Delta y$$

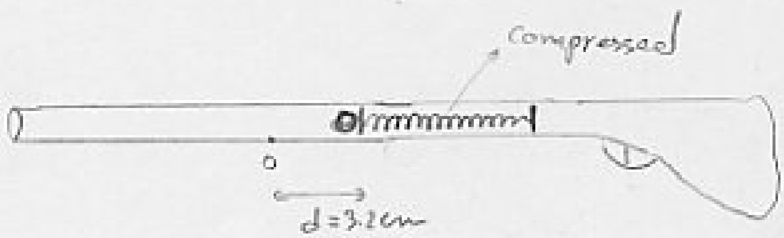
$$\tfrac{1}{2} m V_f^2 - 0 = -mg\Delta y$$

$$m \rightarrow m + \Delta m \longrightarrow \tfrac{1}{2}(m + \Delta m) V_f^2 - 0 = -(m + \Delta m) g \Delta y$$

$$V_f^2 = \sqrt{-2g\Delta y} = 9.9\ m/s \qquad \text{(the same)}$$

Sample prob. 8-2

$m = 12\,g \qquad k = 7.5\ N/cm$



$V = ?$ when the gun is fired.

$$E_i = E_f \qquad U_i + K_i = U_f + K_f \qquad \tfrac{1}{2} k d^2 + 0 = 0 + \tfrac{1}{2} m V^2$$

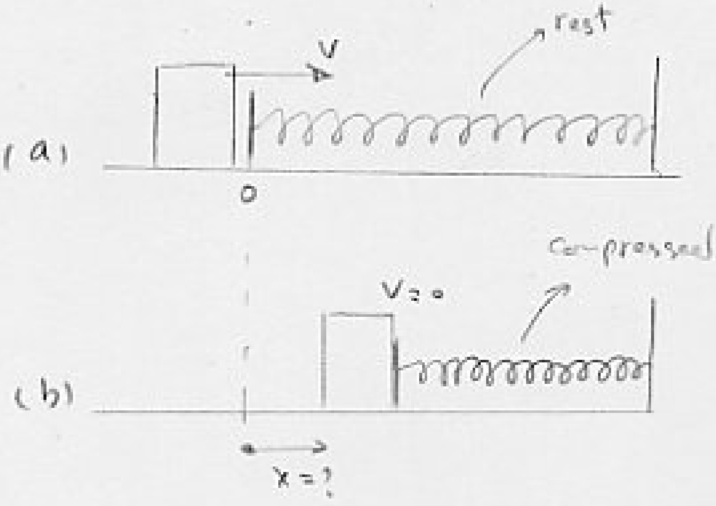
$$V = d\sqrt{\frac{k}{m}} = (0.032\,m)\sqrt{\frac{750\ N/m}{12\times10^{-3}\,kg}} = 8\ m/s$$

Sample prob 8-3

$m = 1.7\ kg$

$V = 2.3\ m/s$

$k = 320\ N/m$



a) $x = ?$

$E_a = E_b \rightarrow U_a + K_a = U_b + K_b$

$0 + \frac{1}{2} m V^2 = \frac{1}{2} k x^2 + 0 \rightarrow x = V\sqrt{\frac{m}{k}} = (2.3)\sqrt{\frac{1.7}{320}} = 0.17\ m$

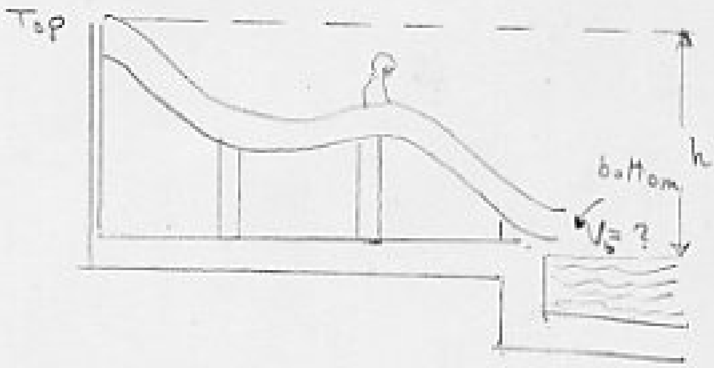
b) For what $\underline{x}$, $U = K = \frac{E}{2}$

