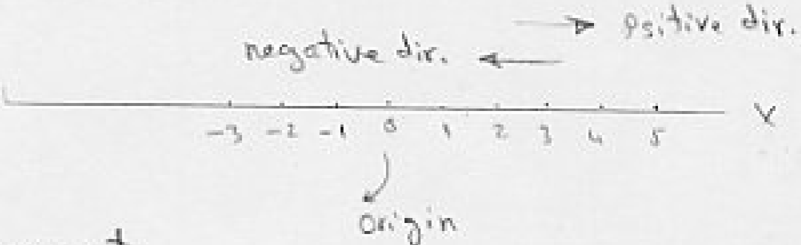
Chapter 2

Motion Along A Straight Line

2-1 Motion

The classification and comparison of motions is called <u>Kinematics</u>.

2-2 Position and Displacement



$\Delta x = x_2 - x_1$ displacement

<u>Ex.</u> $x_2 = 12$ $x_1 = 5$ $\Delta x = +7$ in positive dir.

$x_2 = 12$ $x_1 = 18$ $\Delta x = -6$ " negative "

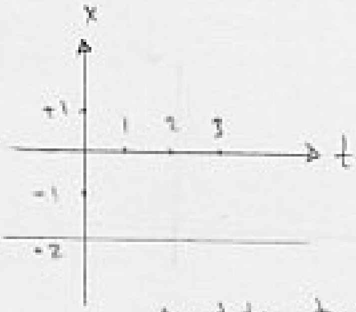
$|\Delta x|$ mag. of displacement

→ Displacement is a vector quantity

A vector quantity has $\begin{cases} \text{1- dir.} \\ \text{2- mag.} \end{cases}$

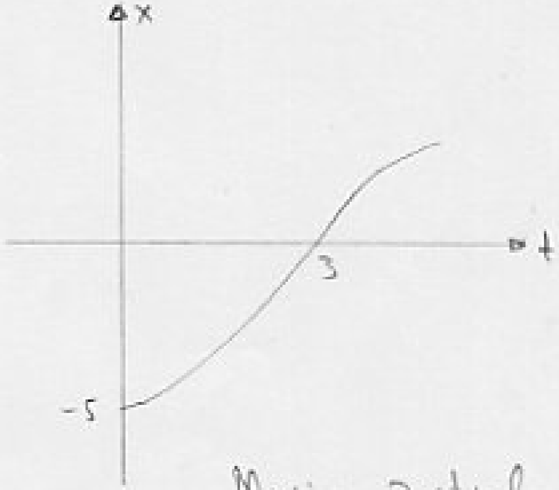
2-3 Average Velocity and Average Speed

$$\bar{V} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1} \quad \text{average velocity}$$



