Chapter 3
Vectors

3-1 Vectors and Scalars:

Def. - A scalar is a quantity that is completely characterized by its <u>magnitude</u>.

Def. - A vector is a quantity that is completely characterized by its <u>magnitude</u> and <u>direction</u>.

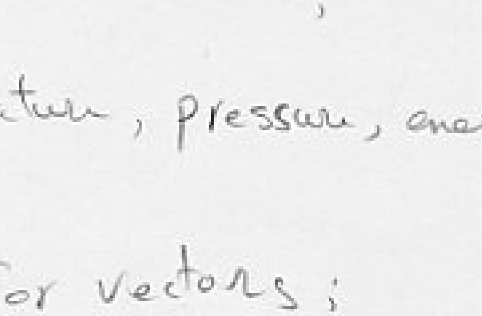
Ex. - For scalars;

Temperature, pressure, energy, mass, time

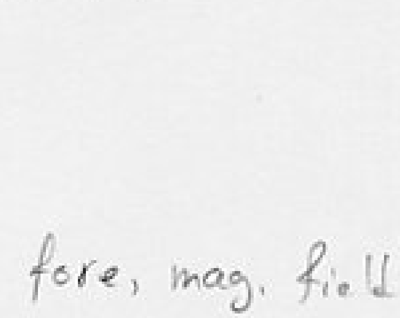
Ex. - For vectors;

Displacement, velocity, acceleration, fore, mag. field

Displacement vector:



All three represent the same displacement



(1) Actual path
(2) " " (another one)
(3) Displacement for (1) and (2)

Sample prob 3-1

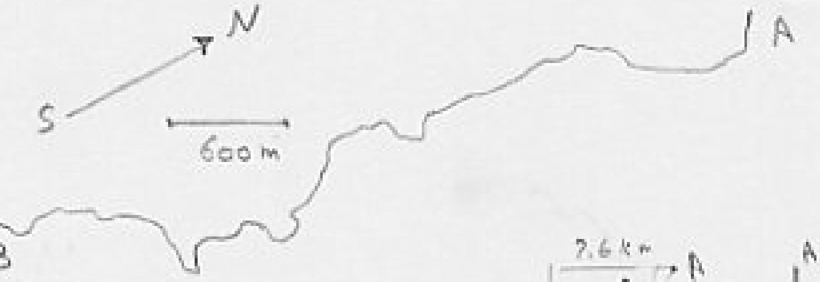
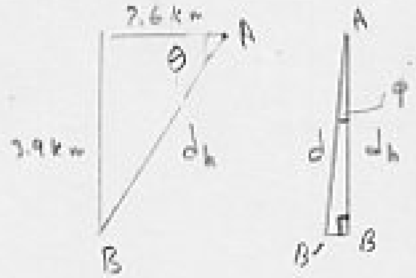


Fig. shows the movement of a team starting from A to B in the following way:



$$\begin{cases} 2.6\text{ km} & \text{westward} \\ 3.9\text{ "} & \text{southward} \\ 25\text{ m} & \text{upward} \end{cases}$$

What is the displacement vector?

Sol.

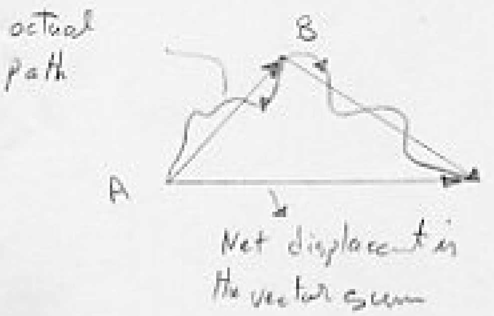
$$d_h = \sqrt{(2.6\text{km})^2 + (3.9\text{km})^2} = 4.69\text{ km} \qquad \tan\theta = \frac{3.9\text{ km}}{2.6\text{ km}} = 1.5$$

