Chap. 2

Nucleon Structure

Neutrons and protons (the two lightest members of the baryon family) are not elementary particles.

They are assumed to be built from quarks.

QCD; is the study of quarks and interaction between them.

Quarks and Leptons:

The accepted view (these days) is that: All matter is made of
- i - quarks
- ii - leptons

The only exceptions are:

- i - photons $\xrightarrow{\text{mediating}}$ El-Mag. interaction
- ii - $W^\pm$ and Z^0 bosons $\longrightarrow$ weak "
- iii) gluons $\longrightarrow$ strong "
- v) gravitons $\longrightarrow$ Gravitational "

Quarks:

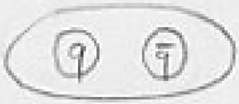
The lightest members of the hadron family { nucleons, pions }

There are six different <u>flavors</u>:
- u (up)
- d (down)
- c (charm)
- s (strange)
- t (top)
- b (beauty, or bottom)

Arrangement of quarks in pairs acc. to their masses:

Quarks			
$Q = \frac{2}{3}e$	u	c	t
$Q = -\frac{1}{3}e$	d	s	b
Leptons			
$Q = -e$	e	μ	τ
$Q = 0$	ν_e	ν_μ	ν_e

Quarks <u>have not</u> been observed in isolation:

They appear either as 

<u>or</u> 

meson baryon

Each flavor has 3-colors { red, green, blue }

Flavors and colors are <u>just</u> quantum numbers specifying the state of the particle.

There is no classical analogues to the flavor and color degrees of freedom

$\longrightarrow$ there are no observables that can be directly associated with them.

In this respect, they are similar to the parity label of a state which must be observed through indirect evidence such as the decay modes of certain particles.

Leptons;

They participate in $\left\{\begin{array}{l}\text{El.-Mag. interactions} \\ \text{Weak} \quad " \end{array}\right.$ but not in strong interactions.

$m_e c^2 = 0.511\ \text{Mev}$ $\quad m_\mu c^2 = 106\ \text{Mev}$ $\quad m_\tau c^2 = 1784\ \text{Mev}$

$m_\nu \approx 0$

$m_{\nu_e} c^2 \lesssim 30\ \text{ev}$ $\quad m_{\nu_\mu} c^2 < 0.25\ \text{Mev}$ $\quad m_{\nu_\tau} c^2 < 70\ \text{Mev}$

Lepton number conservation: The number of Leptons is conserved in a reaction.

Ex. –

$$n \longrightarrow p + e^- + \bar{\nu}_e \qquad (\text{mean life of } n = 896 \pm 10{,}5')$$

$$L = 0 \qquad L = 0 + 1 - 1$$

Remark: Antiparticle + particle $\longrightarrow$ state without particle

Hence, $L_{\bar{\nu}_e} = -1$

Remark: Charge conjugation op transfers a particle to its antiparticle.

Ex. –

$$p_{\text{bound}} \longrightarrow n + e^+ + \nu_e$$

$$L = 0 \qquad L = 0 - 1 + 1$$

Since $M_n c^2 = 939.566$ MeV $\quad M_p c^2 = 938.272$ MeV

$p_{\text{free}} \longrightarrow \ldots$ is not possible.

If $\nu_e = \bar{\nu}_e$

It would be possible $\longrightarrow$ $\nu_e + p \longrightarrow e^+ + n$ for charge conservation

(by the use of $\nu_e = \bar{\nu}_e$ produced in $p \longrightarrow n + e^- + \bar{\nu}_e$)

This is not observed $\longrightarrow$ Lepton number is conserved.

In contrast, the reaction,

$$\bar{\nu}_e + p \longrightarrow e^+ + n$$

is observed.

This stablishes $\begin{cases} \text{i) } \nu \text{ and } \bar{\nu} \text{ are two \underline{different} particles} \\ \text{ii) Lepton number must be \underline{conserved}.} \end{cases}$

Convention: $L = +1$ for electron

The other lepton numbers are determined by <u>conservation</u> requirements in reactions.

Majorana particle: Particle = antiparticle

Dirac " : " $\neq$ "

L_e, L_μ and L_τ are <u>separately</u> conserved.

Ex. –

$$\mu^- \longrightarrow e^- + \bar{\nu}_e + \nu_\mu$$

$L_e = 0,\ L_\tau = 0,\ L_\mu = 1,$ $\qquad L_e = +1 - 1 = 0$

$L_\tau = 0$

$L_\mu = 1$

Baryon number Conservation:

The number of different types of quarks are conserved in strong interactions.

$N_u = \text{const.} \quad N_d = \text{const.} \ldots$

One kind of quark cannot be changed into another except through weak interactions.

$\longrightarrow$ The flavor is a good quantum number if the weak interaction can be ignored.

It is more convenient to deal with baryon number (if we are not concerned with quark contents of hadrons)

Baryon number is conserved under the influence of both $\begin{cases} \text{strong} \\ \text{weak} \end{cases}$ interactions.

The only exception;

$$p \longrightarrow e^+ + \pi^0$$

This int. is allowed under theories for the grand unification of all forces.

At present;

the observed limit on the lifetime of a proton $> 10^{30}$ yr

We ignore this case, and will take baryon number to be conserved.

There is no conservation law for the number of mesons;

mesons $\xrightarrow{\text{can decay}}$ other mesons

" $\longrightarrow$ baryons and antibaryon pairs

" $\longrightarrow$ leptons and antilepton "

meson family $\begin{cases} \text{Pions} & m \sim 140 \text{ MeV} \quad (\text{lightest members}) \\ \vdots & \end{cases}$

Pions $\xrightarrow[\text{strong}]{\text{decay}}\!\!\!\!\!\!\!/\;\;$ another baryon (stable on the time scale of strong ints.

Pions $\xrightarrow[\text{weak}]{\text{decay}}$ muons

$$\pi^+ \xrightarrow{\text{weak}} \mu^+ + \nu_\mu$$

$$\pi^- \xrightarrow{\text{"}} \mu^- + \bar{\nu}_\mu$$

mean life $\approx 2.6 \times 10^{-8}$ s

$$\pi^0 \xrightarrow[99\%]{\text{weak}} \gamma + \gamma$$

mean life $= 8.4 \times 10^{-17}$ s

2.6×10^{-8} and $8.4 \times 10^{-17} \gg 10^{-23}$ (typical time scale of strong int.)

In all three modes of decay the <u>lepton</u> numbers are <u>conserved</u>.

The total number of <u>quarks</u> are also <u>conserved</u>. (we shall see later).

2-2) Quarks, the basic building block of Hadrons:

Quark masses:

Flavor	A	t	t_0	S	C	B	T	Q(e)	M(GeV)
u (up)	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{2}$	0	0	0	0	$+\frac{2}{3}$	≈ 0.39
d (down)	〃	〃	$-\frac{1}{2}$	0	〃	〃	〃	$-\frac{1}{3}$	〃 〃
s (strange)	〃	0	0	-1	〃	〃	〃	〃	〃 0.51
c (charm)	〃	〃	〃	0	1	〃	〃	$+\frac{2}{3}$	〃 1.6
b (beauty)	〃	〃	〃	〃	0	-1	〃	$-\frac{1}{3}$	〃 5.4
t (top)	〃	〃	〃	〃	0	0	1	$+\frac{2}{3}$	〃 20

A: baryon number t: isospin S: strangeness

C: charm B: beauty T: top

$\left\{\begin{matrix} u \\ d \end{matrix}\right.$ are the <u>lightest</u> quarks $\xrightarrow{\text{then}}$ the lightest <u>baryons</u> $\left\{\begin{matrix} \text{nucleons} \\ \Delta\text{-particles} \end{matrix}\right.$

and the lightest mesons $\{$pions must be made of these <u>two quarks</u>.

S quark carries a <u>non-zero</u> strangeness $\xrightarrow{\text{then}}$

K-mesons (kaons) and Λ-baryon with <u>nonzero</u> strangeness must carry this quark.

C-quark is discovered in;

$$\bar{e} + \overset{+}{e} \xrightarrow[\substack{3.1\ \text{Gev} \\ \text{Centre of mass} \\ \text{energy}}]{\substack{\text{narrow} \\ \text{resonance}}} J/\psi \quad \text{meson}$$

Since; meson = $(q\ \bar{q})$ $\xrightarrow{\text{then}}$ a new quark $\underset{\text{heavier}}{>}$ u, d, s is needed.

⟶ This is C- or charm quark.

It was confirmed also by other experiments, including the discovery of excited state of J/ψ.

b-quark is discovered in the Υ-meson (at 10 Gev).

t-quark: There is no experimental evidence.

Perhaps higher energy accelators are needed.

Is there other heavier pair of quark? (open problem)

For any quark $\xrightarrow{\exists}$ an antiquark

Hadrons = f(six q and six $\bar{q}$)

The properties of quarks $\xrightarrow[\text{from measurements}]{\text{can be deduced}}$ on hadrons $\begin{cases}\text{mesons}\\ \text{baryons}\end{cases}$

Reason: Observation on individual quarks cannot be carried out.

The assignment of $\begin{cases}\text{masses}\\ \text{magnetic moments}\\ \vdots \\ \text{other properties}\end{cases}$ to the quarks is therefore

inferred from what we know of the properties of $\begin{cases}\text{mesons}\\ \text{baryons}\end{cases}$

Our ability to make such deductions $\xrightarrow{\text{depends on}}$ our understanding of QCD. (which is incomplete especially at low energies)

$\xrightarrow{\text{then}}$ we resort to models.

For Example; mass of quark can be obtained from $\begin{cases}\text{1- Hadron mass}\\ \text{2- Strength of int. between quarks}\end{cases}$

(poorly known)

model

Constituent masses: The masses as they appear in the hadrons which may or may not be closely related to their true masses.

Fermions and Bosons:

Hadrons : $\begin{cases} \text{Baryons (Fermions)} \xrightarrow{\text{then}} \text{quarks are Fermions} \\ \text{Mesons} \end{cases}$

Remark: Fermions can be constructed from <u>odd</u> numbers of <u>fermions</u>.

The fact $\begin{cases} \text{1-Quark cannot exist as a free particle} \\ \text{2-Baryons are fermions} \end{cases}$

$\Longrightarrow$ The <u>lightest</u> baryon = f(3-quarks)

$\begin{cases} S_{quark} = \text{half-int.} & \text{intrinsic spin} \\ l_{quark} \underset{\text{should be}}{=} \text{low} & \text{(orbital angular momentum) in lightest hadrons} \end{cases}$

Charge of proton $= e^+$ $\longrightarrow$ The only possibility Proton $= f(2u, 1d)$

its charge $= (2\times\frac{2}{3} - 1\times\frac{1}{3})e^+$

charge of neutron $= 0$ $\longrightarrow$ neutron $= f(1u, 2d)$

its charge $= (1\times\frac{2}{3} - 2\times\frac{1}{3})e^+ = 0$

$|p\rangle = |uud\rangle$

$|n\rangle = |udd\rangle$

P and n are the <u>lightest</u> baryons $\xrightarrow{\text{then}}$ The lightest quark must form them.