In (a) position $E = K = \frac{1}{2} m v^2 = \frac{1}{2}(1.7)(2.3)^2 = 4.5\ J$

$U(x) = \frac{1}{2} k x^2 = \frac{1}{2} E \longrightarrow x = \sqrt{\frac{E}{k}} = \sqrt{\frac{4.5}{320}} = 0.12\ m$

Sample prob. 8-4

$\begin{cases} h = 8.5\ m \\ V_t = 0 \end{cases}$ $\quad V_b = ?$



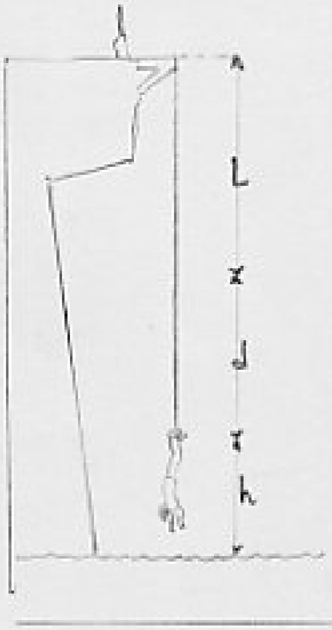
$\frac{1}{2} m V_b^2 + m g y_b = \frac{1}{2} m V_t^2 + m g y_t$

$\rightarrow V_b^2 = V_t^2 + 2g(y_t - y_b)$

$V_b = \sqrt{2gh} = \sqrt{2(9.8)(8.5)} = 13\ m/s$

sample prob. 8.5

A 61.0 kg bungee-cord jumper is on a bridge 45 m above a river. In its relaxed state, the elastic bungee cord has length $L = 25$ m. Assume that the cord obeys Hooke's law with a spring const of 160 N/m.



a- If the jumper stops before reaching the water, what is the height h of her feet above the water at her lowest point.

sol.

$$\Delta U_g = mg\Delta y = -mg(L+d)$$

$$\Delta U_e = \frac{1}{2}kd^2$$

$(v_i = 0,\ v_f = 0)$

$$\Delta K + \Delta U = 0 \rightarrow \Delta U_e + \Delta U_g + \Delta K = 0$$

$$\frac{1}{2}kd^2 - mg(L+d) + 0 = 0 \rightarrow \frac{1}{2}kd^2 - mgd - mgL = 0$$

$$\frac{1}{2}(160\ N/m)d^2 - (61\ kg)(9.8\ m/s^2)d - (61\ kg)(9.8\ m/s^2)(25\ m) = 0$$

$$\rightarrow d = 17.9\ m$$

$$L+d = 42.9 \rightarrow h = 45 - 42.9 = 2.1\ m$$

b- what is the net force on her at her lowest point (in particular is it zero)?

$F = -kx = -(160\ \text{N/m})(-17.9\text{m}) = 2864\ \text{N}$

$mg = 597.8\ \text{N}$

$\Sigma F = 2864 - 597.8 \simeq 2270\ \text{N}$ the net force

This force will yank her back upward (at the lowest point).

8-4 Conservative and Nonconservative Forces:

When a force changes the state of a system, if a change in potential energy can be associated with that change in state, the force is said to be <u>conservative</u>.
Otherwise, the force is said to be <u>nonconservative</u>.

<u>Ex.</u>

Spring force, gravitational force are conservative.

Drag force, frictional force are non- " .

How to know the force is conservative?

First Test:

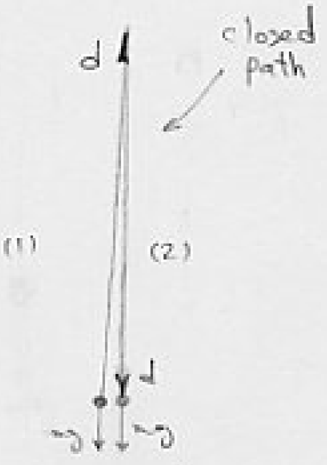
A force is conservative if the work it does on a particle that moves through a <u>closed</u> path is <u>zero</u>, otherwise the force is nonconservative.

