Chapter 3

Interpolation and Polynomial Approx.:

Def.: Fitting a func. to a given data is called interpolation, (the func. satisfies exactly at all given data).

Most useful funcs. mapping;

$$(\text{set of Real numbers}) \xrightarrow{\text{into}} (\text{inself})$$

are algebraic Polynomials;

$$P_n(x) = a_n x^n + a_{n-1} x^{n-1} + \dots a_1 x + a_0$$

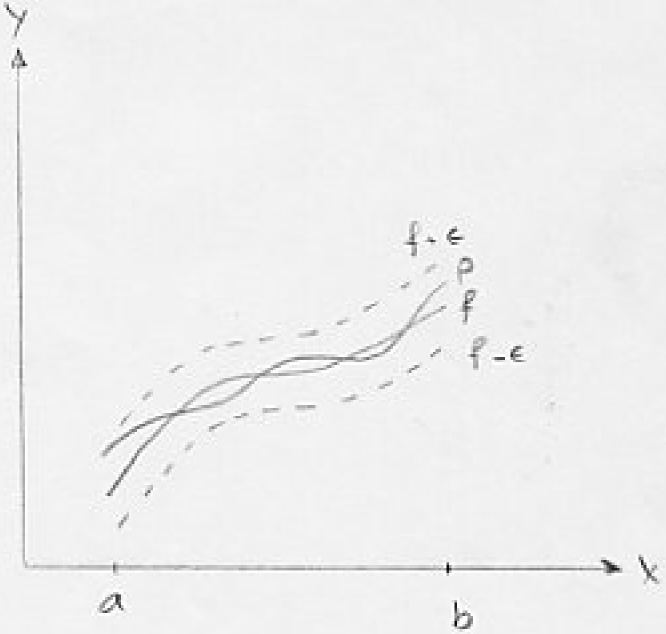
Their derivatives and integrals can be taken easily.

Theo. 3.1 (Weierstrass Approx. Theo.)

If f is defined and continuous on $[a,b]$ and $\epsilon > 0$ is given $\xrightarrow{\text{then}}$ $\exists$ $P(x)$ defined on $[a,b]$ with the property

$$|f(x) - P(x)| < \epsilon$$

$$\forall \; x \in [a,b]$$



(P(x): Polynomial)

3.1 The Taylor polynomials:

We consider the prob. of finding a polynomial of a specific deg. that is close to a given func. at a point x_0, (not all points).

P agrees with f at x_0 precisely when $P(x_0) = f(x_0)$

P has the same direction as f $(x_0, f(x_0))$ if $P'(x_0) = f'(x_0)$

P_n of deg. n best approximates f near x_0 if;

$$P''(x_0) = f''(x_0), \quad P'''(x_0) = f'''(x_0) \quad \ldots$$

This is precisely the cond. satisfied by nth Taylor polynomial for f at x_0

$$P_n(x) = f(x_0) + f'(x_0)(x-x_0) + f''(x_0)\frac{(x-x_0)^2}{2!} + \cdots + f^{(n)}(x_0)\frac{(x-x_0)^n}{n!}$$

$$f(x) - P_n(x) = R_n(x) = \frac{f^{(n+1)}(\xi(x))}{(n+1)!}(x-x_0)^{n+1}$$

Ex.

a) Calculate the third Taylor polynomial about $x_0 = 0$ for $f(x) = (1+x)^{\frac{1}{2}}$

b) Use the polynomial in part (a) to approximate $\sqrt{1.1}$ and find a bound for the error envolved.

c) Use the polynomial in part (a) to approximate $\int_0^{0.1} (1+x)^{\frac{1}{2}}\,dx$ and find a bound for the error of this approx.

$f(x) = (1+x)^{1/2} \qquad f(0) = 1$

$f'(x) = \frac{1}{2}(1+x)^{-1/2} \qquad f'(0) = \frac{1}{2}$

$f''(x) = -\frac{1}{4}(1+x)^{-3/2} \qquad f''(0) = -\frac{1}{4}$

$f'''(x) = \frac{3}{8}(1+x)^{-5/2} \qquad f'''(0) = \frac{3}{8}$

$f^{(iv)}(x) = -\frac{15}{16}(1+x)^{-7/2} \qquad f^{(iv)}(\xi) = -\frac{15}{16}(1+\xi)^{-7/2}$ $\qquad \xi$: between $x_0 = 0$ and x

a) $P_3(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 = 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3$

b) $\sqrt{1.1} = (1+0.1)^{1/2} = f(0.1) = P_3(0.1) = 1 + \frac{1}{2}(0.1) - \frac{1}{8}(0.1)^2 + \frac{1}{16}(0.1)^3 = 1.0488125$

$$|R_3(0.1)| = \frac{\left|-\frac{15}{16}(1+\xi)^{-7/2}\right|}{4!}(0.1)^4 \leqslant \frac{15}{(16)(24)}(0.1)^4 \max_{\xi \in [0, 0.1]} (1+\xi)^{-7/2}$$

$$= \frac{0.0005}{128}(1) \leqslant 3.91 \times 10^{-6}$$

True value is 1.0488088 $\longrightarrow$ the actual error $\approx 3.7 \times 10^{-6}$

c) $$\int_0^{0.1} (1+x)^{1/2}\,dx \approx \int_0^{0.1} P_3(x)\,dx = \int_0^{0.1} \left(1 + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{16}\right)dx$$

