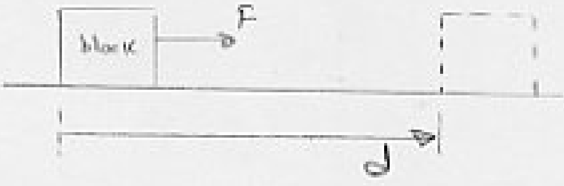
Chapter 7

Work and Kinetic Energy

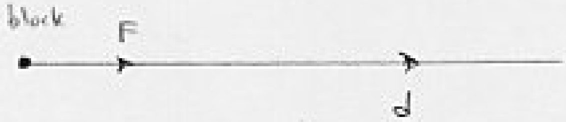
7-2 Work: Motion in One dim. with a Const. Force

$$W = Fd$$



$$W = Fd\cos\varphi$$

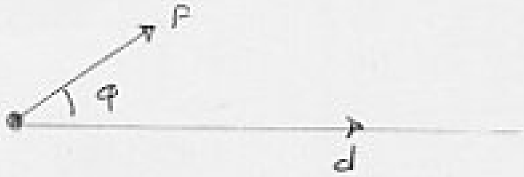
$$W = (d)(F\cos\varphi) = (F)(d\cos\varphi)$$



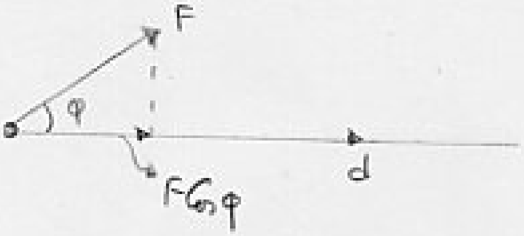
$$W = \bar{F}\cdot\bar{d}$$

Free body diagram

For $0 < \varphi < \frac{\pi}{2} \rightarrow W > 0$



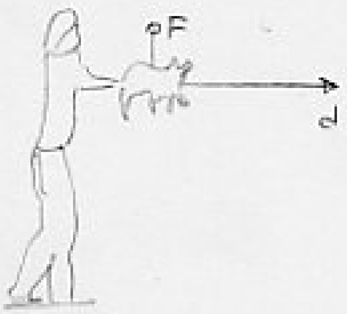
" $\frac{\pi}{2} < \varphi < \pi \rightarrow W < 0$



Suppose you pick up a cat (1), carrying it across a room at const speed (2), and then lower it back to the floor (3)

(1)

$W > 0$



(2)

$W = 0$

(3)

$W < 0$

F: The force excerted to the cart by man.

W: The work done by this F

Unit:

1 Joule = 1 J = N·m = 1 kg $\frac{m^2}{s^2}$ = 0.738 ft·lb

Also, 1 eV = 1.60×10^{-19} J

If $\bar{F} = \sum \bar{F}_i$

$W = \bar{F} \cdot \bar{d}$

Or $W = \sum \bar{F}_i \cdot \bar{d} = \sum W_i$

sample prob 7-1

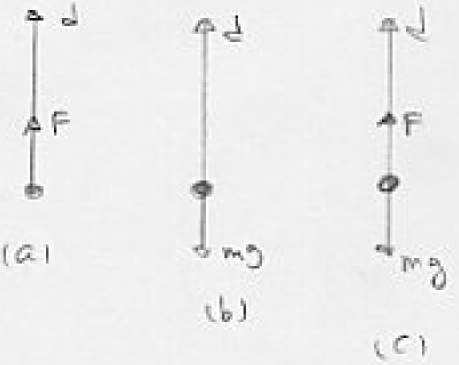
a) How much work is done by force F in lifting a weight of 2500 N a distance of d = 2.0 m?

Sol.

$\bar{F} = -m\bar{g}$ $\quad |\bar{F}| = |m\bar{g}| = 2500$ N

$W = Fd\cos\phi = (2500)(2)(\cos 0)$

$= 5000$ J



Here we neglected brief acceleration during the beginning and end, and assume V = const. Even though the results are the same but calculation will be complicated ($W = \int F dx$)

b) The work done by the weight?