because $\Delta U = -W$ $\quad \Delta U = 0 \rightarrow W = 0$

<u>Ex.</u>

Gravitational force on a ball, is conservative.

$$W = W_1 + W_2 = 0$$



ascent $\quad$ descent

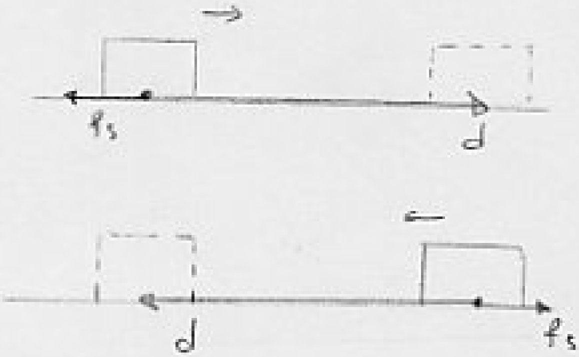
$W_1 < 0$ $\quad$ $W_2 > 0$

<u>Ex.</u> Frictional force is nonconservative.

$$W = W_1 + W_2 < 0$$

$w_1 < 0$ (1)

$w_2 < 0$ (2)



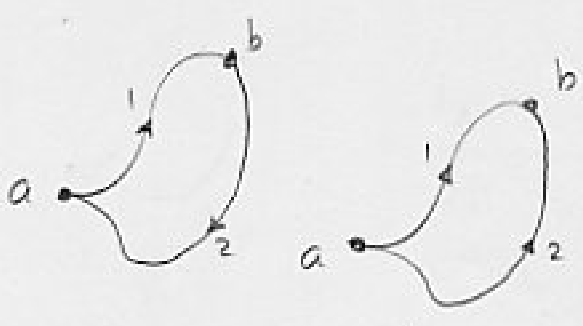
Second Test

A force is conservative if the work it does on a particle that moves between two points is the <u>same</u> for all paths <u>connecting</u> those points; otherwise, the force is non-conservative.

The reason:

Acc. to the first test; for a conservative force

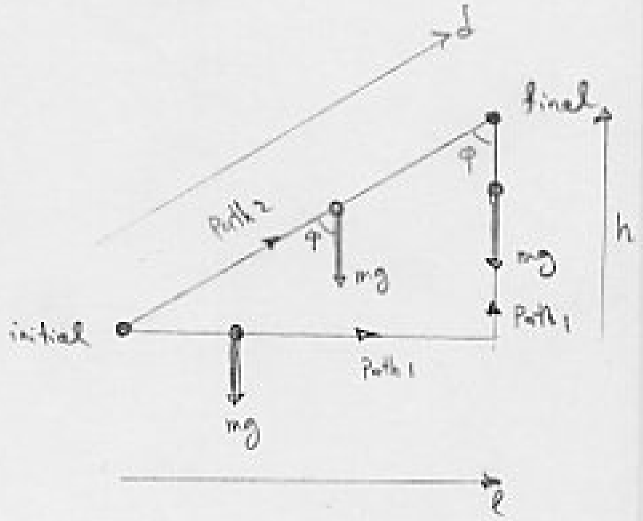
$$W_{ab,1} + W_{ba,2} = 0$$



(a) $\quad$ (b)

$$\rightarrow \begin{cases} W_{ab,1} = -W_{ba,2} \\ \text{but} \quad W_{ab,1} + W_{ba,2} = 0 \longrightarrow \quad W_{ab,1} = W_{ab,2} \end{cases}$$

$\uparrow$ acc. to the first test



Ex.

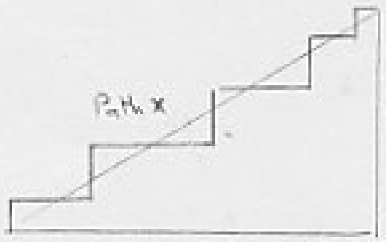
$$W_{Path_1} = m\bar{g}\cdot\bar{l} + m\bar{g}\cdot\bar{h}$$

$$= 0 - mgh$$

$$W_{path_2} = m\bar{g}\cdot\bar{d} = mgd\cos(\pi-\varphi)$$

$$= -mgd\cos\varphi = -mgh$$

$$W_{Path_1} = W_{Path_2} \underset{\text{also}}{=} W_{Path\,x}$$



8-5 Using A Pot. Energy Curve

Finding the Force Analytically

We know how to find $U(x)$ from $F(x)$

i.e. $U(x) = U(x_0) + \Delta U = U(x_0) - \int_{x_0}^{x} F(x)\,dx$

Now we want to go the other way.