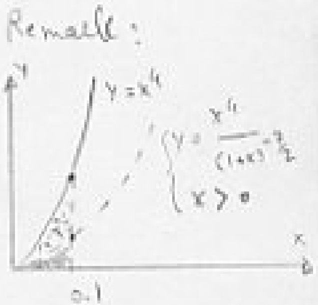
$$= \left[x + \frac{x^2}{4} - \frac{x^3}{24} + \frac{x^4}{64}\right]_0^{0.1} = 0.1024598958$$

$$\left|\int_0^{0.1} R_3(x)\,dx\right| = \frac{15}{(16)4!}\int_0^{0.1} (1+\xi)^{-7/2} x^4\,dx$$

$$\leqslant \frac{5}{128}(1)\int_0^{0.1} x^4\,dx = \frac{5}{128} \cdot \left[\frac{x^5}{5}\right]_0^{0.1} \leqslant 7.82 \times 10^{-8}$$

Since $\int_0^{0.1} R_3(x)\,dx$ = negative

Remark:



$$\int_0^{0.1} \frac{x^4\,dx}{(1+x)^{7/2}} < \int_0^{0.1} x^4\,dx$$

for $x > 0$

$$\rightarrow \int_0^{0.1} P_3(x)\,dx - \left|\int_0^{0.1} R_3(x)\,dx\right| \leq \int_0^{0.1} (1+x)^{\frac{1}{2}}\,dx \leq \int_0^{0.1} P_3(x)\,dx$$

$$0.102459817 6 \leq \int_0^{0.1} (1+x)^{\frac{1}{2}} \leq 0.1024598958$$

The actual value of $\int_0^{0.1} (1+x)^{\frac{1}{2}}\,dx = 0.10245 9822$

$\rightarrow$ true error $= 7.4 \times 10^{-8}$

Ex.

$f(x) = (1+x)^{\frac{1}{2}}$ and $P_3(x)$, $x_0 = 0$

x	0.1	0.5	1	2	10
$P_3(x)$	1.048813	1.2266	1.438	2.00	56.00
$f(x)$	1.048809	1.2247	1.414	1.73	3.32
$\lvert P_3(x) - f(x)\rvert$	0.000004	0.0019	0.024	0.27	52.68

$x_0 = 0 \xrightarrow{\text{close}} 0.1$ large departure

$x_0 = 0 \ll 10$

Ex. (Extreme example)

$f(x) = \frac{1}{x}$ $x_0 = 1$ to approximate $f(3) = \frac{1}{3}$

Sol.

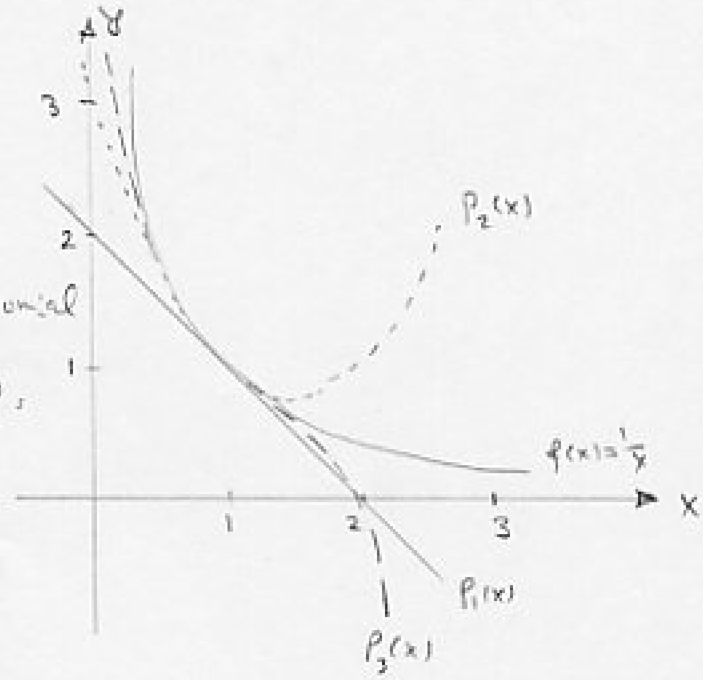
$f^{(n)}(x) = (-1)^n\, n!\, x^{-n-1}$ $\quad P_n(x) = \sum_{k=0}^{n} \frac{f^{(k)}(1)}{k!}(x-1)^k = \sum_{k=0}^{n} (-1)^k (x-1)^k$

n	0	1	2	3	4	5	6	7
$P_n(3)$	1	-1	3	-5	11	-21	43	-81

$f(3) = 0.3333\bar{3}$

$$R_n(x) = \frac{(-1)^{n+1}(x-1)^{n+1}}{\xi^{n+2}}$$

ξ: between 1 and x



Since information in Taylor polynomial is concentrated at a single point x_0, this kind of difficulties arise in this method.