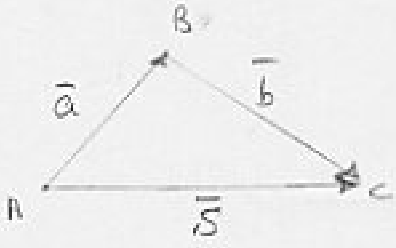
$$\theta = \tan^{-1} 1.5 = 56°$$

$$d = \sqrt{(4.69\text{km})^2 + (0.025\text{km})^2} \simeq 4.7\text{ km} \qquad \varphi = \tan^{-1}\frac{0.025\text{km}}{4.69\text{ km}} = 0.3°$$

3-2 Adding Vectors: Graphical Method,

Vector Sum:

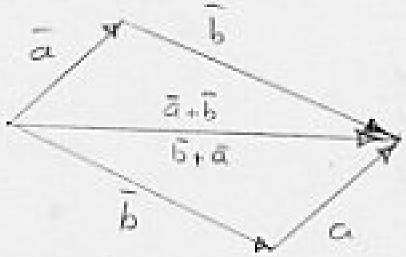




$$\bar{S} = \bar{a} + \bar{b}$$

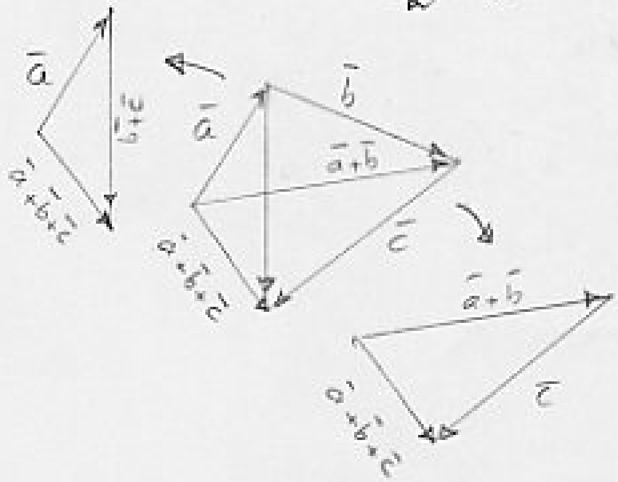
Properties:

i) $\bar{a}+\bar{b}=\bar{b}+\bar{a}$ commutative law (i)



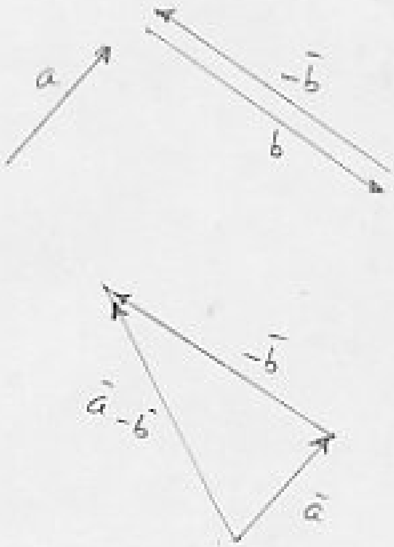
ii) $(\bar{a}+\bar{b})+\bar{c}=\bar{a}+(\bar{b}+\bar{c})$ (ii)

associative law



(iii) $\bar{b}+(-\bar{b})=0$

$d=\bar{a}-\bar{b}=\bar{a}+(-\bar{b})$ subtraction



(iii)