$\Delta U(x) = -W = -F(x)\,\Delta x$

$\Delta x \to 0 \quad \longrightarrow \quad F(x) = -\dfrac{dU(x)}{dx}$ (one-dim)

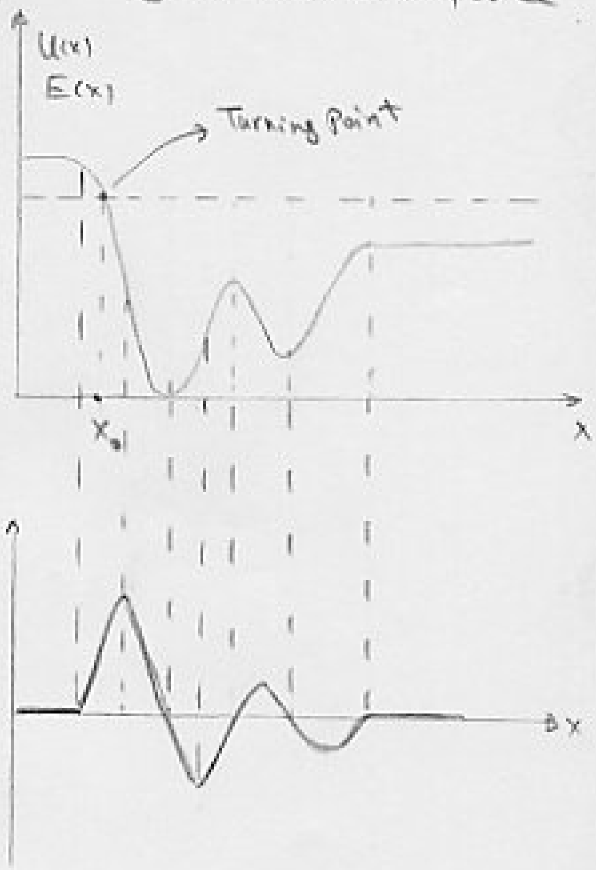
A check:

$U(x) = \frac{1}{2}kx^2 \quad \to \quad F(x) = -kx$ For spring

$U(x) = mgx \quad \to \quad F(x) = -mg$ gravitational force

The Potential Energy Curve:

$\rightarrow$ Fig.(a), Fig.(b)



(a)

(b)

Turning Points:

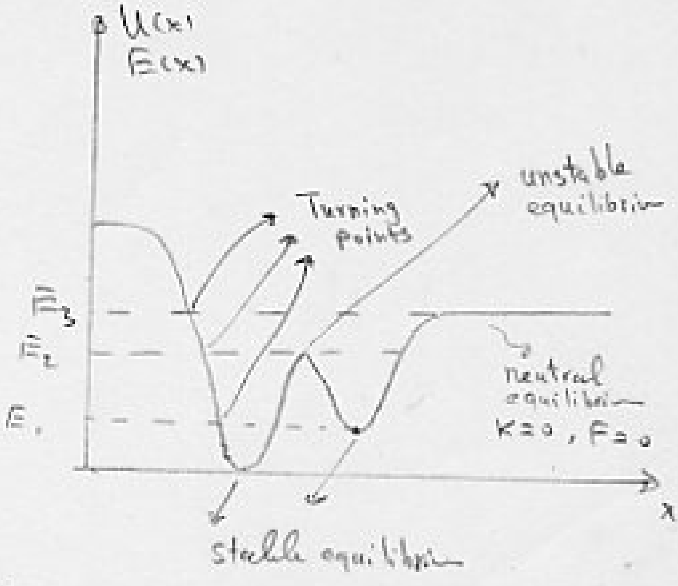
$U(x) + K(x) = E$

for conservative sys.