Bosons; may be made from any even number of fermions.

Bosons can be created or annihilated under suitable conds.
Number of quarks must be conserved under strong interactions.

$\longrightarrow$ Mesons $= f(nq, n\bar{q})$

Quark Charge:

Most of hadrons are charged particles.

$\longrightarrow$ quarks must also carry electric charge.

The most convenient assignment: $\begin{cases} \text{charge of } u, c, t = +\frac{2}{3}e \\ \quad '' \quad , d, s, b = -\frac{1}{3}e \end{cases}$

Fundamental unit of charge $\longrightarrow \frac{1}{3}e$

Isospin:

Proton and neutron specification:

	spin	mass	charge	mag. diple moment
p	1/2	938.272 MeV	e^+	2.793
n	1/2	939.566 MeV	0	-1.913

The electromagnetic properties which are different, don't appear in the strong ints.

$\longrightarrow$ In the absence of electromagnetic ints a proton can not be destinguished from a neutron.

Thus the neutron and proton are two different state of a single particle "the nucleon".

Ex.

$m_s = \frac{1}{2}$
$m_s = -\frac{1}{2}$ (degenerate-indistinguishable)

(I) No mag. field $\bar{B}$

$m_s = \frac{1}{2}$
$m_s = -\frac{1}{2}$ (degeneracy removed - distinguishable)

(II) mag. field $\bar{B}$ Present

The difference between a proton and neutron is analogous to difference between particles with $m_s = \pm\frac{1}{2}$, if we substitute the Coulomb field with a magnetic field.

New lable ⟶ Isospin t

$$t = \frac{1}{2} \qquad \begin{cases} t_z = \frac{1}{2} & \text{for } p \\ t_z = -\frac{1}{2} & \text{" } n \end{cases} \qquad \text{by convention}$$

$$|p\rangle \equiv |t = \tfrac{1}{2}, t_z = +\tfrac{1}{2}\rangle \qquad |n\rangle \equiv |t = \tfrac{1}{2}, t_z = -\tfrac{1}{2}\rangle$$

For a nucleus:

$$T = \sum_{i=1}^{A} t(i)$$

$\xrightarrow{\text{in the absence of electromagnetic int.}}$ T = const. of motion

$\xrightarrow{\text{i.e.}}$ $[H, T^2] = [H, T_z] = 0$

$\xrightarrow{\text{As a result}}$ $\psi_{T, T_z \cdots}$ (eigen-states)

$\uparrow$ may be labled

Main source of isospin symmetry breaking $\longrightarrow$ Coulomb field

Another source less severe but noticeable $\longrightarrow$ difference between the masses of neutral and charged mesons exchanged between two nucleons.

The possibility of more fundamental isospin breaking terms in the <u>nuclear</u> force $\longrightarrow$ Possible small difference between the masses of u and d quarks.

Mathematical feature:

$$|p\rangle = |t=\tfrac{1}{2}, t_z=+\tfrac{1}{2}\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}_t$$

$$|n\rangle = |t=\tfrac{1}{2}, t_z=-\tfrac{1}{2}\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}_t$$

Charge number:

$$Q = t_z + \frac{1}{2} \qquad \text{in units of } e \quad (\text{for a nucleon})$$

$$Q = t_z + \frac{1}{2} A \qquad \text{several nucleons}$$

$$\tau_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \tau_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

where; $\tau_i \tau_j = \delta_{ij} I + i \epsilon_{ijk} \tau_k$

$$\tau_z \begin{pmatrix} 1 \\ 0 \end{pmatrix} = + \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad \tau_z \begin{pmatrix} 0 \\ 1 \end{pmatrix} = - \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\tau^2 \begin{pmatrix} 1 \\ 0 \end{pmatrix} = 3 \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad \tau^2 \begin{pmatrix} 0 \\ 1 \end{pmatrix} = 3 \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Remark:

$$T^2 |T, T_z\rangle = T(T+1) |T, T_z\rangle \qquad T_z |T, T_z\rangle = T_z |T, T_z\rangle$$

$$\tau_+ = \frac{1}{2} (\tau_1 + i \tau_2) = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$

$$\tau_- = \frac{1}{2} (\tau_1 - i \tau_2) = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

$$\tau_\pm |t, t_z\rangle = \sqrt{t(t+1) - t_z(t_z \pm 1)} \; |t, t_z \pm 1\rangle$$

$$T_3 = \frac{1}{2} \sum_{i=1}^{A} \tau_3(i) \qquad \text{for nucleus}$$

The usefulness of isospin:

1- Not restricted to the economy gained in treating $\underline{p}$ and $\underline{n}$ as two-diff. states of the same particle.

2- It is a fundamental $\underline{\text{symmetry}}$ (since it is const. of motion in $\underline{\text{strong}}$ int.)

2-4) Isospin of antiparticles:

$$\text{particle} + \text{antiparticle} \xrightarrow{\text{annihilation}} \underbrace{\begin{matrix}\text{(another pair of particle + antiparticle)} \\ \text{or} \quad \gamma + \gamma\end{matrix}}_{\text{electrically neutral}}$$

$\longrightarrow$ charge of particle = − charge of antiparticle

Conserved quantities:

Scalar quantities:

1- Energy

2- charge $\longrightarrow Q_{particle} = -Q_{anti\text{-}p}$

3- Lepton No. $\longrightarrow L_p = -L_{anti\text{-}p}$

4- Baryon No. $\longrightarrow B_p = -B_{anti\text{-}p}$

Vector quantities:

1- Spin

2- Isospin

The rules of vector addition $\longrightarrow |t_p| = |t_{anti-p}|$

$\xrightarrow{\text{to form}}$ scalar quantities.

Ex. $p + \bar{p} \longrightarrow \gamma + \gamma$

For p: $t = \frac{1}{2}$, also for $\bar{p}$: $t = \frac{1}{2}$

" : $t_z = +\frac{1}{2}$ $\longrightarrow$ " : $t_z = -\frac{1}{2}$

Because; for an isoscalar system:

$$\sum t_z(i) = 0$$

The charge number relation works for antiparticles as well.

$$Q = t_z + \frac{1}{2} A$$

Ex. For anti-proton; $t_z = -\frac{1}{2}$ $A = -1$ Baryon No

$Q = -1$

$$\psi \xrightarrow[C]{\text{charge conjugation}} \psi' \text{ (slightly more complicated)}$$

i.e. Not simply changing the sign of third component of isospin.

Remark: Charge conjugation:

$$\text{Particle} \xrightarrow[C]{\text{charge conj.}} \text{antiparticle} \quad \text{(in rel. quantum Mech.)}$$

As an example;

$$|p\rangle = a^{+}_{\frac{1}{2},+\frac{1}{2}}|0\rangle \quad , \quad |n\rangle = a^{+}_{\frac{1}{2},-\frac{1}{2}}|0\rangle$$

(only the isospin ranks are displayed, and the other lables for simplicity has been suppressed.)

Also;

$$|\bar{n}\rangle = b^{+}_{\frac{1}{2},+\frac{1}{2}}|0\rangle \quad , \quad |\bar{p}\rangle = b^{+}_{\frac{1}{2},-\frac{1}{2}}|0\rangle$$

Since particles and antiparticles are related to each other by charge conjugation C

$$\longrightarrow \quad a^{+} \overset{\text{related}}{\rightleftarrows} b^{+}$$

In addition to changes in the isospin (and spin), the wave funcs. of a particle and antiparticle may also differ by a Phase factor.

If $a^+_{tt_z}$ and $b^+_{tt_z}$ are operators with a definite irreducible spherical tensor rank t, the phase factor can be obtained by the tr. properties under a rotation in the isospin space.

It can be shown: $b^+_{tt_z} = (-1)^{t-t_z} a_{t,-t_z}$

$$|p\rangle \xrightarrow{C} (-1)^{\frac{1}{2}-(-\frac{1}{2})} |\bar{p}\rangle = -|\bar{p}\rangle$$

$$|n\rangle \xrightarrow{C} (-1)^{\frac{1}{2}-\frac{1}{2}} |\bar{n}\rangle = +|\bar{n}\rangle$$

For the quarks:

$$|u\rangle \xrightarrow{C} (-1)^{\frac{1}{2}-(-\frac{1}{2})} |\bar{u}\rangle = -|\bar{u}\rangle$$

$$|d\rangle \xrightarrow{C} (-1)^{\frac{1}{2}-\frac{1}{2}} |\bar{d}\rangle = +|\bar{d}\rangle$$

Intrinsic parity of an antiparticle:

$$P_{anti\text{-}p} = -P_{particle}$$

2-5) Isospin of Quarks:

In isospin formalism:

$$\tau_+ |n\rangle = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} = |p\rangle$$

$$\tau_- |p\rangle = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} = |n\rangle$$

In terms of quarks:

$$|p\rangle = |uud\rangle \qquad |n\rangle = |udd\rangle$$

$\rightarrow \tau_+ |udd\rangle = |uud\rangle \;, \quad \tau_- |uud\rangle = |udd\rangle$

Since there is no other quarks involved here $\longrightarrow$ u and d-quarks form an isospin <u>doublet</u>

$\rightarrow$ If we assign $t_z = +\frac{1}{2}$ for u-quark

$t_z = -\frac{1}{2}$ " d- "

$\longrightarrow$

$$\underset{+\frac{1}{2}}{u}\;\underset{+\frac{1}{2}}{u}\;\underset{-\frac{1}{2}}{d} = \frac{1}{2} \quad (p) \qquad \underset{+\frac{1}{2}}{u}\;\underset{-\frac{1}{2}}{d}\;\underset{-\frac{1}{2}}{d} = -\frac{1}{2} \quad (n)$$

The charge number; $Q = t_z + \frac{1}{2}A$

$$Q_p = \frac{1}{2} + \frac{1}{2}\left(3 \times \frac{1}{3}\right) = 1$$

$$Q_n = -\frac{1}{2} + \frac{1}{2}\left(3 \times \frac{1}{3}\right) = 0$$

Note: Baryon number of quark $= \frac{1}{3}$

Reason: 3-quarks = Baryon

Number of Baryons in Pion : $A = 3 \times \frac{1}{3}$

More formally:

$$\tau_{\pm}(\text{nucleon}) \longrightarrow \sum_{i=1}^{3} \tau_{\pm}(q_i) \qquad q_i : \text{ith-quark}$$

$\tau_{\pm}(q_i)$ acts on the isospin of i-th quark only.

Ignoring for the moment any antisymmetrization requirement between the three quarks in a nucleon

$$\tau_+ |n\rangle = [\tau_+(q_1) + \tau_+(q_2) + \tau_+(q_3)]\,|u(1)\,d(2)\,d(3)\rangle$$

first quark

$$\tau_+(q_1)|u(1)\rangle = 0$$

$$\tau_+(q_2)|u(1)\,d(2)\,d(3)\rangle = |u(1)\,u(2)\,d(3)\rangle$$

$$\tau_+(q_3)| \quad " \quad \rangle = |u(1)\,d(2)\,u(3)\rangle$$

Upon antisymmetrization these two terms produce identical results which we shall represent generically as $|uud\rangle$.

