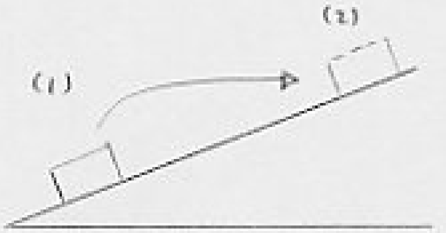
Chapter 6
Force and Motion II

6-1 Friction



i) In order to move a block from position (1) to position (2), we use energy;

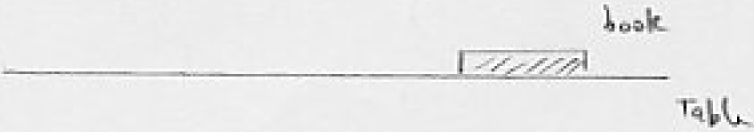
$$E = E_1 + E_2$$

E_1: Equal to the potential energy difference

E_2: waste by the firction fore

ii) On the other hand life without friction is impossible.

1- First experiment;



i- Send a book sliding accros a table top.

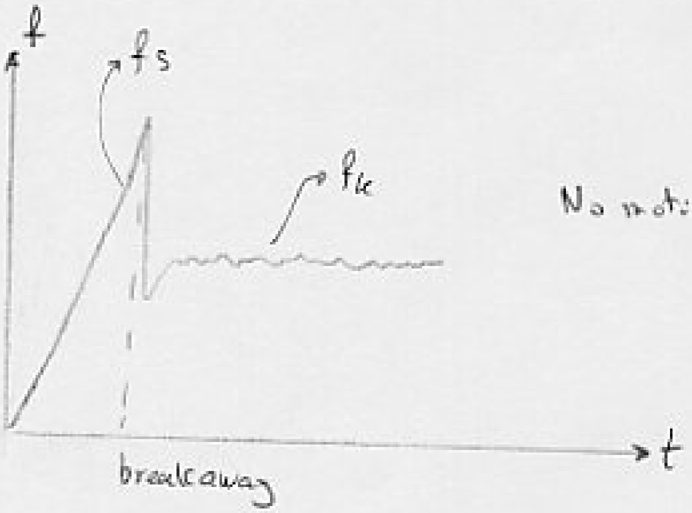
ii- The friction force slows the book

iii - If you want the book to move with a const. velocity you must exert a force with a mag. of the friction force -