$K(x) = E - U(x)$

$\longrightarrow x_0$: Turning Point

Equilibrium Points:



8-6 Conservation of Energy

$\Delta K + \Delta U = 0$ (no frictional force)

$\Delta K + \Delta U + \Delta E_{int.} = 0$ (with frictional force)

$E_{int} \equiv$ internal energy

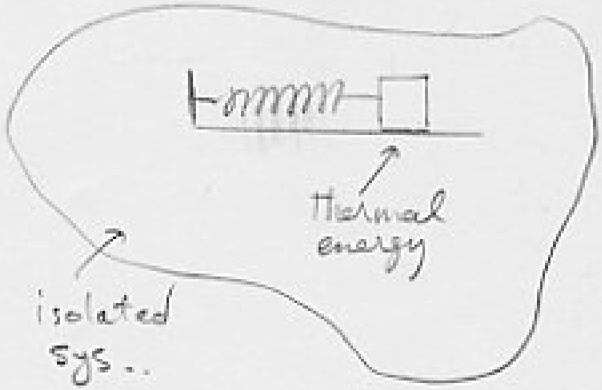
Thermal energy is a form of internal energy, since it is associated with the random motions of the atoms and molecules within an object.

In general: $\Delta K + \Delta U + \Delta E_{int} + \begin{pmatrix}\text{changes in other}\\ \text{forms of energy}\end{pmatrix} = 0$

In an isolated system, energy can be transfered from one form to another, but the total energy of the system remains const.

If forces act on the system boundary and do work W on bodies within the system, the system is not isolated. Then;

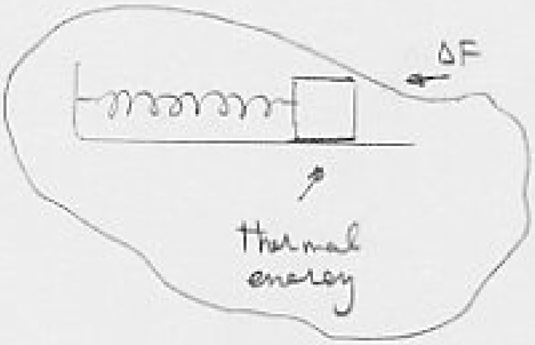
$$W = \Delta K + \Delta U + \Delta E_{int}$$



$$\underbrace{\Delta K + \Delta U}_{\Delta E} + \Delta E_{int} = 0$$

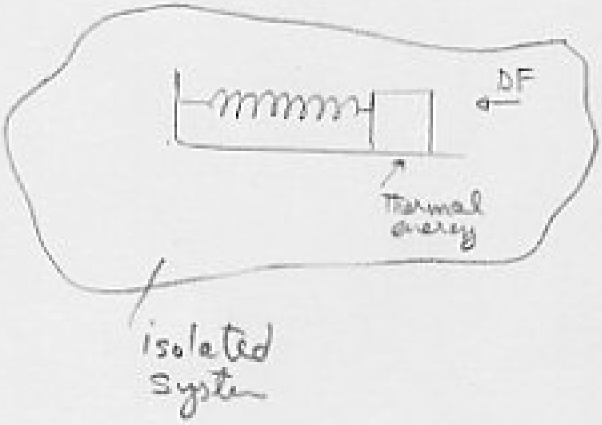
Ex.

We exert ΔF in each oscillation in such a way to compensate the thermal energy loss.
So the system is not isolated and the work done by ΔF is W.



$$W = \Delta K + \Delta U + \Delta E_{int}$$

If we include ΔF agent to our system then the system again is isolated

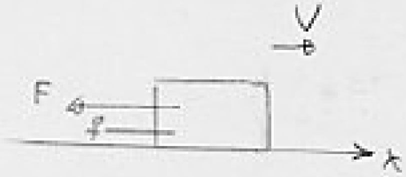


$$\Delta K + \Delta U + \Delta E_{int} = 0$$

8-7 Work Done by Frictional Forces

Assume;

F: conservative force (const)
f: Frictional " (const.)
Both in the same dir.



Our system: the block (not the block-floor)

$$\sum F = -F - f = ma$$

Since $F = $ const. $f = $ const. $\longrightarrow$ $a = $ const.