A stationary particle

$\bar{V}$: slop of the straight line that connects two points on the $x(t)$



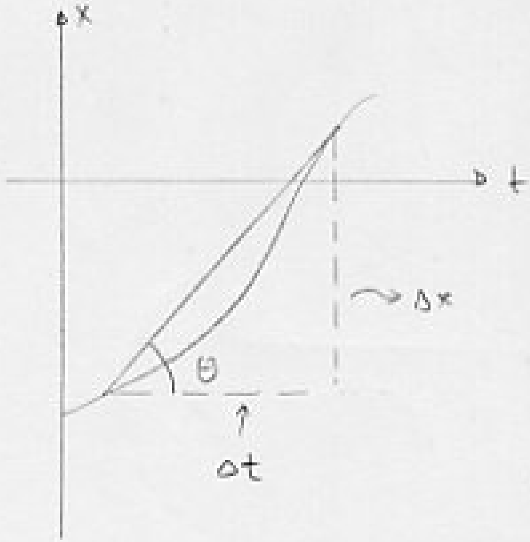
Moving Particle

$\bar{V}$: vector quantity

Since $\Delta t > 0$

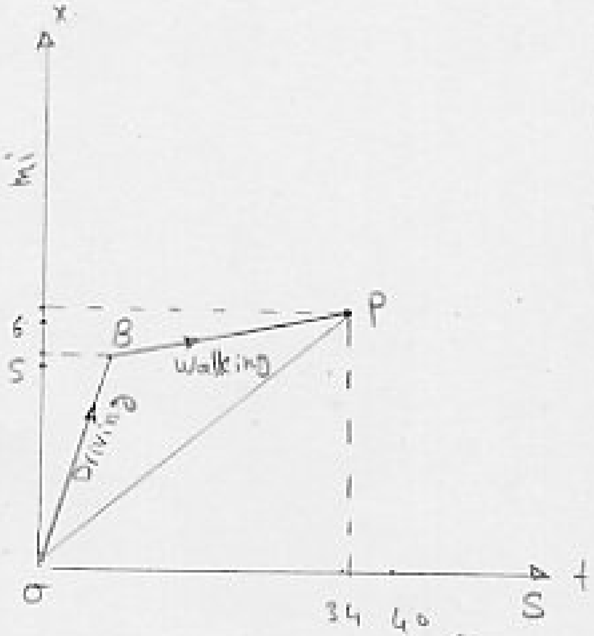
$\rightarrow \bar{V} > 0$ if $\Delta x > 0$

$V < 0$ " $\Delta x < 0$



$$|\bar{V}| = \tan\theta = \frac{\Delta x}{\Delta t}$$

Sample prob. 2-1



Driving from O to B;

$\bar{v} = 43\ mi/h \qquad \Delta x_{OB} = 5.2\ mi$

Walking from B to P;

$\Delta x_{BP} = 1.2\ mi \qquad \Delta t_{BP} = 27\ min (= 0.450 h)$

$\bar{v}_{tot} = ?$

Sol.

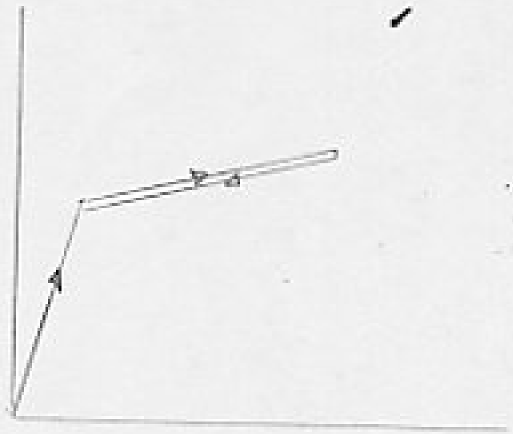
$$\Delta t_{OB} = \frac{\Delta x_{OB}}{\bar{v}} = \frac{5.2\ mi}{43\ mi/h} = 0.121\ h$$

$$\Delta t_{tot} = 0.121\ h + 0.450\ h = 0.571\ h$$

$$\bar{v} = \frac{\Delta x_{tot}}{\Delta t_{tot}} = \frac{5.2 + 1.2}{0.571\ h} = 11\ mi/h$$

Sample prob. 2-2

In the above prob. suppose you return from P to B in 35 min.



$\bar{v} = ?$

Sol.

$$\Delta t_{tot} = \frac{5.2\ mi}{43\ mi/h} + 27\ min + 35\ min$$

$$= 0.121\ h + 0.450\ h + 0.583\ h = 1.15\ h$$

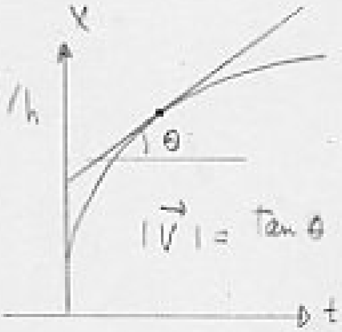
$$\bar{v} = \frac{5.2\ mi}{1.15\ h} = 4.5\ mi/h$$

Average Speed

$$\bar{S} = \frac{\text{total distance}}{\Delta t} \qquad (\text{no dir.})$$

Sample prob. 2-3

In the previous prob, $\bar{S} = ?$



Sol. $\bar{S} = \dfrac{\text{total dist.}}{\Delta t} = \dfrac{(5.2+1.2+1.2)}{1.15\ h} = 6.6\ mi/h$

2-4 Instantaneous Velocity and speed:

Mathematically: Instantaneous velocity is the rate at which the particle's x is changing with t at a given instant.