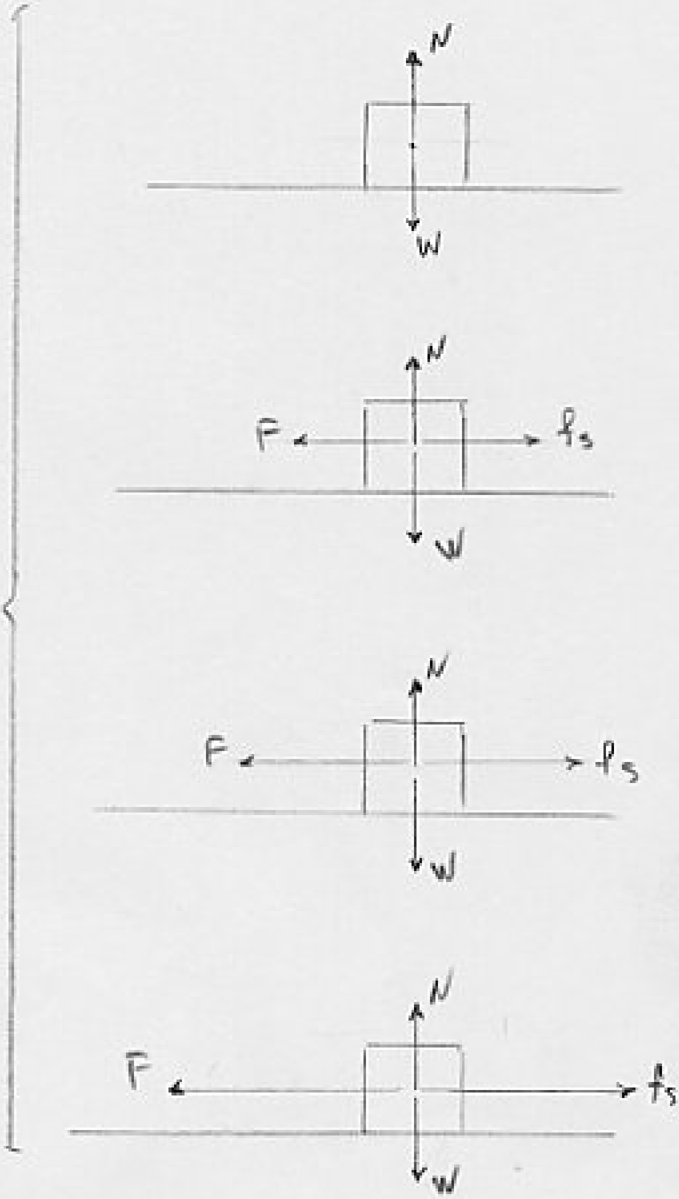
2- Second experiment

f_s: static frictional force

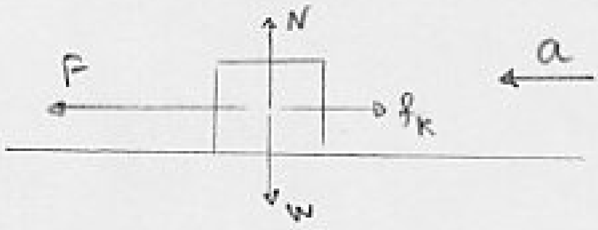
f_k: Kinetic frictional force



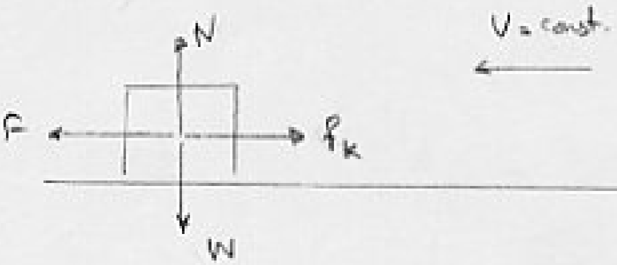
No motion



Acceleration



Const. Velocity

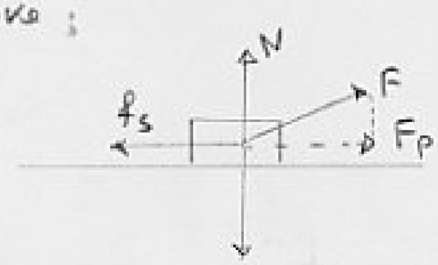


6-2 Properties of Friction

Experiment: When a body is pressed against a surface and a force F attempts to slide the body along the surface, the resulting frictional force has 3-properties:

Property 1 – If the body does not move:

$$f_s = -F_p$$



Property 2 – $|f_{s,max}| = \mu_s N$

μ_s: coeff. of static friction

Property 3 – If the body begins to slide along the surface,

$$|f_k| = \mu_k N$$

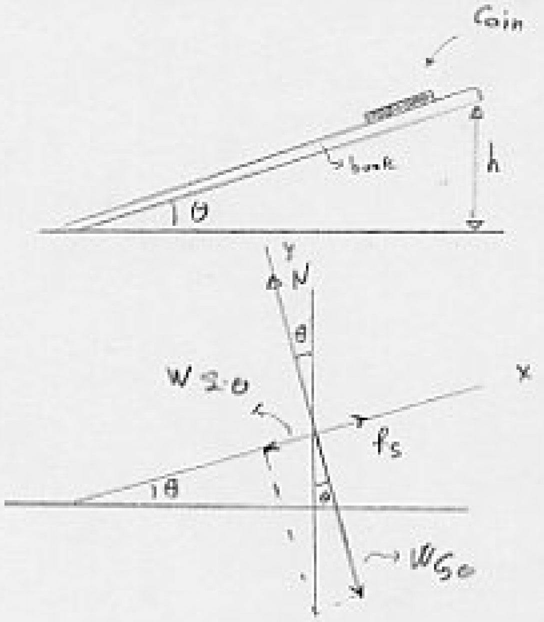
μ_k: coeff of kinetic friction

where $|f_k| < |f_s|$

μ_s, μ_k depend on the body and surface.

$\mu_k = \mu_k(\text{velocity})$ in general.