$\rightarrow V_f^2 = V_i^2 + 2ad \rightarrow a = \dfrac{V_f^2 - V_i^2}{2d}$

$-F - f = m\left(\dfrac{V_f^2 - V_i^2}{2d}\right) \rightarrow -Fd - fd = \frac{1}{2}mV_f^2 - \frac{1}{2}mV_i^2 = \Delta K$

$-Fd = W$ the work done by the conservative force

we had $\Delta U = -W \rightarrow \Delta U = -(-Fd)$

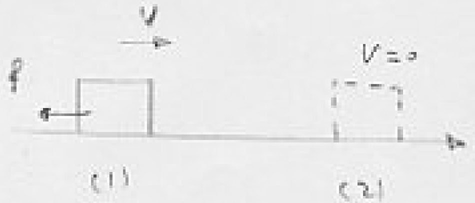
$\rightarrow -Fd = -\Delta U$

$\rightarrow -fd = \Delta K + \Delta U = \Delta E$ change in the mechanical energy of the system

Since $\Delta E < 0 \rightarrow$ the energy of the sys decrease.

Ex.

A block with an initial velocity of V comes to the rest due to the frictional force. (on a horizontal surface). (our sys: only the block)



Acc. to $W = \Delta K + \Delta U + \Delta E_{int}$

Since $\Delta U = 0 \rightarrow W_f = \Delta K + \Delta E_{int}$ (1) work done on the block by the frictional force.

Acc. to $-fd = \Delta K + \Delta U = \Delta E$

$\rightarrow -fd = \Delta K$ (2) the loss of the mechanical energy dissipated by the frictional force.

(1) (2) $\rightarrow W_f \neq -fd$

W_f: The portion of $(-fd)$, leaving the system.

Ex.

A block with the initial kinetic energy $K_i = 100\,J$ slides on a horizontal surface with nonzero friction and finally stops and the internal energy of the block increases by $40\,J$.

$\Delta K = K_f - K_i = -100\,J$ $\qquad (\Delta U = 0)$

$-fd = \Delta K = -100\,J$

$W_f = \Delta K + \Delta E_{int} = -100 + 40 = -60\,J$

i.e. $-(-60)\,J$ is transfered from the block to the floor (external system).

We see that work-energy theorem ($W = \Delta K$), does not hold

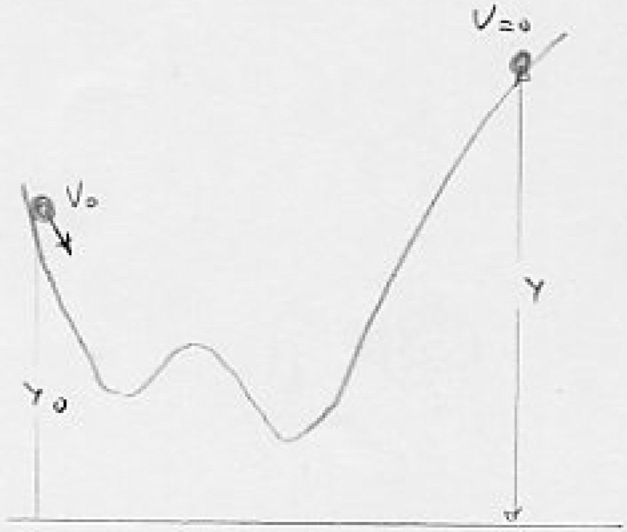
i.e. $W_f \neq \Delta K$

Sample prob. 8-6

$m = 6.0\,kg$

$y_0 = 8.5\,m \quad y = 11.1\,m \quad v_0 = 7.8\,m/s$

Increase of the thermal energy of the ball and ramp = ?



Sol.