3-3 Vectors and Their Components

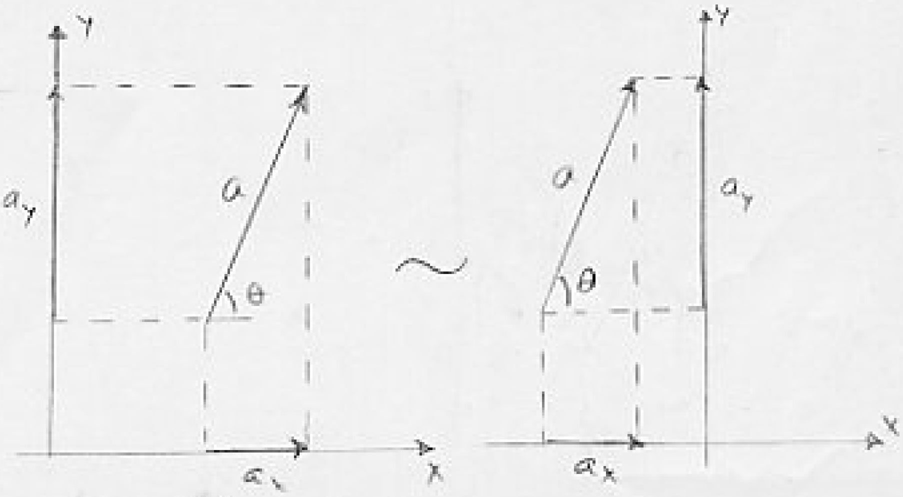
Resolving a vector;

$a_x = a\cos\theta$

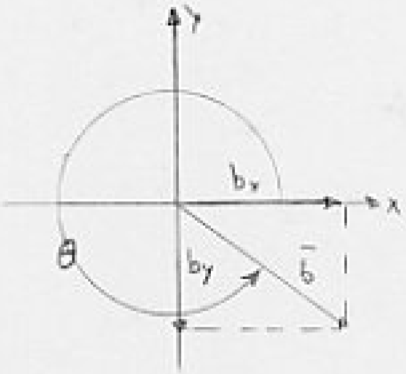
$a_y = a\sin\theta$

$a=\sqrt{a_x^2+a_y^2}$

$\tan\theta = \dfrac{a_y}{a_x}$

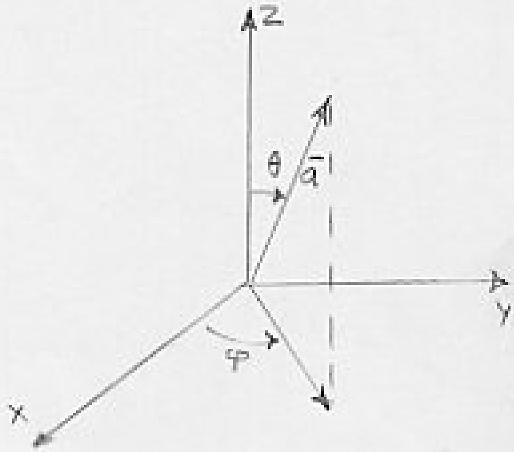


3-dim. Case;



$$\begin{cases} a_x = a \sin\theta \cos\varphi \\ a_y = a \sin\theta \sin\varphi \\ a_z = a \cos\theta \end{cases} \qquad (a_{xy} = a \sin\theta)$$

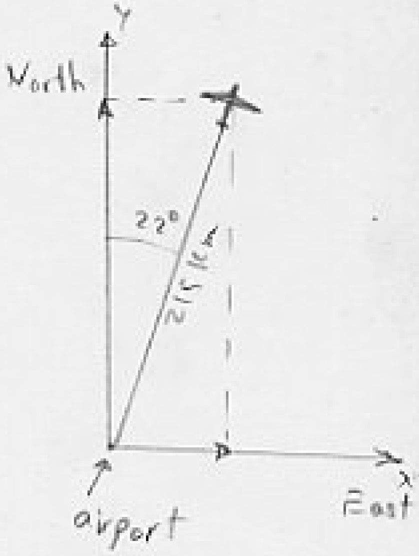
$$a = \sqrt{a_x^2 + a_y^2 + a_z^2}$$



Sample Prob. 3-2

How far east from the airport ?