Quark wave funcs. for Pions:

$t = 1$ for pions $\begin{cases} t_z = +1 \\ t_z = 0 \\ t_z = -1 \longrightarrow \pi^- \end{cases}$

The only quarks (antiquarks) we can use (lightest quarks) are: $u, d, \bar{u}, \bar{d}$

$t_z = -1 \xrightarrow{\text{corresponds}} \bar{u}d$ combination

$t_1 = \frac{1}{2}, t_2 = \frac{1}{2} \xrightarrow{\text{can be coupled}} t = 0$ or 1

$\bar{u}d$ system can not be $t=0$ (because $t_z = -1$)

$\xrightarrow{\text{then}}$ $\bar{u}d$ system is $t = 1$

Thus: $|\pi\rangle = |\bar{u}\, d\rangle$

In general;

It is possible to find several different linearly independent components corresponding to the same t and t_z.

$$|t, t_z\rangle = a|q_i, \bar{q}_j\rangle + b|q_\ell, \bar{q}_m\rangle + \cdots$$

The wave func. must be antisymmetrized having: $\begin{cases} \text{spin part} \\ \text{isospin} \text{ "} \\ \text{radial} \text{ "} \end{cases}$ $\xrightarrow{\text{being}}$ eigenstate of H

Here we just consider isospin coupling;

$$|\pi^0\rangle = ?$$

$$\frac{1}{N}|\pi^0\rangle = \tau_+|\pi^-\rangle = \sum_{i=1}^{2}\tau_+(q_i)|\bar{u}d\rangle \quad \text{(N: normalization factor)}$$

$$\tau_+|d\rangle = |u\rangle \qquad , \tau_+|\bar{u}\rangle = -|\bar{d}\rangle$$

from symmetry requirement under charge conjugation.

The normalization factor;

$$\tau_\pm|t, t_z\rangle = \sqrt{t(t+1) - t_z(t_z\pm1)}\;|t, t_z\pm1\rangle$$

For $|\pi^-\rangle$: $t=1$, $t_z=-1$

$$\tau_+|1,-1\rangle = \sqrt{2}|1,0\rangle \quad \rightarrow N=\sqrt{2}$$

$$|\pi^0\rangle = \frac{1}{\sqrt{2}}\left[|u\bar{u}\rangle - |d\bar{d}\rangle\right]$$

Coupling of isospin $\frac{1}{2}$ particles using Clebsch-Gordan coeffs. gives the same result;

$$|t=1, m_t=0\rangle = \sum_{m_{t1} m_{t2}} (\tfrac{1}{2} m_{t1} \tfrac{1}{2} m_{t2} / 10)\, |\tfrac{1}{2} m_{t1}\rangle |\tfrac{1}{2} m_{t2}\rangle$$

But we have to include an extra (-) sign for charge-conjugation ($|d\rangle \rightarrow |\bar{d}\rangle$).

$|u\rangle \rightarrow |\bar{u}\rangle$?

For π^+:

$$|\pi^+\rangle = -|u\bar{d}\rangle$$

can be obtained either by: $\tau^+|\pi^0\rangle$

or : $|t=1, m_t=1\rangle = \sum_{m_{t_1}, m_{t_2}} (\frac{1}{2} m_{t_1}, \frac{1}{2} m_{t_2} | 1\,1) |\frac{1}{2} m_{t_1}\rangle |\frac{1}{2} m_{t_2}\rangle$

Again; a (−) sign is due to the charge conjugation between $|d\rangle$ and $|\bar{d}\rangle$. ($|u\rangle \to |\bar{u}\rangle$?)

One question ?

$|u\bar{u}\rangle$ and $|d\bar{d}\rangle$ have $t_z = 0$

But they are a mixture of $(t, t_z) = (1,0)$ and $(t, t_z) = (0,0)$

Can we take another combination of these ?

Yes; $|\eta_0\rangle = \frac{1}{\sqrt{2}} \left[|u\bar{u}\rangle + |d\bar{d}\rangle \right]$ $t=0$ system

which can be deduced from

$$|t=0, m_t=0\rangle = \sum_{m_{t_1}\, m_{t_2}} (\tfrac{1}{2} m_{t_1}, \tfrac{1}{2} m_{t_2} | 0\,0) |\tfrac{1}{2} m_{t_1}\rangle |\tfrac{1}{2} m_{t_2}\rangle$$

Rest mass of $\eta_0 \simeq 550$ MeV

$\left\{ \begin{array}{l} \text{Remark:} \\ |u\bar{u}\rangle = \frac{\sqrt{2}}{2}\left[|\eta_0\rangle + |\pi^0\rangle\right] \\ |d\bar{d}\rangle \ldots \end{array} \right\}$

π^+, π^0, π^- and η particles are the observed mesons that can be construced out of $u, d, \bar{u}, \bar{d}$ in the lowest possible energy state.

To obtain other mesons $\longrightarrow$ $\begin{cases} \text{1- introduce excitation in } q\text{-}\bar{q} \text{ sys.} \\ \text{2- invoke the s- and other more massive quarks.} \end{cases}$

Other quarks:

What about the isospin of c, s, t, b and their antiparticles?

They don't form doublets $t = \frac{1}{2}$ like u and d.

These 4-quarks $\xrightarrow[\text{are}]{}$ Isoscalar particles (i.e. $t = 0$)

Why?

If $t = 0$ for these particles, then $Q = t_z + \frac{1}{2} A$ doesn't work for them.

What is the new relation between Q and t_z

Since quarks are not found in isolation, $\xrightarrow{\text{then}}$

Isospin assignment must be done through the hadrons they make up.

For example we consider s-quark:

The lightest strange mesons: K-mesons

$$t=\frac{1}{2}\begin{cases} K^+(u\bar{s}) \\ K^0(d\bar{s}) \end{cases} \qquad \begin{cases} K^-(\bar{u}s) \\ \bar{K}^0(\bar{d}s) \end{cases}$$

Since for u and d $\in t=\frac{1}{2} \longrightarrow$ S must $\in t=0,1$

$\xrightarrow{\text{to form}} t=\frac{1}{2}$ for kaon-sys.

$$|T_1-T_2| \leqslant T \leqslant |T_1+T_2| \qquad |T_u-T_s| \leqslant T_k \leqslant T_u+T_s$$

$$t_s=1 \longrightarrow t_k=\frac{1}{2},\frac{3}{2} \qquad t_s=0 \longrightarrow t_k=\frac{1}{2}$$

$t_k=\frac{3}{2}$ have not been found (made of s-quark and an antiquark, either $\bar{u}$ or $\bar{d}$).

$\Longrightarrow t_s=0$

For the other heavy quarks we may continue in a similar way.

$$t_s=t_c=t_t=t_b=0$$

$$Q=t_z+\frac{1}{2}(A+S+C+B+T) \quad \text{the general form}$$

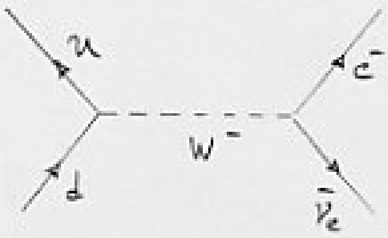
2-6 Strangeness and other Quantum Numbers:

The total number of each type of quarks = Const. in strong int.

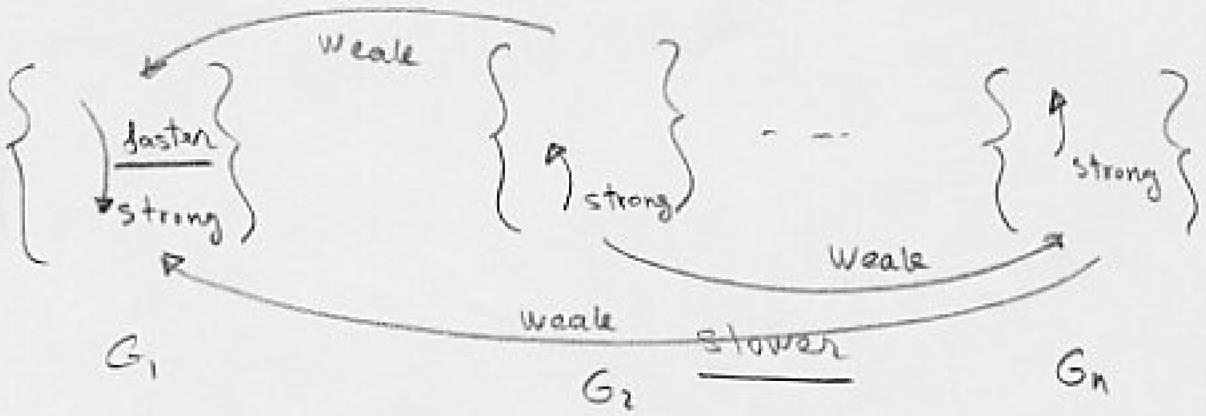
But through weak int.:

$$q_i \xrightarrow{\text{weak}} q_j \qquad i \neq j = 1, \cdots 6$$

Ex.



In terms of observed particles, the flavor degree of freedom in quarks show its presence by separating hadrons into different groups:



Each group of hadrons (G_i) is characterized by a definite number of quarks of a particular flavor.

For example:

i) $K^+(u\bar{s}) \xrightarrow{63.5\%} \mu^+ + \bar{\nu}_\mu$

ii) 〃 $\xrightarrow{21.2\%} \pi^+ + \pi^0$

N_q = const. (total number of quarks) in both cases

However, there is no s-quark among the end products of the decay.

One way to eliminate s-quark:

$\bar{s} \xrightarrow{\text{decay}} \bar{u} + ..$ $\qquad u + \bar{u} \xrightarrow{\text{annihilation}} ...$

Alternatively:

$\bar{s} \xrightarrow{\text{decay}} \bar{d} + ..$ $\longrightarrow$ to form the pion product.

Mean life of the $K^+ \simeq 1.2 \times 10^{-8}$ s (typical for weak decays)

Strangeness, Charm and Beauty.

Λ - baryon with quark structure (uds) , m = 1116 MeV

$$\underbrace{\pi^- + p}_{S = 0 + 0 = 0} \xrightarrow{\text{strong}} \underbrace{\Lambda + K^0}_{S = -1 + 1 = 0} \qquad K^0 \, (d\bar{s})$$

In <u>strong</u> int. <u>strangeness</u> is <u>a conserved</u> quantity.

For historical reasons: $S = -1$ for s-quark

$S = +1$ " $\bar{s}$

$S = 0$ " other quarks.