Sol.

$$W = mgd\cos\phi = (2500)(2)(\cos 180^\circ) = -5000\ J$$

c) The work done by the net force?

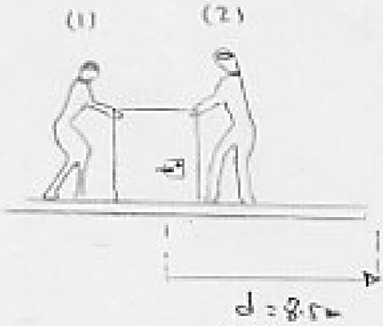
$$\sum F = 0 \rightarrow W = 0$$

Sample prob. 7-2

$d = 8.5\ m \quad \theta_1 = 30^\circ \quad \theta_2 = 40^\circ$

$F_1 = 320N \quad F_2 = 250\ N$

$W = ?$



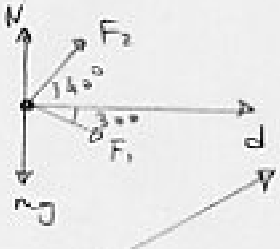
Sol.

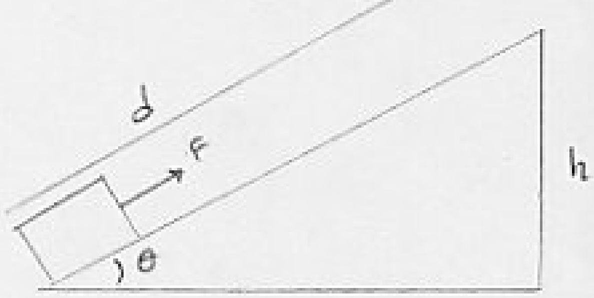
$W = W_1 + W_2$

$W_1 = F_1 d\cos\theta_1 = (320)(8.5)(\cos 30^\circ) = 2356\ J$

$W_2 = F_2 d\cos\theta_2 = (250)(8.5)(\cos 40^\circ) = 1628\ J$

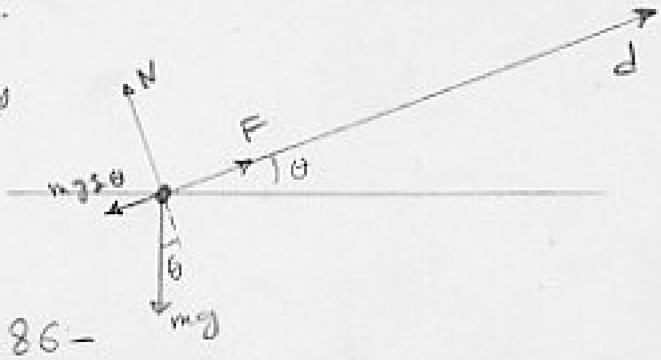
$W = 4000\ J$





Sample prob. 7-3

A 15-kg crate is pulled at v = const. a distance d = 5.7 m up a frictionless ramp to a height of 2.5 m



a) F = ?   b) W = ?

c) If h is the same but θ different W = ?

d) The work done by mg

Sol.

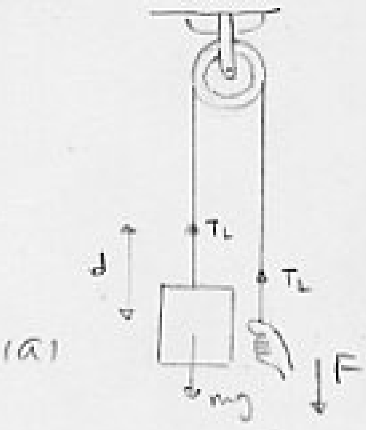
a) $F = mg\sin\theta = (15)(9.8)\left(\frac{2.5}{5.7}\right) = 64.5\ N$

b) $W = Fd\cos\phi = (64.5)(5.7)(\cos 0) = 368\ J$

c) $W = Fd\cos\phi = mg\sin\theta\, d(\cos 0) = mgh$ the same as (b)