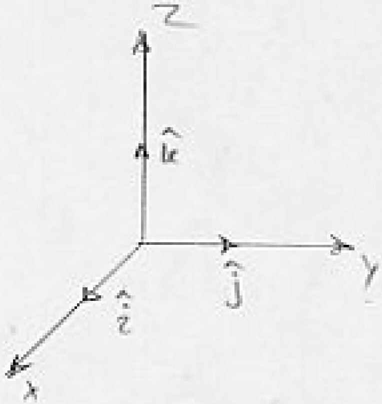
" " north " " " ?



$$d_x = d\cos\theta = (215\text{ km})(\cos(90-22)) = 81\text{ km}$$

$$d_y = d\sin\theta = (215\text{ km})(\sin(90-22)) = 199\text{ km}$$

3-4 Unit Vectors



$$\bar{a} = \begin{cases} a_x \\ a_y \\ a_z \end{cases} \sim \bar{a}(a_x, a_y, a_z)$$

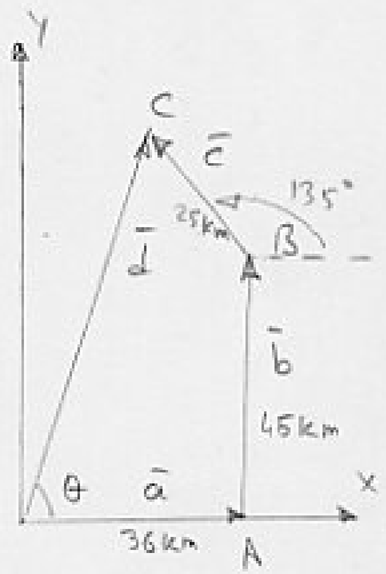
$$\sim \bar{a} = a_x \hat{i} + a_y \hat{j} + a_z \hat{k}$$

3-5 Adding Vectors by Components:

$$\bar{r} = \bar{a} + \bar{b} \qquad \begin{aligned} r_x &= a_x + b_x \\ r_y &= a_y + b_y \\ r_z &= a_z + b_z \end{aligned}$$

$$\bar{r} = (a_x + b_x)\hat{i} + (a_y + b_y)\hat{j} + (a_z + b_z)\hat{k}$$

Sample prob. 3-3



$\bar{d} = ?$ $\quad \theta = ?$

S.I.

$$d_x = a_x + b_x + c_x = 36\,\text{km} + 0 + (25\,\text{km})(\cos 135°)$$

$$= (36 + 0 - 17.7)\,\text{km} = 18.3\,\text{km}$$

$$d_y = a_y + b_y + c_y = 0 + 45\,\text{km} + (25\,\text{km})(\sin 135°)$$

$$= (0 + 45 + 17.7)\,\text{km} = 62.7\,\text{km}$$

$$d = \sqrt{d_x^2 + d_y^2} = \sqrt{(18.3\,\text{km})^2 + (62.7\,\text{km})^2} = 65\,\text{km}$$

$$\theta = \tan^{-1}\frac{d_y}{d_x} = \tan^{-1}\frac{62.7\,\text{km}}{18.3\,\text{km}} = 74°$$

Sample prob. 3-4

$$\begin{cases} \bar{a} = 4.2\hat{i} - 1.6\hat{j} \\ \bar{b} = -1.6\hat{i} + 2.9\hat{j} \\ \bar{c} = -3.7\hat{j} \end{cases}$$

$\bar{r} = \bar{a} + \bar{b} + \bar{c}$?