Similarly:

charm quantum number: $C = 1$ for c-quark

$C = -1$ " $\bar{c}$

$C = 0$ " other quarks

Also;

Beauty quantum number; $B = -1$ for b-quark

$B = +1$ " $\bar{b}$

$B = 0$ " other quarks.

long lifetime $\sim 10^{-8}$ s for Kaons $\longrightarrow$ S-conservation

Similarly:

Long lifetime $\sim 10^{-13}$ s for D-meson $\longrightarrow$ C- "
" " " B- " $\longrightarrow$ B- "

D (1869 Mev) $\begin{cases} D^+ (c\bar{d}) \\ D^- (\bar{c} d) \end{cases}$ $\quad$ B (5278 Mev) $\begin{cases} B^+ (\bar{b} u) \\ B^- (b \bar{u}) \end{cases}$

Also;

Mesons with relatively long life Time (in view of high energies).

Meson	Rest mass Mev	width Γ (Mev)	Mean life τ (s)	quark content
φ	1019.41 ± 0.01	4.41 ± 0.05	1.49×10^{-22}	$s\bar{s}$
J/ψ	3096.93 ± 0.09	0.068 ± 0.010	0.97×10^{-20}	$c\bar{c}$
Υ	9460.32 ± 0.22	0.052 ± 0.003	1.27×10^{-20}	$b\bar{b}$

Since these particles are not stable, they are observed as resonances when their production cross sections are plotted as functions of the bombarding energy.

For this reason $\longrightarrow$ their stabilities are characterized by Γ of their resonance curves;

$$\Gamma = \frac{\hbar}{\tau} \qquad (\hbar = 6.58 \times 10^{-22} \text{ Mev.s})$$

In view of high energies:

Lifetimes of J/ψ and Υ : very long

$\longrightarrow$ Γ: very narrow

This is because;

$$\psi/J(c\bar{c}) \overset{\text{cannot}}{\not\longrightarrow} D^+(c\bar{d}) + D^-(\bar{c}d)$$

$m = 3097$ $\qquad$ 1869×2

Similarly;

$$\Upsilon(b\bar{b}) \overset{\text{cannot}}{\not\longrightarrow} B^+(\bar{b}u) + B^-(b\bar{u})$$

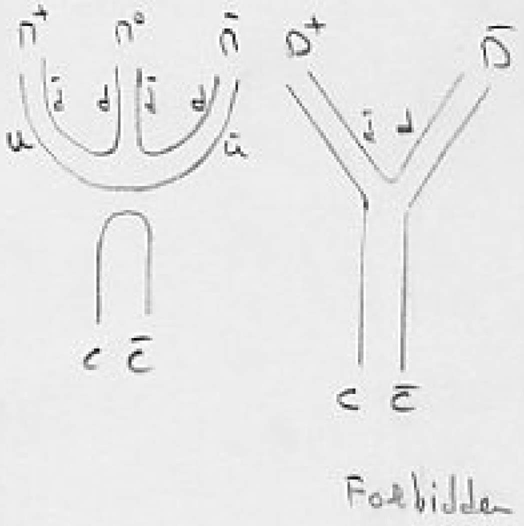
$m = 9460$ $\qquad$ 5278×2

As a result:

$$J/\psi \xrightarrow[\text{Processes}]{\text{slower}} \pi^+ + \pi^0 + \pi^-$$

86%

$$J/\psi \xrightarrow[14\%]{} e^+ + e^- \text{ and } \mu^+ + \mu^-$$



In contrast:

$$\varphi(s\bar{s}) \longrightarrow K^+ + K^-$$

$m = 1019$ $\qquad$ 493.6×2

Ex. - Parity determination:

$$Y_{\ell m}(\theta,\varphi) \xrightarrow[P]{} Y_{\ell m}(\pi-\theta),\pi+\varphi) = (-)^{\ell}\, Y_{\ell m}(\theta,\varphi)$$

As an example, we shall see how the intrinsic parity of a pion is determined to be negative.

$$\pi^- + d \longrightarrow \underbrace{\overset{\text{in s-state}}{\overset{\uparrow}{\pi^- + d}}}_{\pi\text{-mesic Atom}} \longrightarrow n + n$$

$$\left\{\begin{array}{l} \left\{\begin{array}{l} S_{\text{pion}} = 0 \quad (\text{intrinsic spin}) \\ S_d = 1 \end{array}\right. \longrightarrow S = 1 \\ \ell_{\pi d} = 0 \quad (\text{the } \pi^- \text{ is in the atomic s-state}) \end{array}\right.$$

$$\left\{\begin{array}{l} S = 1 \\ \ell_{\pi d} = 0 \end{array}\right. \longrightarrow J = 1 \quad \text{left-hand side } (\pi\text{-mesic atom})$$

$\Longrightarrow$ for right-hand side $J=1$ (due to ang. momentum cons.)

Also, the two n in the final state must be in an antisymmetric state (they are fermions).

The symmetry of wave func of Two neutrons is determined by:

and $\begin{cases} L & \text{the relative ang. momentum} \\ S & S = S_1 + S_2 \end{cases}$

To have antisymmetric wave func.: or $\begin{cases} L = \text{even} \text{ and } S = 0 & (\text{spatial part sym.}, \text{spin part anti sym}) \\ L = \text{odd} \text{ and } S = 1 & (\text{antisym.}, \text{sym.}) \end{cases}$

Since $J = L + S = 1$

$\rightarrow$ $(L,S) = (0,1)$, or $(1,0)$, or $(1,1)$, or $(2,1)$

(0,1) and (2,1) can be ruled out. (because both $\begin{cases}\text{intrinsic spin} \\ \text{spatial}\end{cases}$ wave funcs. are sym.)

(1,0) also is not allowed (both parts are antisym.)

$\xrightarrow{\text{The only possible combination}}$ (1,1) $\longrightarrow$ spatial part antisym.

$\xrightarrow{\text{The parity of right hand side}}$ $(-)^L = (-)^1 = -1$ (indep. of intrinsic parity of neutrons.)

$\xrightarrow{\text{Since parity is conserved}}$ Parity of left-hand side = negative on the left hand side there are 3-components contributing to the parity:

i) The parity of the ground state of deutron which is known to be even ($L = 0$ or 2, and both neutron and proton have the same intrinsic parity)

ii) The parity of orbital wave func. of the π-mesic atom which is positive.

iii) The intrinsic parity of π^-

$\longrightarrow$ Parity of left hand side $= (+)(+)(P_{\pi^-}) = -1$

$P_{\pi^-} = -1$

For fermions:

$$P_{particle} = - P_{antiparticle} \qquad P: \text{intrinsic parity}$$

(This can be seen from the structure of Dirac equ.)

For bosons:

$$P_{particle} = P_{antiparticle}$$

Color:

The need of this additional quantum number can be seen by examining the quark wave func. of a Δ-particle.

Δ is an isospin $t=\frac{3}{2}$ particle $\left\{\begin{array}{l}\Delta^{++}\\ \Delta^{+}\\ \Delta^{0}\\ \Delta^{-}\end{array}\right.$

$S=0$ for Δ-particle (strangeness)

$\xrightarrow{\text{then}}$ It must be made of u- and d-quarks alone (since it is the lightest baryon after nucleon)

For Δ^{++} (with highest charge) $\xrightarrow{\text{only one possibility}}$ (uuu)-structure to form $Q=2$.

Intrinsic parity of Δ, $P=+$ (experiment)

Intrinsic spin of Δ, $S=\frac{3}{2}$ $\longrightarrow$ $\chi_{s,\,uuu} = $ sym.

" Isospin " Δ $T=\frac{3}{2}$ $\longrightarrow$ $\chi_{t,\,uuu} = $ sym.

$$\xrightarrow{\text{Then}} \quad \Psi_\Delta = \Psi(x)\,\chi_s\,\chi_t \qquad \text{sym.} \qquad (1)$$

On the other hand quarks are <u>fermions</u> $\xrightarrow{\text{then}}$

$$\Psi_{\substack{\text{3-identical}\\ \text{quark}}} \overset{\text{must be}}{=\!=} \text{antisym.} \quad (2)$$

(1) and (2) are in contradiction.

$\xrightarrow[\text{possible ways}]{\text{There are two}}$ $\begin{cases} \text{i) Pauli principle is } \underline{\text{wrong}}. \text{ (very unlikely)} \\ \text{or} \\ \text{ii) There is another degree of freedom for quarks.} \end{cases}$

This new degree of freedom $\longrightarrow$ color

QCD: Theory dealing with strong ints. involving <u>colored</u> quarks.

Color $\begin{cases} R \text{ (red)} \\ G \text{ (green)} \\ B \text{ (blue)} \end{cases}$

Thus, from the example of Δ^{++}:

$\xrightarrow{\text{we can deduce}}$ Quarks in <u>hadrons</u> must be antisym. in color

$\xrightarrow{\text{i.e.}}$ The <u>net color</u> in hadron must <u>vanish</u>.

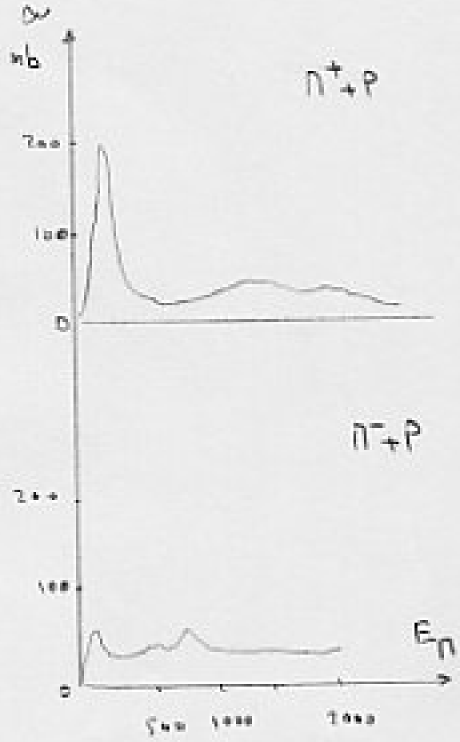
$\xrightarrow{\text{in other words}}$ Since color <u>has not</u> been an observed property

$\Longrightarrow$ All hadrons must be colorless objects.

For mesons; $(q\bar{q})$: $\quad \text{Color}_q = -\text{Color}_{\bar{q}}$

For baryons; The different colors cancel by being antisym. with respect to each other.

Δ-particle: was discovered by Fermi and Anderson in 1949 as a resonance in π^+ scattering off protons at pion kinetic energy $T_\pi = 195$ MeV



corresponds → to a mass of pion-proton system of 1232 MeV

This takes place at $\begin{cases} \ell = 1 \text{ channel} \\ S = \frac{3}{2} \\ T = \frac{3}{2} \end{cases}$

i.e. → P_{33} resonance.

Because it is a very strong resonance at a relatively low energy ⟶ nucleons inside a nucleus may be excited relatively easily to become Δ-particle.

2-7 Static Quark Model of Hadrons

In principle, a quark model of hadrons should involve all six different flavors.

This can be a rather complicated picture as a large number of hadrons can be constructed from $\begin{cases} \text{six } q \\ \text{six } \bar{q} \end{cases}$.

Fortunately:

$$m_c \text{ and } m_b \gg m_u, m_d, m_s$$

Hadrons involving c and b-quarks are rarely observed experimentally (because of high energy required for their production)

Hadrons involving t-quark also have not been reported to date.