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

$$S = |\bar{v}|$$

Sample prob 2-4

Elevator motion

Fig (a) is given

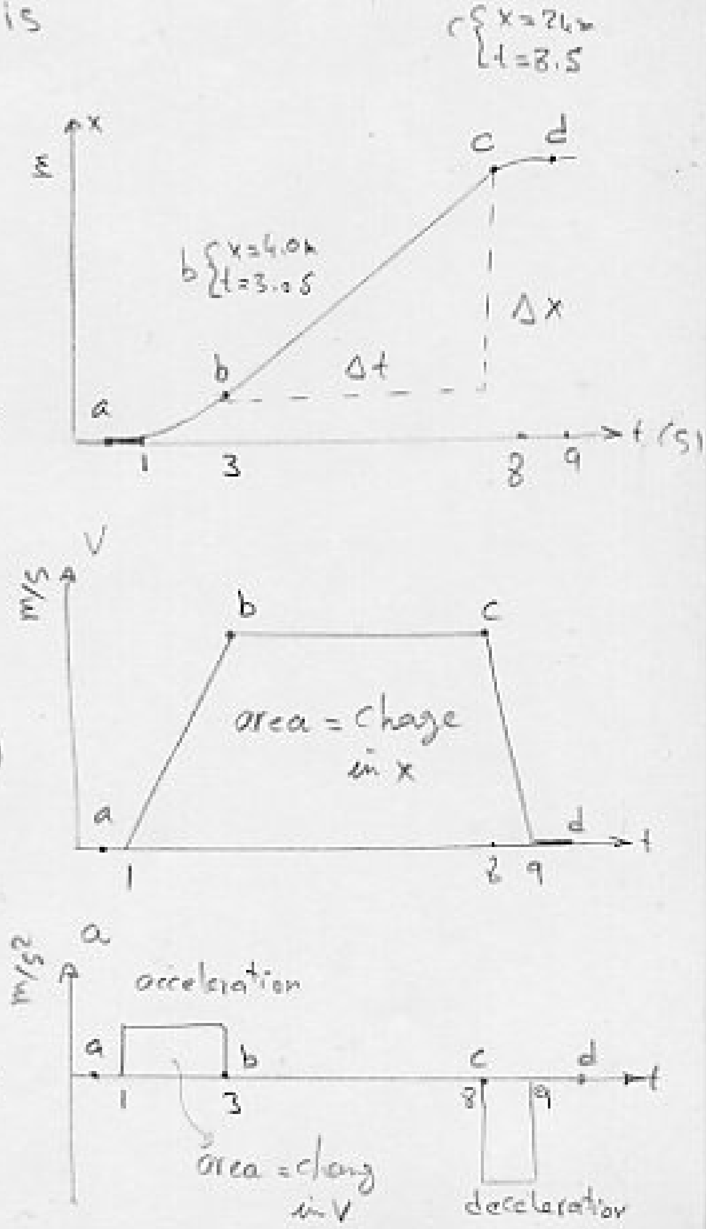
Plot $\bar{v}$

$$\vec{v}_{bc} = \frac{\Delta x}{\Delta t} \quad \frac{24-4.0}{8.0-3.0} = +4.0\ m/s$$

(a)

(b)

(c)



Having Fig (a) ⟶ Fig (b) can be produced

" " (b) ⟶ The shape of Fig (a) can be reproduced (not the values of x, because $\bar{V}(t)$ graph gives only changes in x)

To find Δx;

$$\underset{tc}{\Delta x} = \underset{(\text{under } v(t))}{\text{area}} = (\bar{V})(\Delta t) = \left(\frac{\Delta x}{\Delta t}\right)(\Delta t) = (4.0\ \text{m/s})(8.05-3.0)$$
$$= +20\ \text{m}$$

Sample prob. 2-5

$x = 7.8 + 9.2t - 2.1t^3$

$V(t) = V(3.5s) = ?$

Sol.

$$\vec{v} = \frac{dx}{dt} = 0 + 9.2 - 3(2.1)t^2 = 9.2 - 6.3t^2$$

$V(3.5) = -68\ \text{m/s}$

2-5 Acceleration

$$\bar{a} = \frac{\Delta V}{\Delta t} \quad \text{average acc.}$$

$$\vec{a} = \frac{d\vec{v}}{dt} \quad \text{instantaneous acc.}$$

acceleration: the rate of change in velocity

the slope of $V(t)$ at any point

Sample prob 2-6

$\Delta V = 392.54\ mi/h \qquad \Delta t = 3.72\,s$

$\bar{a} = ? \qquad \bar{a} = \frac{\Delta V}{\Delta t} = \frac{392.54\ mi/h}{3.72\,s} = +106\ \frac{mi}{h.s} = 47.1\ m/s^2$

(average acc.)