Sample prob. 1 -



The coin begins to slide when

$\theta = 13°$

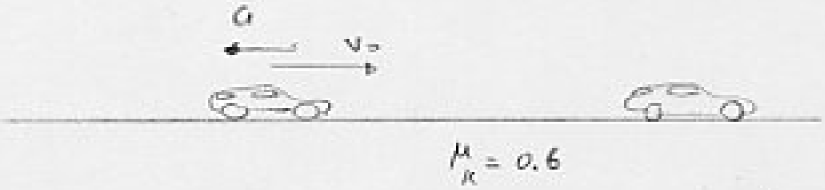
$\mu_s = ?$

Sol. $\quad \Sigma \bar{F} = \bar{f}_s + \bar{W} + \bar{N} = 0$

$$\begin{cases} \Sigma F_x = f_s - W\sin\theta = 0 \\ \Sigma F_y = N - W\cos\theta = 0 \end{cases} \rightarrow \begin{cases} f_s = W\sin\theta \\ N = W\cos\theta \end{cases}$$

$$\frac{f_s}{N} = \frac{\mu_s N}{N} = \frac{W\sin\theta}{W\cos\theta} = \tan\theta \qquad \mu_s = \tan\theta = \tan 13° = 0.23$$

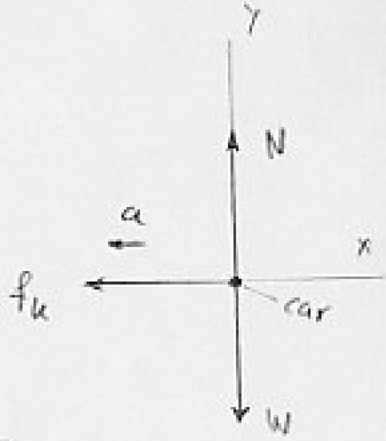
Sample prob. 6.2



A car's wheels are locked during emergency breaking.

The car stops after a displacement $d = 290\ m$

$V_0 = ?$



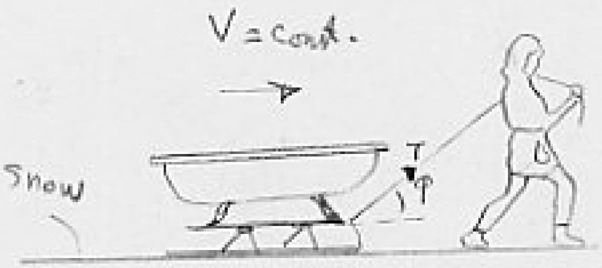
Sol. $\quad V^2 = V_0^2 + 2a_x(x - x_0)$

$V = 0 \quad , \quad x - x_0 = d \qquad V_0 = \sqrt{-2a_x d} \quad , \quad \bar{f}_k = m\bar{a}$

$|f_k| = -ma_x \rightarrow a_x = -\frac{f_k}{m} = \frac{-\mu_k N}{m} = -\frac{\mu_k mg}{m} = -\mu_k g$

$V_0 = \sqrt{2\mu_k g d} = \sqrt{(2)(0.6)(9.8\ m/s^2)(290\ m)} = 58\ m/s = 210\ km/h$

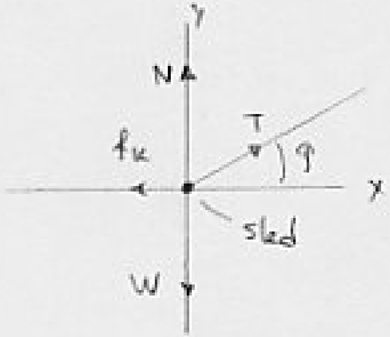
Sample prob. 6.3



A woman pulls a loaded sled of mass $m = 75\,kg$. $\mu_k = 0.10$

$\varphi = 42°$ $\quad V = const.$

a) $T = ?$ $\quad$ b) $N = ?$



Sol.

a) $\begin{cases} T\cos\varphi - f_k = ma_x = 0 & (1) \\ T\sin\varphi + N - mg = ma_y = 0 & (2) \end{cases}$

$f_k = \mu_k N \xrightarrow{(1)} N = \dfrac{T\cos\varphi}{\mu_k}$ $\quad (3)$

(3) in $(2) \rightarrow T = \dfrac{\mu_k mg}{\cos\varphi + \mu_k \sin\varphi} = \dfrac{(0.10)(75\,kg)(9.8\,m/s^2)}{\cos 42° + (0.10)(\sin 42)} = 91\,N$ $\quad (4)$

b) (4) in $(3) \rightarrow N = \dfrac{\cos\varphi}{\cos\varphi + \mu \sin\varphi}\, mg$