For most of the hadrons (in particular those of interest in nucl. Phys.) $\begin{cases} u \\ d \\ s \end{cases}$ and $\begin{cases} \bar{u} \\ \bar{d} \\ \bar{s} \end{cases}$ are involved.

Mesons:

meson = any pair of $(q\bar{q})$

But most of mesons may be understood by considering:

only a <u>single</u> pair of $(q\bar{q})$.

l: relative orbital ang. momentum

S: $S = S_q + S_{\bar{q}}$ $S_q = S_{\bar{q}} = \frac{1}{2}$

$S = 0$ (singlet) $S = 1$ (triplet)

J: $J = l + S$

$l = 0$ lower energy (low-lying mesons)

Pseudoscalar mesons:

For pions; $S = l = 0 \rightarrow J = 0$

$\longrightarrow$ <u>scalar</u> particles $\longrightarrow \varphi$: invariant under spatial rotation

However:

$\varphi \xrightarrow{P} -\varphi$ (unlike ordinary scalars)

Parity of pion = (intrinsic parity of q)(intrinsic parity of $\bar{q}$)(parity of spatial wave-func. of $q\bar{q}$)

$+$ $-$ $(-)^{l=0} = +$

Parity of Pion $=$ $-$

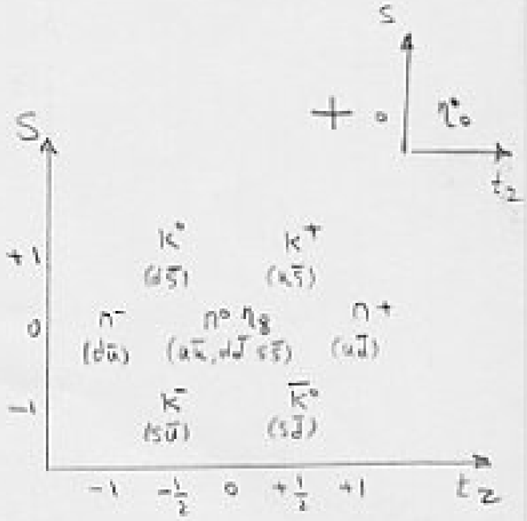
→ Pion wave-func. behaves like a <u>pseudoscalar</u> quantity

pseudoscalar: $\begin{cases}\text{Invariant under a rotation}\\ \text{But changes sign under a spatial inversion}\end{cases}$

pseudoscalar Mesons $\begin{cases} J=0 \\ \text{negative-Parity}\end{cases}$

$\begin{Bmatrix} u, d \\ \bar{u}, \bar{d}\end{Bmatrix} \longrightarrow$ 2×2 pseudoscalar mesons $\begin{cases}\pi^+\\ \pi^0\\ \pi^-\\ \eta_0\end{cases}$

$\begin{Bmatrix} u, d, s \\ \bar{u}, \bar{d}, \bar{s}\end{Bmatrix} \longrightarrow$ 3×3 " " octet $\begin{cases}K^0\\ K^+\\ \pi^-\\ \pi^0\\ \eta_8\\ \pi^+\\ K^-\\ \bar{K}^0\end{cases}$ + η_0



Pseudoscalar Mesons

Mesons of the <u>octet</u> form an irreducible representation in flavor space of u, d, and s.

That is, the tr. is from quarks of <u>one</u> flavor to <u>another</u>.

i.e. under an interchange among u, d and s, the wave-funcs. of 8-mesons transforms into each other as an irreducible representation of SU3 group.

$$\underset{\text{one meson}}{\psi} \xrightarrow{\text{tr. in flavor space}} \underset{\text{another meson}}{\psi'}$$

Remark:

$$\psi_{JM'}(r') = \sum_{M} \psi_{JM}(r)\, D^{J}_{MM'}(\alpha,\beta,\gamma)$$

The $(2J+1)$ components of spherical tensors ψ_{JM} for all possible values of M form an irreducible group under rotation.

The set $(2l+1)$ spherical harmonics $Y_{lm}(\theta,\varphi)$ is an example of spherical tensors with l = integer.

$Y_{lm}(\theta',\varphi')$ can be expressed as a combination of $Y_{lm'}(\theta,\varphi)$.

The remaining meson η_0:

$$\psi \xrightarrow{\text{tr. in flavor space}} \psi \quad \text{(invariant)}$$

i.e. η_0 is invariant under any interchange among u, d, and s.

$\longrightarrow$ η_0 forms an irreducible representation by itself.

In this way:

9-Mesons in model flavor space of u, d, and s-quarks and $\bar{u}$, $\bar{d}$, and $\bar{s}$ antiquarks $\xrightarrow{\text{may be classified into}}$ $\begin{cases}\text{octet} \\ \text{singlet}\end{cases}$ + acc. to their SU3-symmetry in flavor tr..

$$[3] \otimes [\bar{3}] = [8] \oplus [1]$$

Remarks:

As an example;

$$[\tfrac{1}{2}] \otimes [\tfrac{1}{2}] = [1] \oplus [0]$$

$$\begin{matrix} S_1 = \tfrac{1}{2} \\ S_2 = \tfrac{1}{2} \end{matrix} \quad S = S_1 + S_2 \longrightarrow S = \begin{cases} 0 \\ 1 \end{cases}$$

Also,

$$[\tfrac{1}{2}] \otimes [\tfrac{1}{2}] \otimes [\tfrac{1}{2}] = [\tfrac{3}{2}] \oplus [\tfrac{1}{2}] \oplus [\tfrac{1}{2}]$$

$$S = \begin{cases} 0 \\ 1 \end{cases} + S_3 = \tfrac{1}{2} \longrightarrow \begin{cases} \tfrac{1}{2} \\ \begin{cases} \tfrac{1}{2} \\ \tfrac{3}{2} \end{cases} \end{cases}$$

The wave-funcs:

The wave funcs. of $\pi^{\pm}$ and π^0 are the same as before (with $u, d, \bar{u}, \bar{d}$).

The wave funcs. of Kaons, have either s or $\bar{s}$. The charge carried by the Kaon determines the other partner of meson (i.e. $u, d, \bar{u}$ or $\bar{d}$).

The wave-func. of η_0:

Since $\Psi_{\eta_0} \xrightarrow{\text{tr. in flavor space}} \Psi_{\eta_0}$ (invariant)

$\longrightarrow$ the wave-func. must have the flowing form:

$$|\eta_0\rangle = \frac{1}{\sqrt{3}} \left\{ |u\bar{u}\rangle + |d\bar{d}\rangle + |s\bar{s}\rangle \right\}$$

This is the extension of the η_0-meson constructed before.

The wave-func. of η_8: can be constructed by requiring that:

$$\begin{cases} \langle \eta_8 | \eta_0 \rangle = 0 \\ \langle \eta_8 | \pi^0 \rangle = 0 \end{cases}$$

$$\rightarrow |\eta_8\rangle = \frac{1}{\sqrt{6}} \left\{ |u\bar{u}\rangle + |d\bar{d}\rangle - 2|s\bar{s}\rangle \right\}$$

η and η' -mesons:

There are two $I=0$ pseudoscalar mesons known at low energies: $\begin{cases} \eta & 548.8 \text{ MeV} \\ \eta' & 957.5 \text{ MeV} \end{cases}$

Since the SU3 (flavor) symmetry is not an exact one,

η and η' are mixtures of η_0 and η_8

$$\eta = \eta_8 \cos\theta + \eta_0 \sin\theta$$
$$\eta' = -\eta_8 \sin\theta + \eta_0 \cos\theta$$

θ : Cabibbo angle $\qquad \theta \simeq 10^\circ$ for pseudoscalar mesons

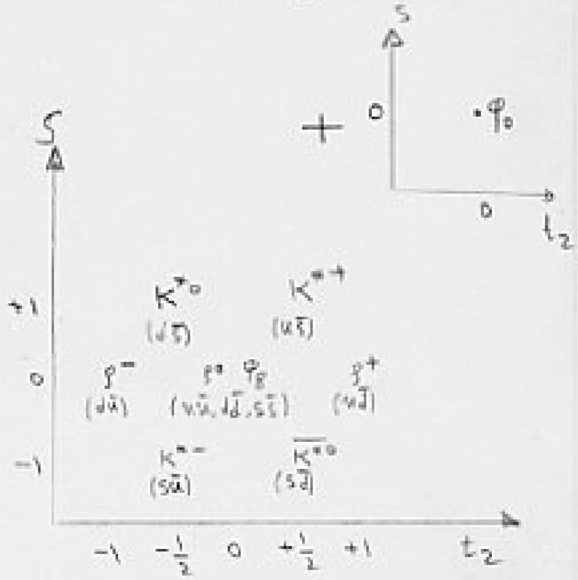
Vector Mesons :

$$\left\{\begin{array}{l} S = S_1 + S_2 = 1 \\ \ell = 0 \end{array}\right. \longrightarrow J = 1$$ but still parity remains negative.

Now we have a set of nine vector mesons

$\psi \xrightarrow{\text{spatial tr.}}$ behaves like an ordinary vector.

They have a structure similar to those of pseudoscalar mesons.



$\begin{cases} \varphi_0 \\ \varphi_8 \end{cases}$ isoscalar ($t=0$) vector mesons ($J=1$) with definite SU3 (in flavor space) symmetry

The observed isoscalar vector mesons $\begin{cases} \varphi \\ \omega \end{cases}$ have much larger SU3(flavor) mixing between φ_0 and φ_8 ($\theta \simeq 40°$)

$\rho^{\pm}$ and ρ^0 are members of isovector ($t=1$) vector ($J=1$) mesons.

$\begin{cases} K^{*0} \\ K^{*+} \end{cases}$ isodoublet vector mesons

$\begin{cases} K^{*-} \\ \overline{K^{*0}} \end{cases}$ " " "

$$M_{\text{vector mesons}} \gg M_{\text{pseudoscalar mesons}} \text{ (counterparts)}$$

For example; $M_\rho \simeq 767$ Mev $\quad M_\omega = 782$ Mev

while $\quad M_{\pi^\pm} \simeq 140$ Mev $\quad M_{\pi^0} = 135$ Mev

The only difference between vector and pseudoscalar mesons is in their S-values:

$\begin{cases} S = 0 \text{ in pseudoscalar Mesons} \\ S = 1 \text{ in vector } \quad " \end{cases}$

$\longrightarrow$ large difference in their masses is the consequence of the difference in the interaction between a quark and antiquark in the $S=0$ and $S=1$ states

Because of their larger masses:

$\rho \xrightarrow{\text{strong}}$ two pions   mean life $= 4\times10^{-24}$ s ($\Gamma = 153$ MeV)

$\omega \xrightarrow{90\%}$ three pions   " " $= 8\times10^{-23}$ s ($\Gamma = 8.5$ ")

$$\frac{\text{lifetime of } \rho \text{ and } \omega}{\text{" " } n} \simeq 10^{-8}$$

ρ and ω-mesons have an important role in nuclear ints..

Baryons:

For a given set of (l, S)-values:

$3\times3\times3 = 27$ baryons can be constructed by 3-flavors.