$\Delta K + \Delta U + \Delta E_{int} = 0$

$(0 - \frac{1}{2}mv_0^2) + mg(y - y_0) + \Delta E_{int} = 0$

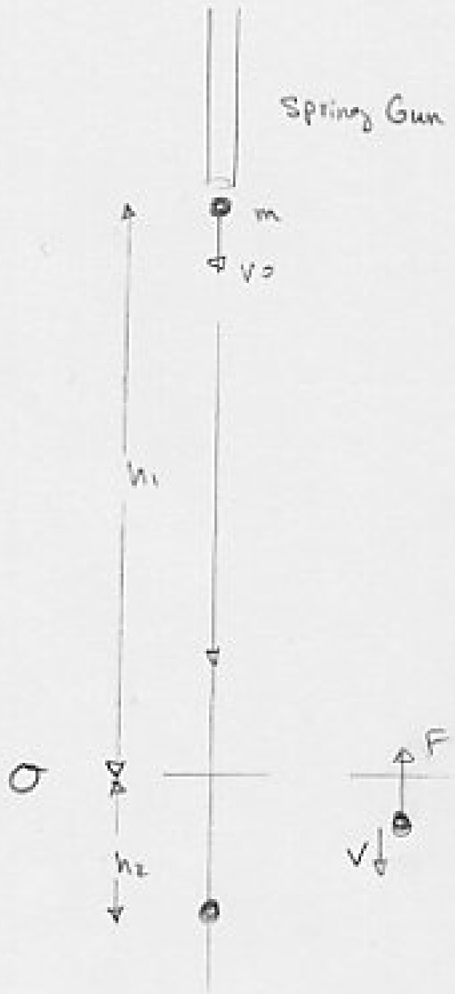
$\Delta E_{int} \simeq 30\,J$

Sample prob. 8-7

$m = 5.2\,gr \quad h_1 = 18\,m \quad h_2 = 21\,cm$

$V_0 = 14\,m/s$

a) Change in the mechanical energy = ?



$\Delta E = \Delta K + \Delta U = (0 - \frac{1}{2} m v_0^2) - mg(h_1 + h_2)$

$-(h_1+h_2)$: downward total displacement

$\Delta E = -\frac{1}{2}(5.2\times10^{-3}\,kg)(14\,m/s)^2 -$

$\quad -(5.2\times10^{-3}\,kg)(9.8\,m/s^2)(18 + 0.21\,m)$

$= -1.437\,J$

b) Change in the internal energy of the ball-Earth system = ?

$\underbrace{\Delta K + \Delta U}_{\Delta E} + \Delta E_{int} = 0$ for an isolated sys.

(The ball-Earth system is an isolated sys.)

$\rightarrow \Delta E + \Delta E_{int} = 0 \quad \rightarrow \Delta E_{int} = -\Delta E = 1.437\,J$

c) $F = ?$ (average force)

Acc. to $-fd = \Delta K + \Delta U = \Delta E \quad \rightarrow -F h_2 = \Delta E$

$F = \frac{\Delta E}{-h_2} = \frac{-1.437\,J}{-0.21\,m} = 6.84\,N$

We could also solve this prob. using Newton's second law.

8-8 Mass and Energy

$E = mc^2$

$Q = -\Delta m c^2$ energy released or absorbed in the nuclear or chemical reaction.

Object	Mass (kg)	Energy Equivalent
Electron	9.1×10^{-31}	8.2×10^{-14} J
Proton	1.7×10^{-27}	1.5×10^{-10} J
Dust particle	1×10^{-13}	1×10^{4} J
Penny	3.1×10^{-3}	2.8×10^{14} J

Units $1u = 1.66 \times 10^{-27}$ kg atomic mass unit

$1eV = 1.6 \times 10^{-19}$ J

$c^2 = 9.32 \times 10^{8}$ eV/u $= 932$ MeV/u

Sample prob. 8-8

$2H_2 + O_2 \longrightarrow 2H_2O$

$Q = 4.85 \times 10^{5}$ J (the energy released)

$\frac{\Delta m}{M} = ?$

Sol.

$\Delta m = \frac{-Q}{c^2} = \frac{-4.85 \times 10^{5}\text{ J}}{(3 \times 10^{8}\text{ m/s})^2} = -5.39 \times 10^{-12}$ kg

$M = 2(2.02) + 32 = 36$ gr $= 0.036$ kg $\frac{|\Delta m|}{M} = 1.5 \times 10^{-10}$

Sample prob. 8-9

$$^{235}U + n \longrightarrow {}^{140}Ce + {}^{94}Zr + 2n$$

$m(^{235}U) = 235.4\,u$ $m(^{140}Ce) = 139.91\,u$

$m(^{94}Zr) = 93.91\,u$ $m(n) = 1.00867\,u$

a) $\frac{|\Delta m|}{M} = ?$

$\Delta m = (139.91 + 93.91 + 2(1.00867)) - (235.4 + 1.00867) = 0.211\,u$

$M = 235.4 + 1.00867 = 236.05\,u$ the mass of interacting particles

$\frac{|\Delta m|}{M} = \frac{0.211}{236.05} = 0.00089$

b) Energy released in fission reaction = ?