$N = \dfrac{\cos 42}{\cos 42 + (0.10)\sin 42}\, mg = 0.917\, mg = 670\,N$

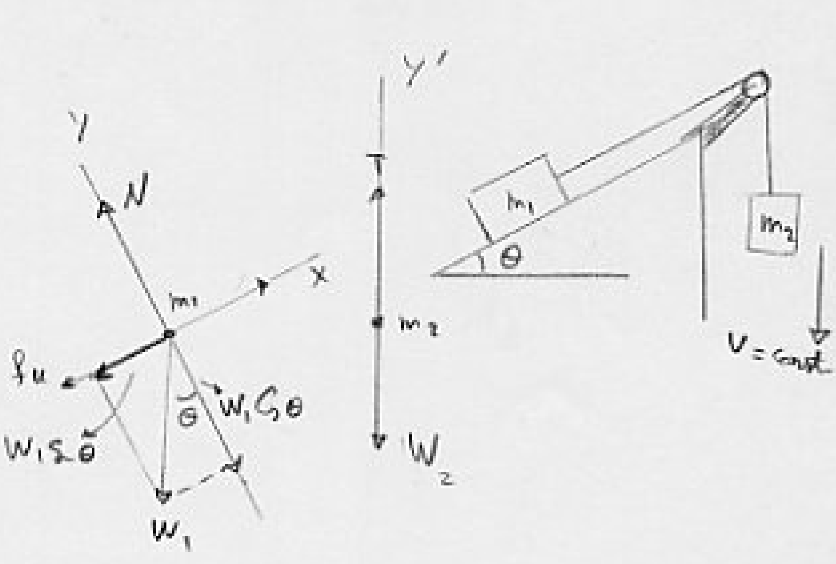
Sample prob 6.4

$m_1 = 14\,kg$ $\quad m_2 = 14\,kg$

$\theta = 30°$

a) $f_k = ?$

b) $\mu_k = ?$



$$\Sigma F_x = T - f_k - m_1 g S\theta = m_1 a_x = (m_1)(0) = 0 \quad (1)$$

$$\Sigma F_y' = T - m_2 g = m_2 a_y = (m_2)(0) = 0 \quad \rightarrow T = m_2 g \quad (2)$$

(2) in (1) $\rightarrow$ $f_k = m_2 g - m_1 g S\theta = (14kg)(9.8 m/s^2) - (14kg)(9.8 m/s^2)(S 30) = 68.6 N$

b) $\Sigma F_y = N - m_1 g Cos\theta = m_1 a_y = 0$

$\rightarrow N = m_1 g C\theta$

$$\mu_k = \frac{f_k}{N} = \frac{f_k}{m_1 g C\theta} = \frac{68.6 N}{(14kg)(9.8 m/s^2)(Cos 30^\circ)} = 0.58$$

6.3 The Drag Force and Terminal Speed

Def. - A fluid is anything that can flow - generally either a gas or a liquid

Drag Force: When there is a relative velocity between a fluid and a body $\rightarrow$ the body experiences a drag force D, that opposes the relative motion and points in the dir. in which the fluid flows relative to the body.

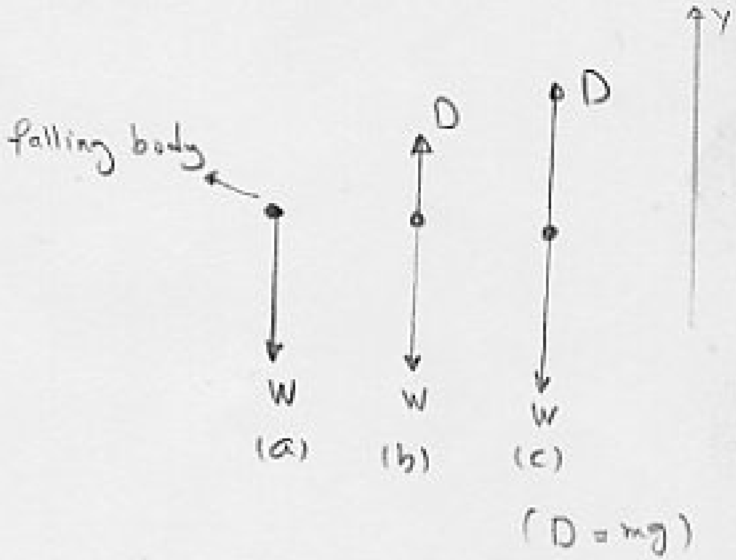
Special case:

$$\begin{cases} \text{Fluid} \longrightarrow \text{air} \\ \text{Body} \longrightarrow \text{like baseball (not Javelin)} \\ V_{rel} \longrightarrow \text{fast enough} \end{cases}$$

$$\rightarrow D = \frac{1}{2} C \rho A v^2$$

C: Drag coef. (In general $C = C(V)$)
ρ: Air density
A: Effective cross-sectional area (perpendicular to V)
V: relative speed