Sample prob. 2-7

$x = 4 - 27t + t^3$

a) $\vec{v}(t) = ? \quad \vec{a}(t) = ?$

Sol. $\vec{v}(t) = \frac{d\vec{x}}{dt} = -27 + 3t^2 \qquad \vec{a}(t) = +6t$

b) $t = ?$ for $\vec{V} = 0$

$V(t) = 0 \rightarrow 0 = -27 + 3t^2 \rightarrow t = \pm 3\,s$

c) Describe the motion of the particle for $t \geq 0$

At $t = 0$: $\quad x = +4\,m \quad V = -27\ m/s \quad a = 0$

$\rightarrow$ moving left

$0 < t < 3$ $\quad a > 0 \longrightarrow$ moving left

V: decreasing

The rate of a: increasing

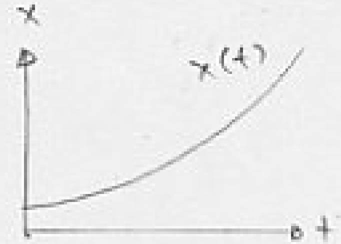
$t = 3$ $\quad V = 0$ $\quad x = -50$ $\quad$ (stops)

It continues to accelerate right at an increasing rate.

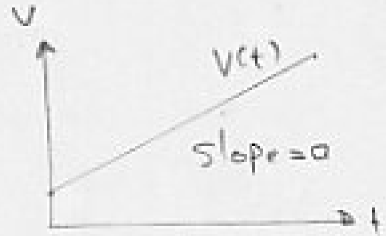
$t > 3$ $\quad a > 0$ $\quad V > 0 \Rightarrow x$: increasing

a, V are increasing.

2-6 Constant Acceleration:

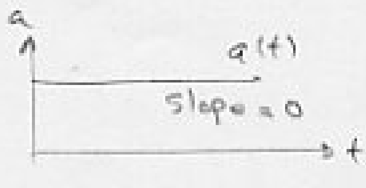


$\vec{a} = \bar{a}$ $\qquad$ for const. acc.



$\Rightarrow a = \dfrac{V - V_0}{t - 0}$ $\qquad (a = \text{const})$

$V = V_0 + at \qquad (1)$

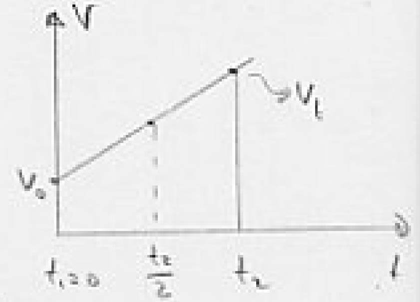


A check:

$$\begin{cases} \text{At } t = 0 \rightarrow V = V_0 \\ \dfrac{dV}{dt} = a \end{cases}$$

$\bar{V} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}$ $\quad (x_1 = x_0 \,, t_1 = 0) \rightarrow X = X_0 + \bar{V} t \quad (2)$

average velocity



$V = V_0 + at$: <u>straight</u> line $\quad (*)$

Under these conds. (a = const. , $(*)$, $t_1 = 0$)

$$\bar{V} = \frac{1}{2}(V_0 + V) \qquad (3)$$

(1) (3) $\rightarrow \quad \bar{V} = v_0 + \frac{1}{2} a t \qquad (4)$

(4) in (2) $\rightarrow \quad x - x_0 = v_0 t + \frac{1}{2} a t^2 \qquad (5)$

A check:

$$\begin{cases} \text{At } t = 0 \rightarrow x = x_0 \\ \frac{d}{dt}(\text{Equ. } 5) = \text{Equ } (1) \end{cases}$$

Eliminating t in (1) and (5) $\rightarrow \quad v^2 - v_0^2 = 2a(x - x_0)$

" a " " " " $\rightarrow \quad x - x_0 = \frac{1}{2}(v_0 + v) t$

" v_0 " " " $\rightarrow \quad x - x_0 = v t - \frac{1}{2} a t^2$

Equation for Motion with Const. Acc.