Acc. to SU3 (flavor) symmetry:

$$[3]\otimes[3]\otimes[3] = [10]\oplus[8]\oplus[8]\oplus[1]$$

[10]: decuplet completely symmetric under a tr. (in flavor)

[8]: octet Mixed sym.

[1]: singlet completely antisym.

Similar to the case of mesons;

By the use of SU3(flavor) symmetry $\rightarrow$ Quark wave-funcs. of Baryons can be constructed.

Because the number of quarks now is $\underline{3}$, the case is slightly more complicated.

We treat quarks as <u>identical</u> particles in different <u>flavor</u>-states.

Since hadrons are <u>color neutral</u> (<u>not</u> observable like spin)

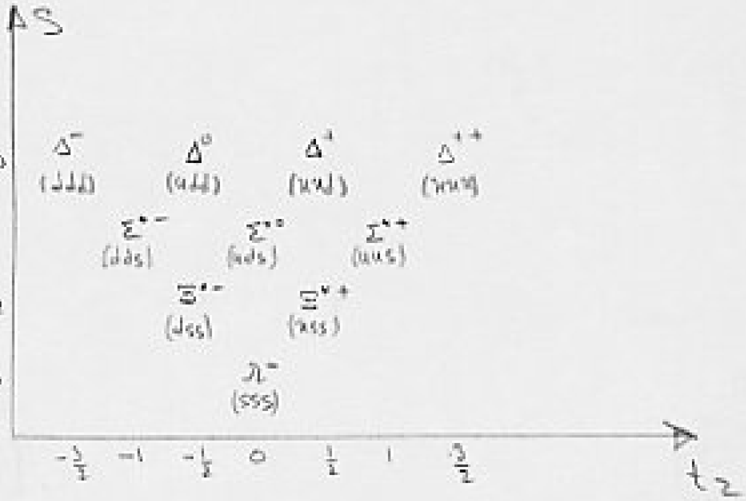
$\longrightarrow$ their quark wave-funcs must be <u>antysym.</u> in <u>color</u>.

$$\rightarrow \quad \Psi_{\text{hadron}} = \underbrace{\Psi_{\text{flavor}}\,\Psi_{\text{spin}}\,\Psi_{\text{isospin}}\,\Psi_{\text{spatial}}}_{\substack{\text{must be sym. under} \\ \text{permutaion of any two} \\ \text{quarks}}}\ \underbrace{\Psi_{\text{color}}}_{\text{anti-sym.}}$$

Decuplet:

Because of SU3 (flavor) sym. it is relatively simple to construct the quark wave-funcs.

(Decuplet is sym. in flavor space)



We start with Δ^{++};

As mentioned before both $\begin{cases} \text{intrinsic spin} = \text{sym.} \\ \text{isotopic spin} = \text{sym.} \end{cases}$

$$\begin{cases} s = \frac{3}{2} \\ t = \frac{3}{2} \end{cases}$$

Furthermore; $\underbrace{\psi_{\text{spatial}}}_{\substack{\text{must be} \\ \text{sym}}} \leftarrow \underbrace{\psi_{\text{spin}} \; \psi_{\text{isospin}}}_{\text{sym.}}$ = if it is to be sym.

(for three u-quarks)

Once the quark wave func. of Δ^{++} is known $\longrightarrow$ the other members of quartet, Δ^{+}, Δ^{0} and Δ^{-} may be obtained by using τ_{-};

$$\tau_{-} | \Delta^{++} > \ldots$$

$\longrightarrow$ this gives correct isospin structure of all Δ-particles.

However, unlike the pion case (two non-identical fermions, a quark and an antiquark), here we are dealing with 3-identical particles. In addition to isospin coupling, we must ensure, the proper symmetry between quarks under a permutation between any two of them.

For two particles:

$$\Psi_S(1,2) = \begin{cases} \Psi_\alpha(1)\Psi_\alpha(2) \\ \frac{1}{\sqrt{2}}\{\Psi_\alpha(1)\Psi_\beta(2) + \Psi_\alpha(2)\Psi_\beta(1)\} \\ \Psi_\beta(1)\Psi_\beta(2) \end{cases}$$

$$\Psi_A(1,2) = \frac{1}{\sqrt{2}}\{\Psi_\alpha(1)\Psi_\beta(2) - \Psi_\alpha(2)\Psi_\beta(1)\}$$

Under permutation;

$$P_{12}\Psi_S(1,2) = \Psi_S(2,1) = \Psi_S(1,2)$$

$$P_{12}\Psi_A(1,2) = \Psi_A(2,1) = -\Psi_A(1,2)$$

For Δ^{++}

$$|\Delta^{++}\rangle = |u(1)\rangle|u(2)\rangle|u(3)\rangle$$ (the same flavor)

To obtain Δ^+ we apply τ_- on $|\Delta^{++}\rangle$

There are 3-posibilities to change u-quark to d-quark

Thus $\rightarrow$

$$|\Delta^+\rangle = \frac{1}{\sqrt{3}}\{|d(1)\rangle|u(2)\rangle|u(3)\rangle + |u(1)\rangle|d(2)\rangle|u(3)\rangle + |u(1)\rangle|u(2)\rangle|d(3)\rangle\}$$

In shorthand form:

$$|\Delta^+\rangle = \frac{1}{\sqrt{3}}\{|duu\rangle + |udu\rangle + |uud\rangle\}$$

normalized and symmetrized

(color degree is omitted)

where we wish only to indicate the quark content of a hadron without having to display the permutation symmetry $\longrightarrow$ $|\Delta^+\rangle = |uud\rangle$

Similarly:

$$|\Delta^0\rangle = \frac{1}{\sqrt{3}}\left\{|ddu\rangle + |dud\rangle + |udd\rangle\right\}$$

$$|\Delta^-\rangle = |ddd\rangle$$

They are obtained by applying τ_- operator successively. Alternatively we could start from $|\Delta^-\rangle$ ($t_z = -\frac{3}{2}$) and apply τ_+

The wave-func. of $S=-1$ baryons in the decuplet;

We may begin with the $|\Delta^{++}\rangle$ and apply lowering operator for strangeness Δ_-

$$\Delta_-|uuu\rangle \longrightarrow \frac{1}{\sqrt{3}}\left\{|suu\rangle + |usu\rangle + |uus\rangle\right\} \equiv |\Sigma^{*+}\rangle$$

Now

$$\tau_-|\Sigma^{*+}\rangle = |\Sigma^{*0}\rangle$$

$$\tau_-|\Sigma^{*0}\rangle = |\Sigma^{*-}\rangle$$

Since $\tau_\pm|s\rangle = 0$ ($t=0$ particle)

$$\rightarrow \quad |\Sigma^{*0}\rangle = \frac{1}{\sqrt{6}}\left\{|dus\rangle + |uds\rangle + |dsu\rangle + |usd\rangle + |sdu\rangle + |sud\rangle\right\}$$

$$|\Sigma^{*-}\rangle = \frac{1}{\sqrt{3}}\left\{|dds\rangle + |dsd\rangle + |sdd\rangle\right\}$$

The wave-func. of $S=-2$ baryons in the decuplet;

$$\Delta_- |\Sigma^{*+}\rangle \longrightarrow |\Xi^{*+}\rangle$$

$$|\Xi^{*+}\rangle = \frac{1}{\sqrt{3}} \{ |uss\rangle + |sus\rangle + |ssu\rangle \}$$

$$\tau_- |\Xi^{*+}\rangle = |\Xi^{*-}\rangle = \frac{1}{\sqrt{3}} \{ |dss\rangle + |sds\rangle + |ssd\rangle \}$$

For $S=-3$, there is only one possibility;

$$|\Omega^-\rangle = |sss\rangle$$

isoscalar particle

$t=0$, $t_z=0$

$$Q = t_z + \frac{1}{2}(A+S+C+B+T)$$

$A=1$, $S=-3$, $t_z=0$, $C=B=T=0$ $\rightarrow Q=-1$

or $3\times(-\frac{1}{3}e) = -e$

Baryon Singlet:

A state of 3-quarks completely antisym. in flavor;

The quark content $\longrightarrow$ uds one of each flavor

There are $3! = 6$ possibilities

<u>However</u> a linear combination of these must be antisymmetric with respect to permutation between any two quarks (requirement of single state).

$$|\Lambda_1\rangle = \frac{1}{\sqrt{6}}\left\{|uds\rangle + |dsu\rangle + |sud\rangle - |dus\rangle - |usd\rangle - |sdu\rangle\right\}$$

$$P_{12}, P_{23}, P_{31} |\Lambda_1\rangle = -|\Lambda_1\rangle \qquad \text{odd permutation}$$

$$P_{12}P_{23},\ P_{31}P_{23} |\Lambda_1\rangle = |\Lambda_1\rangle \qquad \text{even} \quad ''$$

This is the only unique way (except for overall sign) to construct antisym. linear combination.

$\longrightarrow$ $|\Lambda_1\rangle$ forms an <u>irreducible</u> SU3(flavor) representation by <u>itself</u>.

A check on isospin-symmetry;

$$|\Lambda_1\rangle = \frac{1}{\sqrt{6}}\Big\{ \big(|u(1)\rangle|d(2)\rangle - |d(1)\rangle|u(2)\rangle\big)|s(3)\rangle$$
$$+ \big(|u(2)\rangle|d(3)\rangle - |d(2)\rangle|u(3)\rangle\big)|s(1)\rangle$$
$$+ \big(|u(3)\rangle|d(1)\rangle - |d(3)\rangle|u(1)\rangle\big)|s(2)\rangle\Big\}$$

For $|s\rangle$; $t=0$ $\longrightarrow$ the isospin of $|\Lambda_1\rangle$ is determined by the remaining part $|u\rangle$ and $|d\rangle$.

The remaining part is antisym. in isospin indices.

$\longrightarrow \in t=0$

$\longrightarrow$ The singlet SU3(flavor) representation describes an isoscalar particle.

The isospin is however, completely determined by the symmetry in flavor of the quark wave-func. and is not an independent deg. of freedom in the wave-func.

So far, we have not explicitly put in the intrinsic spin part of the wave func.

We mention only the result for Λ_1-baryon;

$$J^{\Pi} = \frac{1}{2}^{+}$$

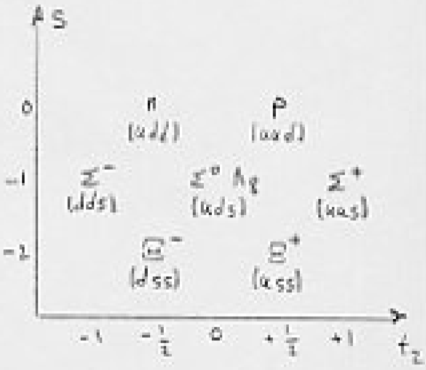
As mentioned earlier;

the SU3(flavor) symmetry is not an exact one.