Terminal speed:



Acc. to Newton's second law;

at the terminal speed; $a = 0$

$$\Sigma F_y = D - mg = 0$$

$$F = ma \rightarrow a = 0 \qquad (V = \text{const})$$

$$\frac{1}{2} C \rho A V_t^2 = mg \qquad V_t = \sqrt{\frac{2mg}{C\rho A}}$$

Sample Prob. 6.5

If a falling cat reaches a first terminal speed of 60 mi/h while it is tucked in and then stretches out doubling A, how fast is it falling when it reaches a new terminal speed?

Sol.

V_{t_0} : first terminal speed — A_0 : original area

V_{tn} : new " " — A_n : new "

$$\frac{V_{tn}}{V_{t_0}} = \frac{\sqrt{2mg/C\rho A_n}}{\sqrt{2mg/C\rho A_0}} = \sqrt{\frac{A_0}{A_n}} = \sqrt{\frac{A_0}{2A_0}} = \sqrt{0.5} \approx 0.7$$

$$V_{tn} \approx 0.7\, V_{t_0} \approx 40 \text{ mi/h}$$

Sample prob. 6.6

A raindrop with radius R = 1.5 mm falls from a cloud that is at height h = 1200 m above the ground.

$C = 0.6$, Assume the drop is spherical, $\rho_w = 1000 \text{ kg/m}^3$ and $\rho_a = 1.2 \text{ kg/m}^3$

a) $V_t = ?$

$m = \frac{4}{3}\pi R^3 \rho_w \qquad A = \pi R^2$

$$V_t = \sqrt{\frac{2mg}{C\rho_a A}} = \sqrt{\frac{8\pi R^3 \rho_w g}{3C\rho_a \pi R^2}} = \sqrt{\frac{8R\rho_w g}{3C\rho_a}}$$

$$= \sqrt{\frac{8(1.5\times10^{-3}\,m)(1000\,kg/m^3)(9.8\,m/s^2)}{3(0.60)(1.2\,kg/m^3)}} = 7.6\,m/s \;(= 17\,mi/h)$$

Note that the raindrop reaches terminal speed after falling just a few meters.

b) What would have been the speed just before impact if there had been no drag force?

$$V^2 = V_0^2 - 2g(y - y_0)$$

$$h = -(y - y_0), \quad V_0 = 0$$

$$\rightarrow V = \sqrt{2gh} = \sqrt{2(9.8\,m/s^2)(1200\,m)}$$

$$= 150\,m/s \;(= 340\,mi/h)$$

6.4 Uniform Circular Motion

The magnitude of the centripetal acc. in uniform circular motion is const. and given by

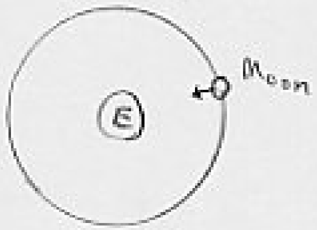
$$a = \frac{V^2}{r}$$

and $\quad F = ma = m\dfrac{V^2}{r} \quad$ Centripetal force

Ex. A hockey puck whirled around on the end of a string

The centripetal force is provided by the tension T in the string.

Ex Moon around the Earth

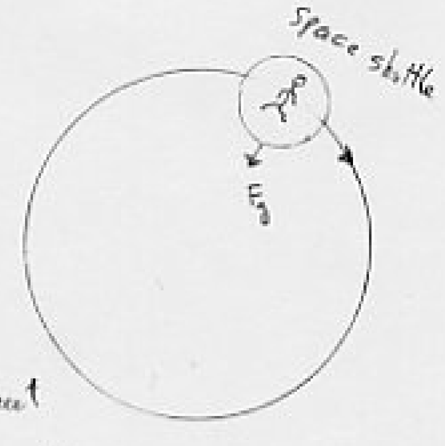


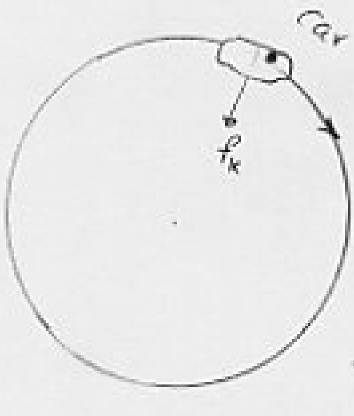
The centripetal force is provided by the gravational attraction force.