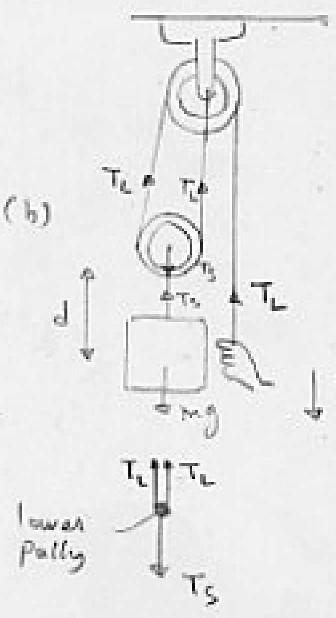
d) $W_g = m\vec{g}\cdot\vec{d} = mgd\cos(\theta+\frac{\pi}{2}) = -mgd\sin\theta = -mgh = -368\ J$

sample prob. 7-4



a) In Fig. (a) what is the magnitude of force F that you must exert on the cord to lift the block

F = mg



b) Through what distance must your hand move to lift the block by a distance d?

answer: d

c) W = ? on the block

$W = Td = mgd$ on th block by th cord.

$W = Fd = mgd$ on th free end of th cord (by hand)

→ The work is done on th block by means of th cord.

d) $F' = ?$ in Fig (b)

$T_S = 2T_L$ $F' = T_L = \frac{T_S}{2} = \frac{mg}{2}$

e) Through what distance must your hand move to lift th block a distance d?

answer: $2d$

f) $W = ?$ on th block in a distance d.

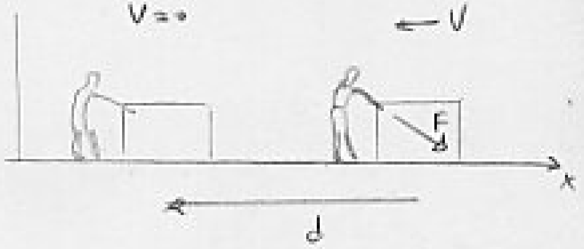
$W = T_S d = mgd$

Also $W = F(2d) = \frac{mg}{2}(2d) = mgd$ on th free end of th cord

Sample prob. 7-5

$\bar{F} = 2\hat{i} - 6\hat{j}$ $\bar{d} = -3\hat{i}$

$W = ?$ by F



stoping th block by F

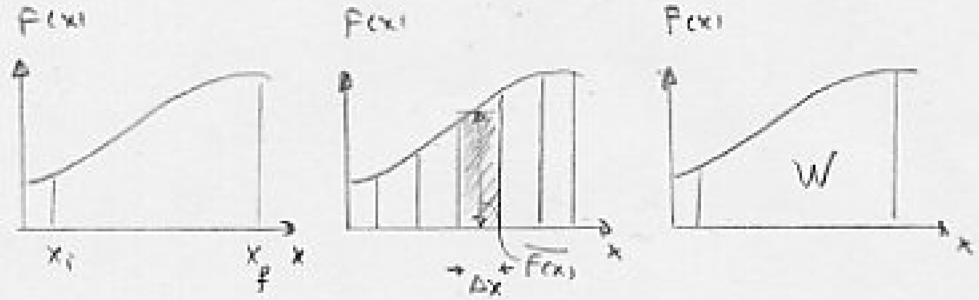
Sol.