$Q = -\Delta m c^2 = (-0.211\,u)(932\,MeV/u) = 197\,MeV$

Sample prob. 8-10

$d \longrightarrow n + p$

How much energy is involved in the process ?

Sol.

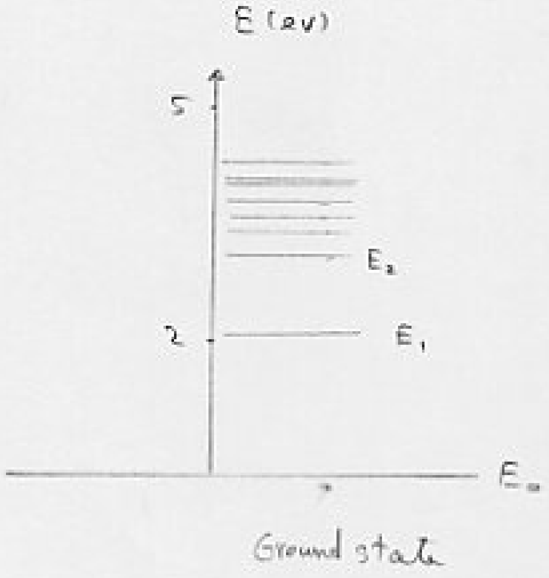
$M_d = 2.01355\,u$ $m_p = 1.00728\,u$ $m_n = 1.00867\,u$

$\Delta m = (m_p + m_n) - M_d = 0.00240\,u$ $Q = -\Delta m c^2 = -2.24\,MeV$

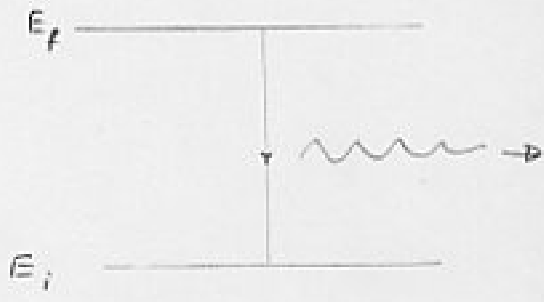
This is the binding energy of the deutron (the energy is absorbed)

E_b (electron) = 13.6 eV in Hydrogen atom

8-9 Energy is Quantized



Quantization and the Emission of Light:



$E_f - E_i = hf$

$h = 6.63\times10^{-34}$ J·s $= 4.14\times10^{-15}$ eV·s

Sample prob. 8-9

$$^{235}U + n \longrightarrow {}^{140}Ce + {}^{94}Zr + 2n$$

$m(^{235}U) = 235.4\,u$ $m(^{140}Ce) = 139.91\,u$

$m(^{94}Zr) = 93.91\,u$ $m(n) = 1.00867\,u$

a) $\frac{|\Delta m|}{M} = ?$

$$\Delta m = (139.91 + 93.91 + 2(1.00867)) - (235.4 + 1.00867) = 0.211\,u$$

$M = 235.4 + 1.00867 = 236.05\,u$ the mass of interacting particles

$$\frac{|\Delta m|}{M} = \frac{0.211}{236.05} = 0.00089$$

b) Energy released in fission reaction = ?

$$Q = -\Delta m c^2 = (-0.211\,u)(932\,\mathrm{Mev}/u) = 197\,\mathrm{Mev}$$

Sample prob. 8-10

$$d \longrightarrow n + p$$

How much energy is involved in the process?

Sol.

$M_d = 2.01355\,u$ $m_p = 1.00728\,u$ $m_n = 1.00867\,u$

$\Delta m = (m_p + m_n) - M_d = 0.00240\,u$ $Q = -\Delta m c^2 = -2.24\,\mathrm{Mev}$

This is the binding energy of the deutron (the energy is absorbed)

$E_b(\text{electron}) = 13.6\,eV$ in Hydrogen atom