Ex.

1 – Rounding a curve in a car (centripetal force by frictional force)

2 – Orbiting the Earth (" " " gravitational ")





In case (1) the man feels a force by the seat or wall of the car

In case (2) the astronaut is floting around with no sensation of force!

The reason: In case (1) the force is a contact force (by the wall) while in case (2) the centripetal force is volume force (acting on the man).

Sample prob. 6.7

The mass of a cosmonaut is $m = 79 kg$. He is in a spacecraft orbiting the Earth at an altitude of $h = 520 km$ with a speed of $V = 7.6 km/s$.

a) $a = ?$ $$a = \frac{v^2}{r} = \frac{v^2}{R_E + h} = \frac{(7.6 \times 10^3 \, m/s)^2}{6.37 \times 10^6 m + 0.52 \times 10^6 m} = 8.38 \, m/s^2$$

This is free fall acc. at h, i.e. $g_h = 8.38$

b) What (centripetal) gravitational force does the Earth exerts on the cosmonaut?

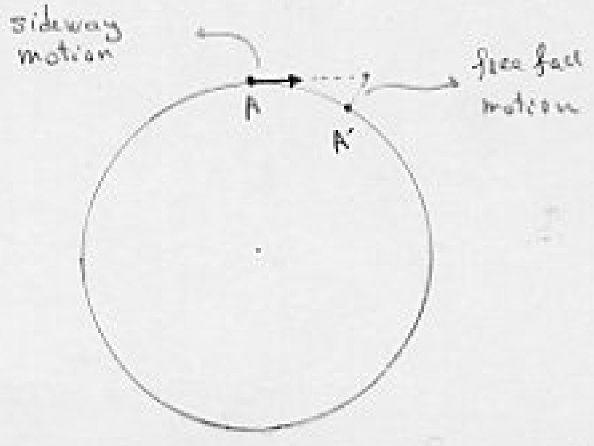
$$F = ma = (79 kg)(8.38 \, m/s^2) = 660 N \approx 150 \, lb$$

i.e.

If the cosmonaut stands on a scale placed on the top of a tower with height $h = 520 m$ $\longrightarrow$ the scale would read 660 N.

While in orbit the scale would read Zero, because the cosmonaut and scale are both in free fall.

sideway motion + free fall motion = circular path

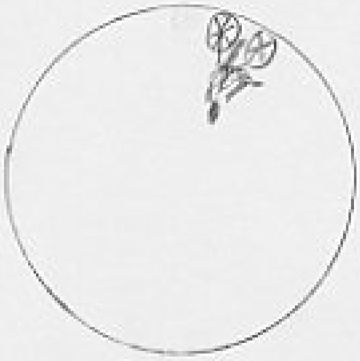


Sample prob. 6.8

$V_{min} = ?$ at the top of the loop

if he is to remain in contact with it there.

$R = 2.7\,m$



Sol.

free body diagram

$$\Sigma F = -N - mg = -ma$$

$$-N - mg = -m\frac{V^2}{R}$$

$N = 0 \rightarrow V_{min}$

$$\rightarrow mg = m\frac{V^2}{R} \qquad V = \sqrt{gR}$$

$$V = \sqrt{(9.8\,m/s^2)(2.7\,m)} = 5.1\,m/s$$

Sample prob. 6.9

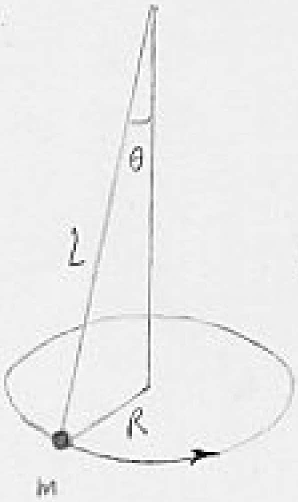
Conical pendulum:

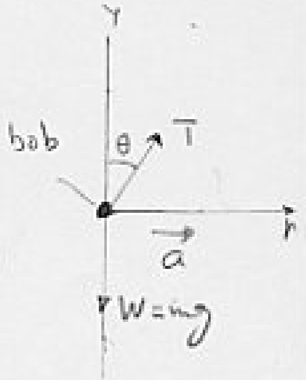
$m = 1.5\,kg$ the mass of pendulum bob

$\theta = 37°$

$L = 1.7\,m$

$\tau = ?$





Sol.