$W = F \cdot d = (2\hat{i} - 6\hat{j}) \cdot (-3\hat{i}) = -6\,J$

7-3 Work Done by a Variable Force

One-Dim. Analysis:

Assume $\bar{F} \parallel \bar{d}$, but $\bar{F} = \bar{F}(x)$



$$\Delta W = \overline{F(x)}\,\Delta x$$

$$W = \sum \Delta W = \sum \overline{F(x)}\,\Delta x$$

$$W = \lim_{\Delta x \to 0} \sum \overline{F(x)}\,\Delta x \quad \rightarrow \quad W = \int_{x_i}^{x_f} F(x)\,dx$$

3-Dim. Analysis:

$$\bar{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$$

$$d\bar{r} = dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k}$$

$$dW = F \cdot dr = F_x\,dx + F_y\,dy + F_z\,dz$$

$$W = \int_{r_i}^{r_f} dW = \int_{x_i}^{x_f} F_x\,dx + \int_{y_i}^{y_f} F_y\,dy + \int_{z_i}^{z_f} F_z\,dz$$

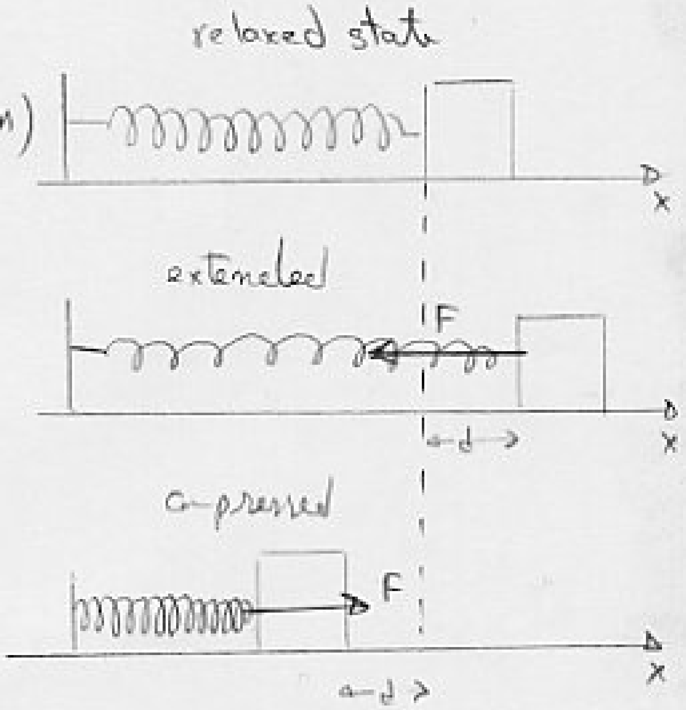
Sample prob 7-6

$$\bar{F} = 3x\hat{i} + 4\hat{j} \qquad r_i(2,3,0) \qquad r_f(3,0,0)$$

$$W = \int_2^3 3x\,dx + \int_3^0 4\,dy + 0 = 3\left[\tfrac{1}{2}x^2\right]_2^3 + 4\left[y\right]_3^0 = -4.5\,J$$

7-4 Work Done by a spring

$\bar{F} = -k\bar{x}$ Hooke's law



K : spring const. (a measure of stiffness) of the spring

In one dim.;

$$W = \int_{x_i}^{x_f} F(x)\,dx = \int_{x_i}^{x_f} (-kx)\,dx$$

$$= \left(-\frac{k}{2}\right)\left[x^2\right]_{x_i}^{x_f} = -\frac{1}{2}k\left(x_f^2 - x_i^2\right)$$

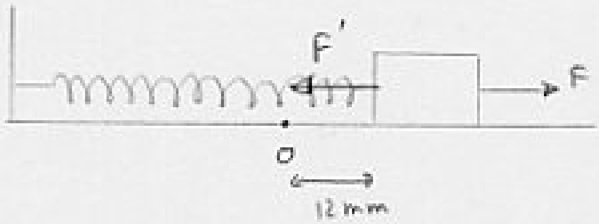
$W = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2$ work by a a spring

If $x_i = 0$

$\Rightarrow W = -\frac{1}{2}kx^2$ work done by the spring on an object

$W = +\frac{1}{2}kx^2$ work done by us

Sample prob. 7-7



$F = 4.9\,N$

$X = 12\,mm$

a) $= K = ?$

b) $F' = ?$ for $X = 17\,mm$

Sol.