	Equ.	Missing Quantity
basic equ.	$v = v_0 + at$	$x - x_0$
basic equ.	$x - x_0 = v_0 t + \frac{1}{2}at^2$	v
	$v^2 = v_0^2 + 2a(x - x_0)$	t
	$x - x_0 = \frac{1}{2}(v_0 + v)t$	a
	$x - x_0 = vt - \frac{1}{2}at^2$	v_0

Sample prob. 2-8

Spotting a police car, you brake a Porsche from 75 Km/h to 45 Km/h over a displacement 88 m.

a) $a = ?$ (if a = const.)

$$a = \frac{v^2 - v_0^2}{2(x - x_0)} = \frac{(45^2 - 75^2)\left(\frac{km}{h}\right)^2}{2\,(.088\ km)} = -2.05 \times 10^4\ km/h^2 \simeq -1.6\ m/s^2$$

b) $t = ?$ the elapsed time

$$t = \frac{2(x - x_0)}{v_0 + v} = \frac{2(0.088\ km)}{(75 + 45)\ km/h} = 1.5 \times 10^{-3}\ h = 5.4\ S$$

c) $t = ?$ required the car to come in rest.

$$t = \frac{v - v_0}{a} = \frac{0 - 75\ km/h}{(-2.05 \times 10^4\ km/h^2)} = 3.7 \times 10^{-3}\ h = 13\ S$$

d) $x - x_0 = ?$ in (c)

important

$$x - x_0 = v_0 t + \frac{1}{2}at^2 = (75\ km/h)(3.7\times10^{-3}\ h) + \frac{1}{2}(-2.05\times10^{4}\ km/h^2)\cdot$$

$$\cdot(3.7\times10^{-3}\ h)^2 = 0.137\ km \simeq 140\ m$$

e) If $\begin{cases} a = -1.6\ m/s^2 \\ x - x_0 = 200 \\ V_{final} = 0 \end{cases}$ $\quad t = ?$

V_0 = different

$$t = \left(-\frac{2(x-x_0)}{a}\right)^{1/2} = \left(-\frac{2(200\,m)}{-1.6\ m/s^2}\right)^{1/2} = 16\ s$$

2-7 Another look at Const. Acc.:

The basic equs. in Table P17 can be obtained in the following way (cond.: a = const.)

$$a = \frac{dv}{dt} \rightarrow dv = a\,dt \rightarrow \int dv = \int a\,dt$$

$$\rightarrow V = \int a\,dt + c \qquad \rightarrow \qquad V = at + c$$

$C = ?$

$$\begin{cases} \text{at } t = 0 \\ V = V_0 \end{cases} \longrightarrow V_0 = a(0) + c \rightarrow c = V_0$$

$$\longrightarrow V = V_0 + at$$

Also;

$$V = \frac{dx}{dt} \rightarrow dx = v\,dt \rightarrow \int dx = \int v\,dt + c'$$

$$\rightarrow \int dx = \int (v_0 + at)\,dt + c' \rightarrow x = v_0 t + \frac{1}{2}at^2 + c'$$

$$c' = ? \quad \begin{cases} \text{at } t = 0 \\ x = x_0 \end{cases} \rightarrow c' = x_0$$

$$\rightarrow x - x_0 = v_0 t + \frac{1}{2}at^2$$

2-8 Free Fall Acceleration;

If you tossed an object either up or down and could somehow eliminate the effects of air on its flight, you would find that the object accelerates downward at a particular rate.

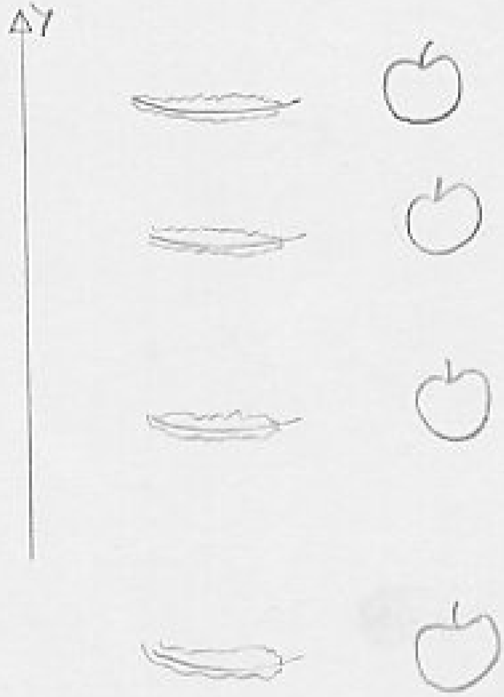
This rate is called the free fall acc. g.

g = indep (object characteristics, such as mass, density, shape.)

$$g = 9.8\ m/s^2 = 32\ ft/s^2$$

Equs. for Free Fall

Equ.	Missing Quantity
$v = v_0 - gt$	$y - y_0$
$y - y_0 = v_0 t - \frac{1}{2} g t^2$	v
$v^2 = v_0^2 - 2g(y - y_0)$	t
$y - y_0 = \frac{1}{2}(v_0 + v)t$	g
$y - y_0 = vt + \frac{1}{2} g t^2$	v_0