$$\Sigma F_y = T\cos\theta - mg = ma_y = 0$$

$$\rightarrow T\cos\theta = mg \qquad (1)$$

There must be a net force along the r axis providing the centripetal acc.

$$\sum F_r = T \sin\theta = m a_r \qquad T S\theta = m\frac{v^2}{R} \quad (2)$$

$$(1)(2) \rightarrow \quad v = \sqrt{\frac{gRS\theta}{C\theta}}$$

But $2\pi R = v\tau \quad \rightarrow \tau = \frac{2\pi R}{v} = 2\pi\sqrt{\frac{RC\theta}{gS\theta}}$

but also $R = LS\theta \quad \rightarrow \tau = 2\pi\sqrt{\frac{LC\theta}{g}} \quad (3)$

$$\tau = 2\pi\sqrt{\frac{(1.7m)(C37°)}{9.8\,m/s^2}} = 2.3\,s$$

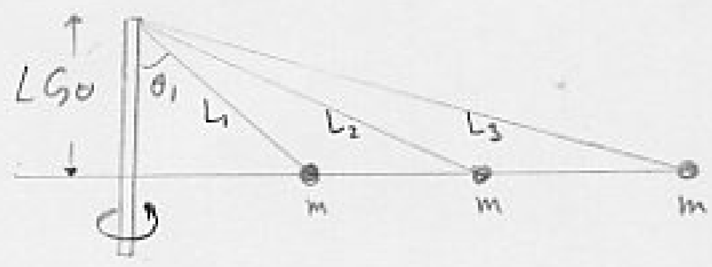
$(3) \rightarrow \tau = \tau(LC\theta) = \tau$(vertical distance of the bob from its point of support)

→ In the following Fig.

If $\tau_1 = \tau_2 = \tau_3$

(but $L_1 \neq L_2 \neq L_3$)

→ Their bobs will all lie in the same horizontal plane.



Sample prob. 6.10

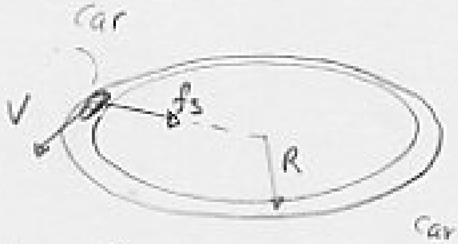
A car of mass $m = 1600\,kg$

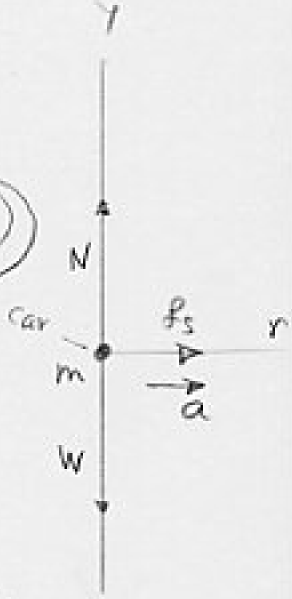
$V = 20\ m/s$

path: flat, circular

$R = 190\,m$

$\mu_{s\,min} = ?$ preventing the car from <u>slipping</u>.





<u>sol.</u>

f_s: the friction force providing the centripetal force.

Remark: Since the car is <u>not</u> sliding we consider μ_s not μ_k.

$$\sum F_y = N - mg = ma_y = 0 \qquad N = mg$$

$$\sum F_r = f_s = ma_r \qquad f_s = m\frac{V^2}{R}$$

A body is on the <u>verge</u> of slipping when f_s <u>reaches</u> its max. value:

$$f_{s\,max} = \mu_s N$$

$$\rightarrow \mu_s N = m\frac{V^2}{R} \rightarrow \mu_s mg = m\frac{V^2}{R} \rightarrow \mu_s = \frac{V^2}{gR}$$

$$\rightarrow \mu_s = \frac{(20\,m/s)^2}{(9.8\,m/s^2)(190\,m)} = 0.21$$

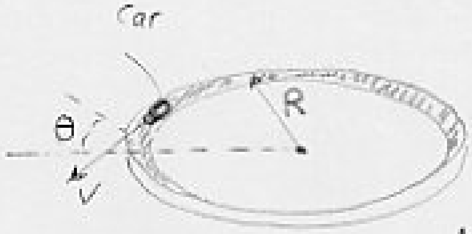
$\rightarrow$ if $\mu_s \geq 0.21 \rightarrow$ The car will be held in a circle by f_s.