a) $F' = -kx \qquad k = -\frac{F'}{X} = -\frac{-4.9}{12\times10^{-3}} = 408\ N/m$

b) $F' = -kx = -(408)(17\times10^{-3}) = -6.9\,N$

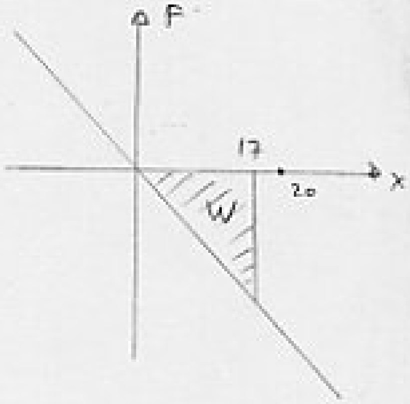
Sample prob 7-8

Stretch the spring in prob. 7-7 by 17 mm. How much work does the spring force do on the block?

Sol.

$$W = -\frac{1}{2}kx^2 = -\frac{1}{2}(408\ N/m)(17\times10^{-3}\,m)^2 = -5.9\times10^{-2}\ J$$

Sample prob 7-9



The spring is initially stretched by 17 mm.
You allow it to return its relaxed state and then compress it by 12 mm.
$W = ?$ by spring during the total displacement.

Sol

$x_i = 17\,mm \quad x_f = -12\,mm \quad k = 408\ N/m$

$$W = \frac{1}{2}k(x_i^2 - x_f^2) = \frac{1}{2}(408)\left[(17\times10^{-3})^2 - (-12\times10^{-3})^2\right] = 0.030\,J$$

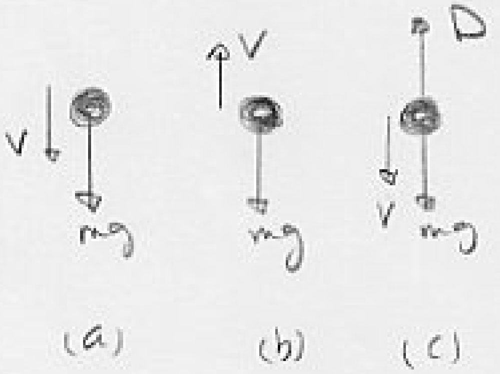
7-5 Kinetic Energy

Def: $K=\frac{1}{2}mv^2$

$W = K_f - K_i = \Delta K \longrightarrow K_f = K_i + W$ work-kinetic energy theorem

W: the work done by a force F on an object

A particle in Free Fall:



a) $W > 0$ the work done by mg

b) $W < 0$ " " " " "

Acc. to $K_f = K_i + W$

$K_f \xrightarrow{\text{will}}$ increase in (a)

$K_f \xrightarrow{\text{"}}$ decrease " (b)

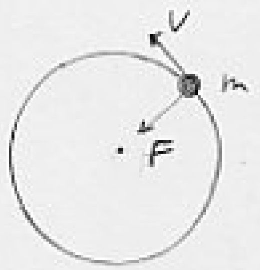
c) $W_{net} = 0$ the work done by the net force ($F_{net} = m\vec{g} + \vec{D} = 0$)

in this case $\Delta K = 0$

A particle in Uniform Circular Motion:

$V = \text{const.}$

$W = F \cdot d = 0 \qquad (F \perp d)$



Proof of the Work-kinetic Energy Theorem:

This theorem is the consequence of Newton's Second law, we prove the theorem in one-dim., but it is valid also in 3-dim.

$$W = \int_{x_i}^{x_f} F(x)\,dx = \int_{x_i}^{x_f} m a\,dx$$

$$m a\,dx = m\frac{dv}{dt}dx = m\frac{dv}{dx}\frac{dx}{dt}dx = m\frac{dv}{dx}v\,dx = mv\,dv$$

$$W = \int_{v_i}^{v_f} mv\,dv = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