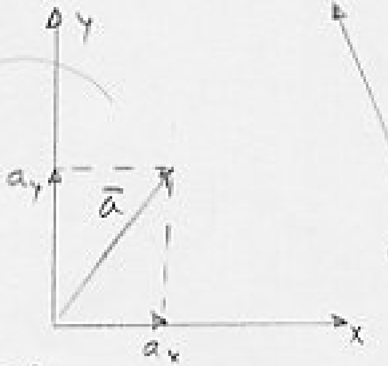
$$r_x = a_x + b_x + c_x = 4.2 - 1.6 + 0 = 2.6$$

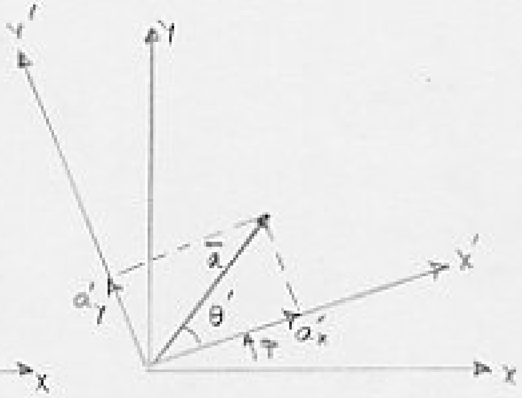
$$r_y = a_y + b_y + c_y = -1.6 + 2.9 - 3.7 = -2.4$$

$$\bar{r} = 2.6\hat{i} - 2.4\hat{j}$$

3-6 Vectors and the Laws of Physics

There are infinite set of components.





Which one is the right one?

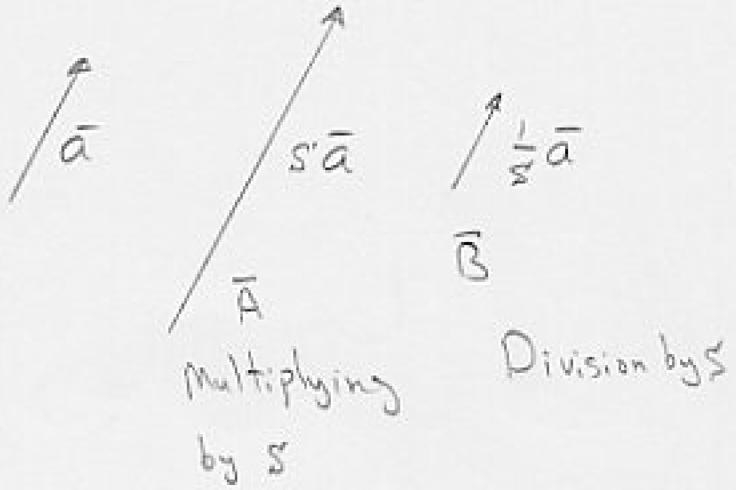
→ They are all equally valid, because each set (with its axis) just give a different way of describing the same vector $\bar{a}$:

All produce the same magnitude and direction for the same vector.

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{a_x'^2 + a_y'^2} \qquad \theta = \theta' + \varphi$$

• The laws of Physics (including vector additions, ..) do not depend on the location of the origin of the coord. sys. or on the orientation of the axes.

Multiplying / Division a Vector by a Scalar:



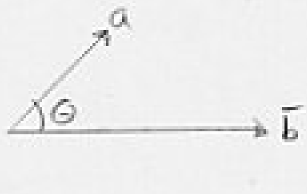
$|\bar{A}| = 5|\bar{a}|$

$|\bar{B}| = \frac{1}{5}|\bar{a}|$

A Glimpse Ahead

$\bar{F} = m\bar{a}$

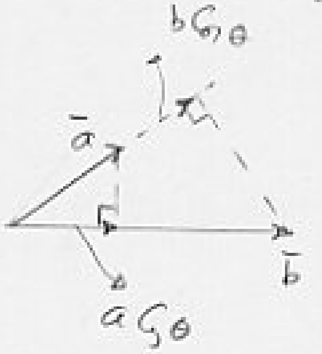
The Scalar product:



$\bar{a}\cdot\bar{b} = ab\cos\theta$

$\bar{a}\cdot\bar{b} = (a\cos\theta)b = (b\cos\theta)a$

$\rightarrow \bar{a}\cdot\bar{b} = \bar{b}\cdot\bar{a}$ Commutative law



$\bar{a}\cdot\bar{b} = (a_x\hat{i} + a_y\hat{j} + a_z\hat{k})\cdot(b_x\hat{i} + b_y\hat{j} + b_z\hat{k})$