The observed strangeness $S=-1$, isoscalar, $J^\pi = \frac{1}{2}^+$ particle Λ $\overset{\text{mixture of}}{=} \begin{cases} \Lambda_1 \\ \Lambda_8 \quad (J^\pi = \frac{1}{2}^+) \end{cases}$

Λ_8 ↑ member of baryon octet

Baryon Octet;



$J^\pi = \frac{1}{2}^+$ baryon-octet

The remaining 16 members of the 27 possible baryons (made of u, d and s) have mixed-sym. in flavor.

They are classified as $\begin{cases} \text{Octet (No.1)} \\ \quad \text{"} \quad \text{(No.2)} \end{cases}$ distinguished by their symmetries under simultaneous interchange of both flavor and spin.

We are interested only in the octect with lower energy (containing n and p)

The waves func. of each member of this group;

$$\psi \xrightarrow[\text{both flavor and intrinsic spin}]{\text{exchange of}} -\psi$$

As an example, let us construct the proton wave-func.;

$J^{\pi} = \frac{1}{2}^{+}$ for proton

We can start by coupling the intrinsic spin of the 3-quarks $\xrightarrow{\text{to the value}} \frac{1}{2}$

One possible way;

Two quarks (spin $\frac{1}{2}$) $\xrightarrow{\text{couple}}$ 0

$\xrightarrow{\text{Then couple the third one with spin UP}} (S, S_z) = (\frac{1}{2}, +\frac{1}{2})$ system

$$|\tfrac{1}{2}, +\tfrac{1}{2}> = \frac{1}{\sqrt{2}}\left(|q(1)\uparrow>|q(2)\downarrow> - |q(1)\downarrow>|q(2)\uparrow>\right)|q(3)\uparrow>$$

A second possibility to form $(S, S_z) = (\frac{1}{2}, +\frac{1}{2})$;

Two quarks (spin $\frac{1}{2}$) $\xrightarrow{\text{couple}}$ 1

$\xrightarrow{\text{couple the third one}} (S, S_z) = (\frac{1}{2}, +\frac{1}{2})$ a mixture of spin $\frac{3}{2}$ and $\frac{1}{2}$

To project out the desired $S = \frac{1}{2}$ part, a linear combination must be taken of the two possibilities;

$$\begin{cases} \{(q_1 q_2)_{1,1}\,(q_3)_{\frac{1}{2},-\frac{1}{2}}\} \\ \{(q_1 q_2)_{1,0}\,(q_3)_{\frac{1}{2},+\frac{1}{2}}\} \end{cases}$$

Determination of combined symmetry of the spin and flavor;

First we assign a flavor to each one of the quarks.

The condition: $\underline{P} = f(uu,d)$

Let us start by assigning the first two quarks with different flavors;

$$|\tfrac{1}{2}, +\tfrac{1}{2}\rangle = \frac{1}{\sqrt{2}}\Big(|u(1)\uparrow\rangle|d(2)\downarrow\rangle - |u(1)\downarrow\rangle|d(2)\uparrow\rangle\Big)|u(3)\uparrow\rangle$$

The combination of spin and flavor may be symmetrized in two stages;

First; we carry out the process only for the first two quarks;

$$|\tfrac{1}{2}, +\tfrac{1}{2}\rangle = \tfrac{1}{2}\Big(|u(1)\uparrow\rangle|d(2)\downarrow\rangle - |u(1)\downarrow\rangle|d(2)\uparrow\rangle$$
$$+|d(1)\downarrow\rangle|u(2)\uparrow\rangle - |d(1)\uparrow\rangle|u(2)\downarrow\rangle\Big)|u(3)\uparrow\rangle$$

Next, we generate the other terms by making the permutation P_{31} and P_{32} on each on the four terms;

This gives $\longrightarrow$ 12-terms

Grouping the identical terms $\longrightarrow$

$$|P\rangle = \frac{1}{\sqrt{18}}\Big\{2\big(|u\uparrow u\uparrow d\downarrow\rangle + |u\uparrow d\downarrow u\uparrow\rangle + |d\downarrow u\uparrow u\uparrow\rangle\big)$$
$$-\big(|u\uparrow u\downarrow d\uparrow\rangle + |u\uparrow d\uparrow u\downarrow\rangle + |d\uparrow u\uparrow u\downarrow\rangle$$
$$+ |u\downarrow u\uparrow d\uparrow\rangle + |u\downarrow d\uparrow u\uparrow\rangle + |d\uparrow u\downarrow u\uparrow\rangle\big)\Big\}$$

This wave-func. is antisymetical under a simultaneous interchange of flavor and spin between two quarks.

The Neutron wave func.;

Can be obtained by prescription of; $\begin{cases} u \to d \\ d \to u \end{cases}$ in the $|P\rangle$ wave func.

The wave func. of $S<0$ members;

Can be derived from $|P\rangle$ in a similar manner that we did in $|\Delta^{++}\rangle$.

2-8 Magnetic Dipole Moment of the Baryon Octet:

The hadron wave funcs. were obtained solely on the basis of symmetry considerations.

They are not eigenfucs. of a realistic Hamitonian involving interaction between quarks,

$$H\Psi \neq E_n \Psi$$

$\longrightarrow$ The don't describe any of the dinamic properties of hadrons with great accuracy.

But still they are useful as Zeroth-order approx. to the true wave funcs.

Besides $\begin{cases} \text{charge number} \\ \text{spin} \\ \text{Isospin} \\ \text{strangeness} \end{cases}$ of which we have already

made use in obtaining the wave funcs., the magnetic dipole moment is one such quantity to consider.

The magnetic dipole moment has two sources:

$\begin{cases} \text{1- intrinsic dipole moment of quarks} \\ \text{2- the orbital motion of quarks} \end{cases}$

For the baryon octet;

$$J^{\pi} = \frac{1}{2}^{+}$$

In the simple model : The three quarks are sym. in the spatial part of their wave fncs, with relative motion between them in the $l = 0$ states.

$\longrightarrow$ No contribution to the magnetic dipole moment from quark orbital motion.

Quark magnetic dipole moments:

Associated with the intrinsic spin of a particle, there is a magnetic dipole moment.

$$\mu = g \, \mu_D \, S$$ mag. dipole moment op. (for quark)

S: intrinsic spin op.

$\mu_D = \dfrac{q \hbar}{2 m_q c}$ (cgs) $\quad \mu_D = \dfrac{q \hbar}{2 m_q}$ (SI)

g: gyromagnetic ratio

For a Dirac particle;

i.e. A particle without internal structure

having $s=\frac{1}{2}$, we have $g=2$

In practice, no particle is observed to be a purely Dirac particle.

For example; electrons and muons emit and absorb virtual photons.

Contributions from these virtual processes $\xrightarrow{\text{give rise}}$ an anomalous mag. dipole moment, such that

$$g_e = 2 \times 1.0011596521 93(10)$$

$$g_\mu = 2 \times 1.001165923(8)$$

observed values

These are well understood and can be calculated to very high accuracy in QED.

We can consider quark as a simple Dirac particle,

i.e. $g = 2$

However; $m_q = ?$ unknown

$\longrightarrow \mu = ?$ (not possible to deduce in a simple way)

However if we assume $m_u = m_d$

$\longrightarrow \mu_u = -2\mu_d$

Nucleons:

Mag. dipole mom. of a baryon $\xrightarrow{\text{dep. on}}$ $\begin{cases} 1\text{- intrinsic spin of quarks} \\ 2\text{- relative } \ell \end{cases}$

Since $\ell = 0 \longrightarrow$ spin orientation of quarks determines the Mag. mom. of baryon.

Since $\ell = 0 \longrightarrow$ The <u>number</u> of quarks of <u>each flavor</u> in each one of the <u>Two possible</u> orientation <u>determines</u> the mag. mom.

For <u>Proton</u>;

Using the wave fuc. of proton we obtained;

If N: number op.

$$\langle p | N_{u\uparrow} | p \rangle = \frac{5}{3} \qquad \langle p | N_{d\uparrow} | p \rangle = \frac{1}{3}$$

$$\langle p | N_{u\downarrow} | p \rangle = \frac{1}{3} \qquad \langle p | N_{d\downarrow} | p \rangle = \frac{2}{3}$$

Net contribution of $= \frac{5}{3} - \frac{1}{3} = \frac{4}{3}$
u-quark to mag mom.

Ne contribution of $= \frac{1}{3} - \frac{2}{3} = -\frac{1}{3}$
d-quark to mag. mom.

$$\mu_p = \frac{4}{3}\mu_u - \frac{1}{3}\mu_d$$

Neutron wave func. can be obtained by $\begin{matrix} u \longrightarrow d \\ d \longrightarrow u \end{matrix}$ in $|p\rangle$.

$$\longrightarrow \qquad \mu_n = \frac{4}{3}\mu_d - \frac{1}{3}\mu_u$$

If we assume $m_d = m_u \rightarrow \quad \mu_u = -2\mu_d$

Then $\dfrac{\mu_n}{\mu_p} = \dfrac{\frac{4}{3}\mu_d - \frac{1}{3}\mu_u}{\frac{4}{3}\mu_u - \frac{1}{3}\mu_d} = \dfrac{-2}{3}$

This is in good agreement with the observed value of $\quad -\dfrac{1.913}{2.793} = -0.685$

Baryon with $S<0$:

For the other six members of the octet, there is at least one S-quark involved.

$\mu_s = ?$ we need.

Since $m_s >> m_u$ and m_d, we cannot relate μ_s to μ_u and μ_d as we did before ($m_u \simeq m_d$).

On the other hand
1- The mag. moments of Baryon-octet are known.
2- They can be given in terms of the intrinsic mag. moments of three quarks.

$\longrightarrow$ A least square fitting procedure may be used to deduce the three unknown quark values from these 8-pieces of known data.

Then we have to find:

$$\mu_{\text{baryon}} = f(\mu_u, \mu_d, \mu_s)$$

as we did for n and P.

Although we didn't find the quark-wave func. in detail for the $S<0$ members (as we did for n and p),

it is possible to count; $\begin{cases} N_{\text{each flavor}} \uparrow = ? \\ N_{\text{each flavor}} \downarrow = ? \end{cases}$

For those $\text{Baryon}_{\text{Strange}} \overset{\text{that}}{=} f(s,d)$ or $f(s,u)$

for example Σ^+ with the structure (uus) is comparable with proton with the structure (uud)

The difference is just in Σ^+, we have s, instead of d in P.

Since all the members of the octet have the same combined symmetry for $\begin{cases} \text{spin} \\ \text{flavor} \end{cases}$

$\longrightarrow$ the P and Σ^+ must have very similar quark wave func. except for the replacement of d with s.

$\xrightarrow{\text{Hence}}$

$$\mu_{\Sigma^+} = \frac{4}{3}\mu_u - \frac{1}{3}\mu_s$$

Similarly for Σ^- with the structure (dds) comparable with neutron, we have

$$\mu_{\Sigma^-} = \frac{4}{3}\mu_d - \frac{1}{3}\mu_s$$

Using similar methods, expressions for the mag. moments of the two $S=-2$ members of the octet; $\begin{cases} \Xi^- (dss) \\ \Xi^+ (uss) \end{cases}$ can be obtained.