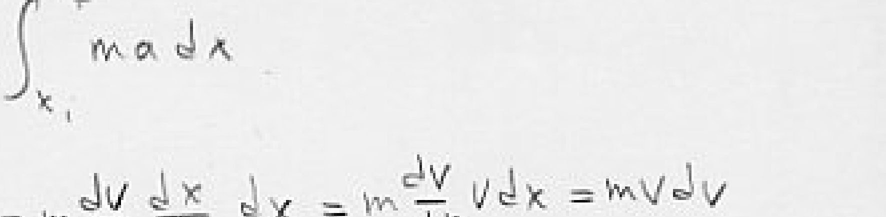
$$\rightarrow W = K_f - K_i = \Delta K$$

Sample prob. 7-10

$w = 1.2\times10^6\ N$ (1896 Texas)

$a = 0.26\ m/s^2$

Total kinetic energy of the two locomotive just before collision?



Collision of two locomotive

Sol.

$v^2 = v_0^2 + 2a(x - x_0) \qquad v_0 = 0 \qquad x - x_0 = \frac{6.4}{2} = 3.2\ km$

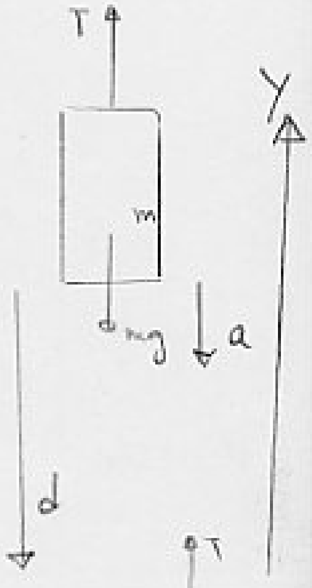
$v^2 = 0 + 2(0.26)(3.2\times10^3) \rightarrow v = 40.8\ m/s \qquad m = \frac{W}{g} = \frac{1.2\times10^6}{9.8} = 1.22\times10^5\ kg$

$K = 2\left(\frac{1}{2}mv^2\right) = 2\times10^8\ J$

Sample prob. 7-11

A 500-Kg elevator cab is descending with speed $v_i = 4.0\ \mathrm{m/s}$.
At this moment the winch system allows it to fall with $a = \frac{g}{5}$

a) $W_1 = ?$ the work done on the cab by mg during the fall through the distance $d = 12\ \mathrm{m}$



$$W_1 = mgd\cos 0^\circ = (500\,\mathrm{kg})(9.8\ \mathrm{m/s^2})(12\,\mathrm{m})(1) = 5.88\times 10^4\ \mathrm{J}$$

b) $W_2 = ?$ the work done on the cab by T during the fall

$$\Sigma F = T - mg = -ma \qquad (a > 0)$$

$$T = m(g-a) = m\left(g - \frac{g}{5}\right) = 3920\ \mathrm{N} \qquad \left(a = \frac{g}{5},\ g > 0\right)$$

$$W_2 = Td\cos 180^\circ = 3920(12)(-1) = -4.7\times 10^4\ \mathrm{J}$$

c) $W = ?$ total work done on cab

$$W = W_1 + W_1 = 1.2\times 10^4\ \mathrm{J}$$

or alternatively,

$$\Sigma F = -ma = -(500)\left(\frac{g}{5}\right) = -980\mathrm{N}$$

$$W = Fd\cos 0 = (-980)(-12)(1) = 1.2\times 10^4\ \mathrm{J}$$

d) $K_f = ?$ at the end of the 12-m fall.