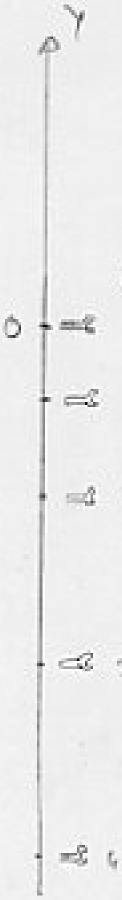
$g > 0$

Sample prob. 2-9

A wrench is dropped down the tall building.

a) $y = ?$ at $t = 1.5\,s$

b) $v = ?$ 〃



t (s)	y (m)	v (m/s)	a (m/s²)
0	0	0	-9.8
1	-4.9	-9.8	〃
2	-19.6	-19.6	〃
3	-44.1	-29.4	〃
4	-78.4	-39.2	〃

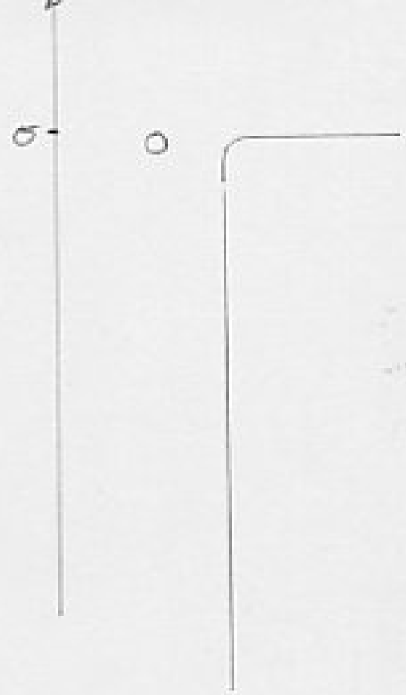
Sol.

a) $y = v_0 t - \frac{1}{2} g t^2 = (0)(1.5\,s) - \frac{1}{2}(9.8\,m/s^2)(1.5\,s)^2$

$= -11\,m$

b) $v = v_0 - gt = 0 - (9.8\,m/s^2)(1.5\,s) = -15\,m/s$

Sample prob. 2-10



A ball is released from the top of a building

$h = \Delta x = 800$ ft

$V_0 = 0$

a) $t = ?$ (Toching the ground)

$Y - Y_0 = v_0 t - \frac{1}{2} g t^2$

$-800\ ft = (0)t - \frac{1}{2}(32\ ft/s^2)t^2$

$16t^2 = 800 \rightarrow t = \pm 7.1\ S \rightarrow t = +7.1\ S$

we choose + sign because toching since the ball reaches the grod after it is released.

b) $V = ?$ at this time.

$v^2 = v_0^2 - 2g(y - y_0) = 0 - 2(32\ ft/s^2)(-800\ ft)$

$= 5.12 \times 10^4\ ft^2/s^2 \qquad v = \pm\ 226\ ft/s \simeq \pm 154\ mi/h$

$\rightarrow V = -226\ ft/s$

Since ball is moving downward, we choose − sign

Sample Prob. 2-11

A pitcher tosses a baseball stright up, with an initial speed of 12 m/s.

a) $t = ?$ to reach the highest point

$$V = V_0 - gt \rightarrow t = \frac{V_0 - V}{g} = \frac{12\,m/s - 0}{9.8\,m/s^2} = 1.2\,s$$

b) $Y_{max} = ?$

$$V^2 = V_0^2 - 2g(y - y_0) \rightarrow y = \frac{V_0^2 - V^2}{2g} = \frac{(12\,m/s)^2 - (0)^2}{2(9.8\,m/s^2)}$$

$y_{max} = 7.3\,m$

c) $t = ?$ for the ball to reach $Y = 5.0\,m$

$$Y - Y_0 = V_0 t - \frac{1}{2} g t^2 \rightarrow Y = V_0 t - \frac{1}{2} g t^2$$

$$5.0\,m = (12\,m/s)t - \frac{1}{2}(9.8\,m/s^2)t^2 \rightarrow 4.9t^2 - 12t + 5.0 = 0$$

$t = 0.53\,s$ $\quad$ $t = 1.9\,s$

on the way up $\quad$ on the way down