$= a_x b_x + a_y b_y + a_z b_z$ (obeys distributive law)

A Glimpse Ahead:

$W = \bar{F}\cdot\bar{d} = Fd\cos\theta$

Sample prob. 3-5

$\bar{a} = 3.0\hat{i} - 4.0\hat{j}$ $\bar{b} = -2.0\hat{i} + 3.0\hat{k}$

$\varphi = ?$ the angle between them

Sol.

$\bar{a}\cdot\bar{b} = ab\cos\varphi = \sqrt{(3.0)^2+(-4.0)^2}\,\sqrt{(2.0)^2+(3.0)^2}\,\cos\varphi$

$= 18.0\cos\varphi$

$\bar{a}\cdot\bar{b} = (3.0\hat{i} - 4.0\hat{j})\cdot(-2.0\hat{i} + 3.0\hat{k}) = -6.0$

$18.0\cos\varphi = -6.0$ $\varphi = \cos^{-1}\frac{-6.0}{18.0} = 109^\circ$

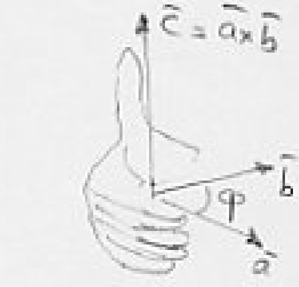
The Vector Product:

$\bar{c} = \bar{a} \times \bar{b}$ $\qquad c = ab \sin\theta$ $\qquad \theta$: smaller of the two angles between $\bar{a}$ and $\bar{b}$

$\bar{b} \times \bar{a} = -\bar{a} \times \bar{b}$

$\bar{a} \times \bar{b} = (a_x \hat{i} + b_x \hat{j} + b_z \hat{k}) \times (b_x \hat{i} + b_y \hat{j} + b_y \hat{k})$

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$$



right-hand rule

$(\bar{a} \times \bar{b})_i = \epsilon_{ijk} a_j b_k$

A Glimpse Ahead

$\bar{\tau} = \bar{r} \times \bar{F}$

Sample prob. 3-6

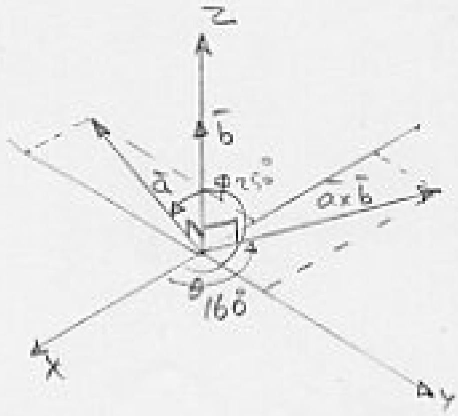
$|\bar{a}| = 18 \quad |\bar{b}| = 12$ $\qquad$ a: in xy-plane
its angle from x-dir $= 250°$

a) $\bar{a} . \bar{b} = ?$

$\bar{a} . \bar{b} = ab \cos\varphi = (18)(12)(\cos 90°) = 0$

b) $\bar{a} \times \bar{b} = ?$ $\qquad c = ab \sin\varphi = (18)(12)(\sin 90°) = 216$

$\theta = 250 - 90° = 160°$



Sample prob 3-7

$\vec{a} = 3\hat{i} - 4\hat{j}$ $\vec{b} = -2\hat{i} + 3\hat{k}$

$\vec{c} = \vec{a} \times \vec{b}$?

Sol.

$\vec{c} = (3\hat{i} - 4\hat{j}) \times (-2\hat{i} + 3\hat{k}) = -12\hat{i} - 9\hat{j} - 8\hat{k}$