$$K_i = \frac{1}{2} m v_i^2 = \frac{1}{2}(500)(4)^2 = 4000 J$$

$$K_f = K_i + W = 4000 + 1.2\times10^4 = 1.6\times10^4 J$$

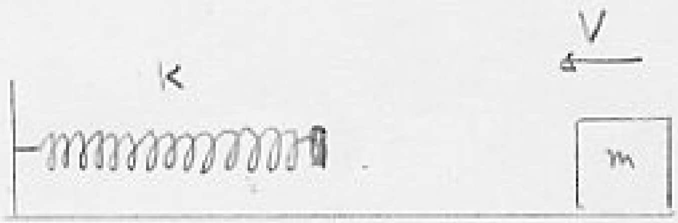
e) $V_f = ?$

$$K_f = \frac{1}{2} m v_f^2 \qquad V_f = \sqrt{\frac{2K_f}{m}} = 7.9 m/s$$

Sample prob. 7-12

$V = 1.2 m/s$ $m = 5.7 kg$

$k = 1500 N/m$



$d_{max} = ?$ compressing distance

Sol.

$W = -\frac{1}{2} k x^2 = -\frac{1}{2} k d^2$ the work done by spring forcee on the block

$\Delta K = K_f - K_i = 0 - \frac{1}{2} m v^2$

$W = \Delta K \rightarrow d = V\sqrt{\frac{m}{k}} = 7.4 cm$

7-6 Power

$$\bar{P} = \frac{W}{\Delta t}$$ average power

$$P = \frac{dW}{dt}$$ instantaneous "

In SI unit; $1\ \text{Watt} = 1\ \text{W} = 1\ \text{J/s} = 0.738\ \text{ft.lb/s}$

$1\ \text{horsepower} = 1\ \text{hp} = 550\ \text{ft.lb/s} = 746\ \text{W}$

$1\ \text{Kilowatt-hour} = 1\ \text{Kw.h} = (10^3\ \text{W})(3600\ \text{s}) = 3.6\times10^6\ \text{J}$

Also; $P = \frac{dW}{dt} = \frac{F\,dx}{dt} = F\left(\frac{dx}{dt}\right) = FV$

In 3-dim. $P = F.V$

Sample prob. 7-13

A load of bricks with $m = 420\ \text{kg}$ is to be lifted a height of $120\ \text{m}$ in 5 min. $P = ?$

Sol.

$W = mgh = (420)(9.8)(120) = 4.94\times10^5\ \text{J}$

$$\bar{P} = \frac{W}{\Delta t} = \frac{4.94\times10^5}{5\times60} = 1650\ \text{W}$$

We have neglected the mass of cable and platform, and also frictional forces. And we have assumed lifting process without appreciable acceleration.

Sample prob 7-14

A 80-hp outboard motor can drive a spaceboat at 22 knots (25 mi/h). What is the forward thrust (force) of the motor?

Sol.

$$F = \frac{P}{V} = \frac{(80\ \text{hp})(746\ \text{W/hp})}{11\ \text{m/s}} = 5400\ \text{N}$$

Note that since $V = \text{const.} \longrightarrow F \overset{\text{must}}{=} D$ (drag force)

7-7 Kinetic Energy at High Speeds

In high speeds Newtonian mechanics fails and must be replaced by Einstein's special Theory of Relativity.

$$K = \frac{1}{2} mV^2 \longrightarrow K = mc^2\left(\frac{1}{\sqrt{1-\frac{V^2}{c^2}}} - 1\right)$$

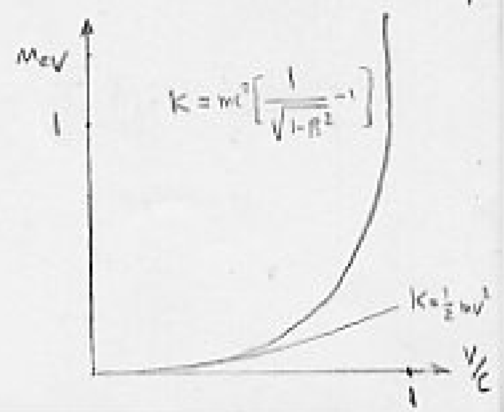
Let $\frac{V}{c} \equiv \beta \quad \rightarrow \quad K = mc^2\left[(1-\beta^2)^{-\frac{1}{2}} - 1\right]$