For Λ_8 and Σ^0; (We make use of their isospin difference) to derive their wave-funcs.
(both with (uds)-structure)

Let us start with Λ_8;

Λ_8: $t=0$ isospin singlet

We ignore s-quark for the moment, since it is an isoscalar particle not involved in any isospin considerations

In order to project out the isospin $t=0$ part;

Antisymmetric linear combination of the two possible arrangement of u and d is needed (our aim is to form a spin-up baryon)

($t=0$ is the common part in both Λ_1 and Λ_8)

$$|(t,t_z)=(0,0)\rangle = \frac{1}{\sqrt{2}}\left\{|u(1)\,d(2)\rangle - |d(1)\,u(2)\rangle\right\}$$

Since

Under simultaneous exchange of $\begin{cases}\text{spin}\\ \text{flavor}\end{cases}$

$\longrightarrow$ Wave func must be sym. or antisym.?

Hence $\longrightarrow$ The spins of two quarks can not be both up (such an arrangement will be antisym under spin & flavor exchange)

The only possibility:

$$|(t,t_z)=(0,0);(S,m_s)=(0,0)\rangle = \frac{1}{2}\left\{(|u(1)\uparrow d(2)\downarrow\rangle - |d(1)\uparrow u(2)\downarrow\rangle) + (|d(1)\downarrow u(2)\uparrow\rangle - |u(1)\downarrow d(2)\uparrow\rangle)\right\}$$

(Note: If in the first line both quarks are in spin-up state, in the second line they must also be both in spin-up state to form a certain resultant spin.)

Note that $S_{total}=0$ as a result of sym. requirement.

We can now couple s-quark $\xrightarrow{\text{to form}}$ Spin $\frac{1}{2}$ system of 3-quarks

$$|(t,t_z)=(0,0);(S,m_s)=(\tfrac{1}{2},+\tfrac{1}{2})\rangle = \frac{1}{2}\left\{\quad // \quad\right\}|s(3)\uparrow\rangle$$

The wave func. is not properly antisymmetized with respect to the third quark.

However; for the purpose of calculating the mag. dipole moment, we need;

$N_{\text{each flavor}\uparrow} = ?$

$N_{\quad \text{"} \quad \downarrow} = ?$

Note: What we need is, to form a spin-up baryon, and we have done it, and any other treatment must not change this character (like anti-symmetrization).

and this is indep. of symmetrization among the three quarks beyond those given we obtained.

$$\longrightarrow \mu_u = \mu_d = 0 \quad \rightarrow \mu_{\Lambda_8} = \mu_s$$

Because of the crudness of the model used here; $\rightarrow$

$\rightarrow$ There is no point in considering any SU3(flavor) symmetry-breaking effects $\xrightarrow{\text{and the resulting}}$ difference between Λ_8 and the observed Λ.

Σ^0 :

$$\Sigma^0 \in t=1 \begin{cases} \Sigma^+ \\ \Sigma^0 \\ \Sigma^- \end{cases}$$

The wave fuc. is more complicated than Λ_8 case, since

$$u\,a\,d\,d \xrightarrow{\text{must couple to}} S=1$$

We may get information in some other way:

$$\Sigma^0 \longrightarrow \Lambda + \gamma$$

is similar to mag. dipole transition

$$\longrightarrow \quad P \sim |\mu_{\Sigma^0 \to \Lambda}|^2 \quad \text{transition probability}$$

where

$$\mu_{\Sigma^0 \to \Lambda} = \langle \Sigma^0 | O(M1) | \Lambda \rangle = -\frac{1}{\sqrt{3}} (\mu_u - \mu_d)$$

$O(M1)$: Mag. dipole tr. op.

$$|\mu_{\Sigma^0 \to \Lambda}| = 1.59 \pm 0.09 \ \mu_N \qquad \text{experimentally}$$

$$\mu_{\Sigma^0 \to \Lambda} = -1.59 \pm 0.09 \ \mu_N \qquad \left(\mu_N = \frac{e\hbar}{2M_p c} \text{ nuclear magneton, cgs}\right)$$

from other sources.

The values μ_u, μ_d and μ_s can be deduced by fitting these three unknown values to the eight measured dipole moments.

The calculated values for the baryon magnetic dipole moments agree quite well with observation.

This close agreement has two implications:

1- The model used is reasonable

2- The values deduced for μ_u, μ_d and μ_s are physically meaningful

Note:

In our simple model we considered only $l=0$ contributions. But we will see later angular momentum is not a const. of motion.

Therefore $\rightarrow$ The ground states of members of the baryon octet is not purely $l=0$. There are mixings from $l>0$ terms.

The calculated values for μ are:

$\mu_u = 1.852 \quad \mu_d = -0.972 \quad \mu_s = -0.581$

$\longrightarrow \dfrac{\mu_u}{\mu_d} = -1.91$ close to the value of -2, we obtained earlier, by assuming $m_u = m_d$ and $g_u = g_d$.

Using these values $\longrightarrow$ $m_u c^2 = 0.34\,\text{GeV}$
$m_d c^2 = 0.32\,\text{GeV}$
$m_s c^2 = 0.54\,\text{GeV}$

Octet member	Quark content u	d	s	Best fit (μ_N)	Observed (μ_N)
p	4/3	−1/3	0	2.793	2.792847386(63)
n	−1/3	4/3	0	−1.913	−1.91304275(45)
Λ	0	0	1	−0.581	−0.613(4)
Σ^+	4/3	0	−1/3	2.663	2.42(5)
Σ^-	0	4/3	−1/3	−1.102	−1.157(25)
Ξ^0	−1/3	0	4/3	−1.392	−1.250(14)
Ξ^-	0	−1/3	4/3	−0.451	−0.69(4)
$\Sigma^0 \to \Lambda$	$-\sqrt{1/3}$	$\sqrt{1/3}$	0	−1.630	−1.59(9)
Ω^-			3	−1.744	
u	1			1.852	
d		1		−0.972	
s			1	−0.581	

2.9) Hadron Mass and Quark-Quark Interaction:

Another striking feature in hadron spectroscopy is the systematics in their masses.

The masses for the Baryon octet are correlated by S-quantum number.

i.e. Δm = small for each group with the same S.

But Δm = large for two members from different groups with different S.

Baryons		Mesons	
$S=0$		Pseudoscalar mesons	
p	938.27231(28)	$\pi^\pm$	139.56755(33)
n	939.56563(28)	π^0	134.9734(25)
$S=-1$		$K^\pm$	493.646(9)
Λ	1115.63(5)	$K^0, \bar{K}^0$	497.671(30)
Σ^+	1189.37(6)	η	548.8(6)
Σ^0	1192.55(9)	η'	957.50(24)
Σ^-	1197.43(6)	Vector mesons	
$S=-2$		ρ	766.9(12)
		K^*	892.09(30)
Ξ^0	1314.9(6)	ω	781.99(13)
Ξ^-	1321.32(13)	ϕ	1019.414(10)

Low lying hadron masses (MeV/c^2)

Conclusion:

$m_u \simeq m_d$, $m_s > m_u, m_d$ by the order of 100-200 MeV

Further support for s-quarks being more massive:

Δm in
- members of baryon decuplet
- " " pseudoscalar mesons
- " " vector "

Evidences for c and b-quarks to be more massive:

$\left\{\begin{array}{l} J/\psi \\ \Upsilon \end{array}\right.$ mesons

The masses of different quarks can be deduced $\longrightarrow$ from a comparison of masses of hadrons and mesons made of different quarks (except for t-quark)

Small Δm between hadrons having the same S-number $\xrightarrow[\text{from}]{\text{can come}}$ $\left\{\begin{array}{l} \text{1- Electromag. effects} \\ \text{2- small } \Delta m \text{ between u and d-quarks} \end{array}\right.$

But our present knowledge of the strong int. is not able to elucidate on this question.

Despite $M_n - M_p$ and $M_{\pi^\pm} - M_{\pi^0}$ are small but are important in understanding some of the nuclear phenomena, like isospin symmetry-breaking in the nuclear forces.

Since quarks have not been observed in isolation outside hadrons, $\longrightarrow$ the masses deduced from hadron spectra are not necessarily their true masses.

Observed hadron masses $= f$ (intrinsic masses of quarks, binding energy between them)

In nuclear phys., having the binding energies and the mass of nucleus, the masses of constituents can be inferred. (Even in nucl. Phys. it is not easy to obtain a great accuracy, because of $\begin{cases} \text{1- incomplete understanding of nuclear int.} \\ \text{2- difficulties of many-body Prob.} \end{cases}$

For quarks, the situation is further complicated by several factors:

1- We are concerned with high energies, and for example π and ρ-mesons with the same quark contents have

$$\underset{\substack{\downarrow \\ S=0}}{M_\pi c^2 \simeq 140\,\text{MeV}}\,, \qquad \underset{\substack{\downarrow \\ S=1}}{m_\rho c^2 = 767\,\text{MeV}}$$

The large difference must be mainly attributed to the spin-dep. of quark-quark int.

This is quite different from the usual situation in quantum phys.;

when the interesting phys. $\xrightarrow{\text{arises}}$ from small parts of the complete int. (using perturbation methods)

For quarks:

Q–Q int. is weak only at high energies (asymptotic freedom).

⟶ Perturbative techniques in QCD apply only at extremely high energies.

Q–Q int. is very strong in low lying energies

⟶ Perturbative methods can not be used. New methods must be found.

2– Question of confinement?

Since quarks are not fount in isolation ⟶

their int. must have a component that grows stronger as the distance of separation between them increases.

This is in contradiction to the usual lawas such as gravitational and electromagnetic ints, that grow weaker as the distance of separation increases.

Remedy:

1- One way is to impose confinement as a bondary cond. (bag model)

2- Lattice gage calculation and soliton models.

Chapter 3

Nuclear Force and Two-Nucleon Systems

3-1 The Deuteron

Binding Energy: — The deuteron is a very unique nucleus in many respects.

1- Only loosely bound nucleus
2- having binding energy much less than the average value between a pair of nucleons in all the other stable nuclei.

$$E_B = \{[\sum M_{p_i} + \sum M_{n_i}] - M_{nucleus}\} c^2 \qquad (M_p \simeq M_H)$$

$$E_B = \{[938.783 + 939.566] - 1876.124\} = 2.225 \text{ Mev}$$

$E_B = 2.22457312(22)$ Mev more precise

This value is obtained, using the radiative capture of a neutron by hydrogen.

$$H + n \longrightarrow \gamma + d \qquad H(n,\gamma)d$$

If $E_n \simeq 0 \longrightarrow E_\gamma \simeq E_B$

Partly because of small E_B, the deutron has no excited state.

Ground state Property		Value
Binding energy	E_B	2.22457312(22) Mev
Spin & Parity	J^π	1^+
Isospin	T	0
Magnetic dipole moment	μ_d	0.857406(1) μ_N
Electric quadrupole "	Q_d	0.28590(30) $e\,fm^2$
Radius	r_d	1.963(4) fm