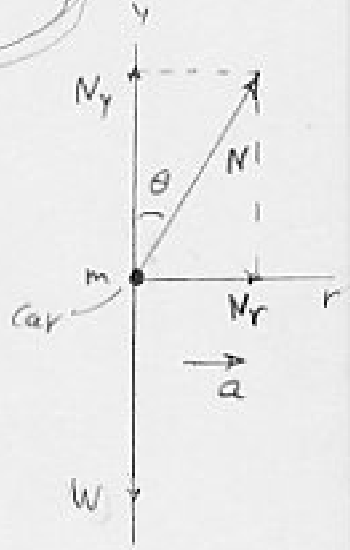
Sample prob. 6.11

A banked roadway:

$\mu = 0$

$V = 20\ m/s$





Sol.

The centripetal force is supplied by N_r

(The effect of banking is to tilt N toward the center of curvature of the road, to produce N_r)

$$\Sigma F_y = N_y - mg = ma_y = 0 \qquad N\cos\theta = mg \quad (1)$$

$$\Sigma F_r = N_r = ma_r \qquad N\sin\theta = m\frac{V^2}{R} \quad (2)$$

$$(1)(2) \rightarrow \tan\theta = \frac{V^2}{gR} \ (3) \qquad \theta = \tan^{-1}\frac{V^2}{gR}$$

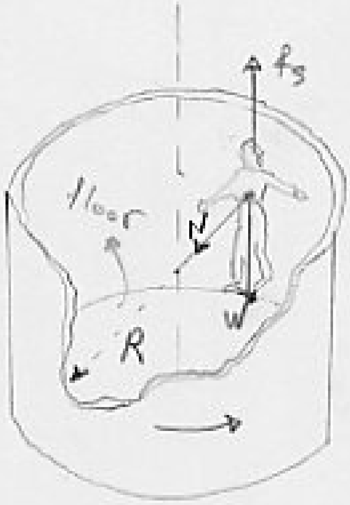
$$\theta = \tan^{-1}\frac{(20\ m/s)^2}{(9.8\ m/s^2)(190\ m)} = 12^\circ$$

Now comparing;

$$\begin{cases} \mu_s = \dfrac{V^2}{gR} & \text{(Previous prob.)} \\ \tan\theta = \dfrac{V^2}{gR} \end{cases} \rightarrow \tan\theta = \mu_s$$

Sample prob. 6.12 $\mu_s = 0.40$, $R = 2.1$m

a) $V_{min} = ?$ (if the rider is not to fall when the floor drops)



Sol.

$\sum F_y = ma_y \overset{must}{=} 0$

$\rightarrow f_s = W$

At V_{min} $\longrightarrow$ the rider is on the verge of slipping.

$\longrightarrow$ f_s must be at its max. value $\mu_s N$

$\longrightarrow$ $\mu_s N = mg$ (1)

N: centripetal force

$\sum F_r = N = ma_r$ $\quad N = m\frac{V^2}{R}$ (2)

(1)(2) $\rightarrow$ $V = \sqrt{\frac{gR}{\mu_s}} = \sqrt{\frac{(9.8\,m/s^2)(2.1\,m)}{0.40}} = 7.17\,m/s$

V: indep of m , i.e. the same from a child to a heavy adult

b) $m = 49$ kg

$N = ?$ $\quad N = m\frac{V^2}{R} = (49\,kg)\frac{(7.17\,m/s)^2}{2.1\,m} = 1200\,N$

Although this force is directed toward the central axis, the rider has a sensation that it is directed radially outward. This is because he is in a noninertial frame and the forces measured from such frames can be illusionary.

6.5 The Forces of Nature

All forces that we can experience directly are:

i) Gravitational
ii) Electromagnetic

forces.

<u>Ex.</u>

1- Frictional forces,
2- Normal "
3- Contact "
4- Drag forces
5- Tention forces
6- push and pull "

involve fundamentally <u>electromagnetic</u> forces excerted by <u>one atom</u> <u>on another</u>.

There are two other fundamental forces,

iii) weak force
iv) Strong "

There was attempt in unification of these forces (but not successful). Just, Glashow, Salam, Weinberg (1979) showed that the <u>weak</u> and <u>electromagnetic</u> forces are different aspects of a single <u>electroweak</u> force.