If $V \ll c \longrightarrow \beta \ll 1$

$$\rightarrow (1-\beta^2)^{-\frac{1}{2}} = 1 + \frac{1}{2}\beta^2 + \cdots \qquad \left((1+x)^n = 1 + \frac{n}{1!}x + \frac{n(n-1)}{2!}x^2 + \cdots\right)$$

$$K = mc^2\left[\left(1 + \frac{1}{2}\beta^2 + \cdots\right) - 1\right]$$

$$\simeq mc^2\left(\frac{1}{2}\beta^2\right) = \frac{1}{2}mV^2$$



7-8 Reference Frames:

We apply Newton's laws of mechanics <u>only</u> in <u>inertial</u> <u>ref.</u> frames.

Observers in different inertial ref. frames will measure,

i) the exact <u>same</u> values for some quantities

In Newtonian mechanics these <u>invariant</u> quantities are,

1- Force, 2- mass, 3- acceleration, 4- Time

ii) different values for some other quantities, such as,

1- Displacement 2- Velocity of a particle <u>(not invariant)</u>

Since for example $W = F \cdot d$

d: depends of the ref. frame of the observer

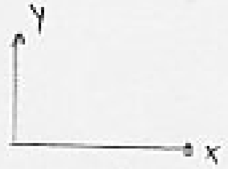
F: the same in all inertial frames

$\longrightarrow$ W: depends on the ref. frame.

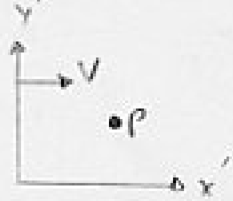
<u>Ex.</u>

$d \neq 0$

$d' = 0$



Fixed



P: stationary in this system

Also $K = \frac{1}{2}mv^2$

frame-indep

frame-dep

K will be different, in different inertial ref. frames

<u>But</u>;

From Galileo to Einstein, physicists have come to believe in the principle of invariance.

The laws of physics must have the same form in all inertial ref. frames.

A check,

Work-Kinetic Energy Theorem (?)

F: The force exerted by hand on book

mg: The weight of the book

F, mg the same in both ref frames

Observer A:

$W_1 = Fd\cos\theta = (+mg)h(1) = mgh$ on book by F

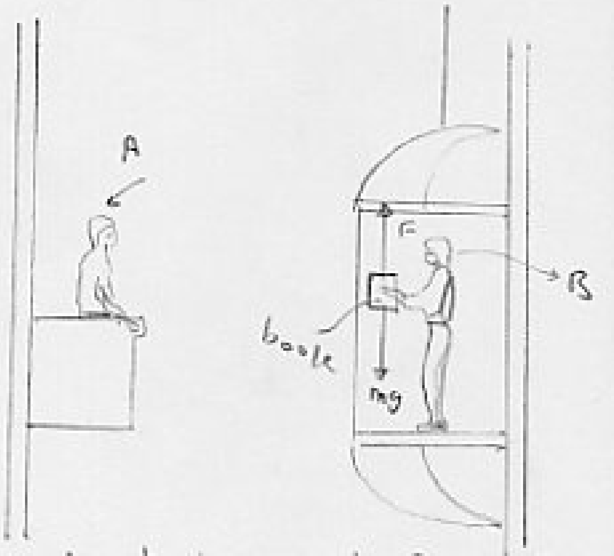
$W_2 = (-mg)d\cos\theta = -mgh$

$W = W_1 + W_2 = 0$

Observer B: $(d' = 0)$

$W_1 = Fd'\cos\theta = 0$ $\quad W_2 = (-mg)d'\cos\theta = 0$

$W = W_1 + W_2 = 0$



Acc. to Work-Kinetic Energy Theorem the kinetic energy of the book <u>does not</u> change.