This method is useful when the approximation is needed at points very close to x_0.

3.2 Interpolation and the Lagrange polynomial;

Determination of polynomial of deg. $n=1$ that passes through (x_0, y_0) and (x_1, y_1):

$$\longrightarrow \begin{cases} y_0 = f(x_0) \\ y_1 = f(x_1) \end{cases}$$

(approximating func. f) which satisfies these conds.

Consider linear polynomial:

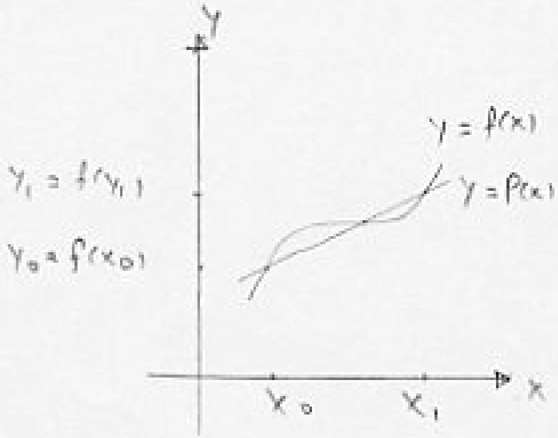
$$P(x) = \frac{(x-x_1)}{(x_0-x_1)} y_0 + \frac{(x-x_0)}{(x_1-x_0)} y_1$$

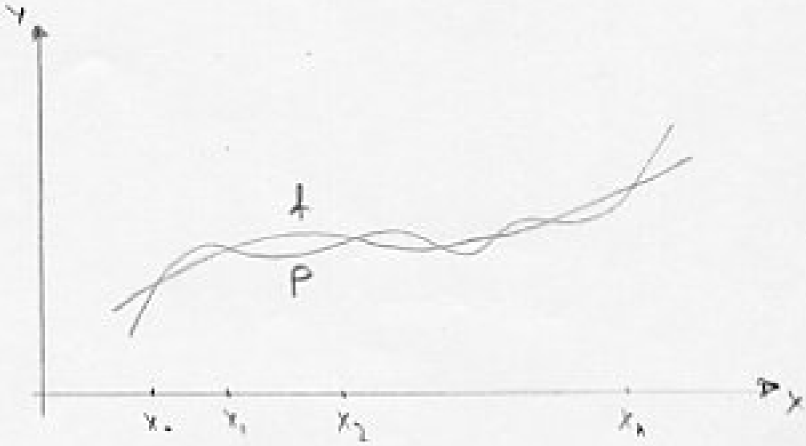
$x = x_0 \rightarrow P(x_0) = 1 \cdot y_0 + 0 \cdot y_1 = y_0 = f(x_0)$

$x = x_1 \rightarrow P(x_1) = 0 \cdot y_0 + 1\ y_1 = y_1 = f(y_1)$

Generalization:

Construction of a polynomial of deg. at most n that passes through $n+1$ points, $(x_0, f(x_0))$, $(x_1, f(x_1)) \dots (x_n, f(x_n))$ (Fig.)





The linear polynomial passing through;

$$\begin{cases}(x_0, f(x_0)) \\ (x_1, f(x_1))\end{cases}$$

is constructed by using the quotients;

$$L_0(x) = \frac{(x-x_1)}{(x_0-x_1)} \quad \text{and} \quad L_1(x) = \frac{(x-x_0)}{(x_1-x_0)}$$

$$x = x_0 \longrightarrow L_0(x_0) = 1 \quad \text{and} \quad L_1(x_0) = 0$$

$$x = x_1 \longrightarrow L_1(x_0) = 0 \quad \text{"} \quad L_1(x_1) = 1$$

For the general case; we have to construct $L_{n,k}(x)$ with the property;

$$L_{n,k}(x_i) = 0 \quad i \neq k$$

$$L_{n,k}(x_k) = 1$$

$$L_{n,k}(x_i) = \frac{A}{B}$$

To satisfy $L_{n,k}(x_i) = 0$ for $i \neq k$

A must contain: $(x-x_0)(x-x_1)\cdots(x-x_{k-1})(x-x_{k+1})\cdots(x-x_n)$

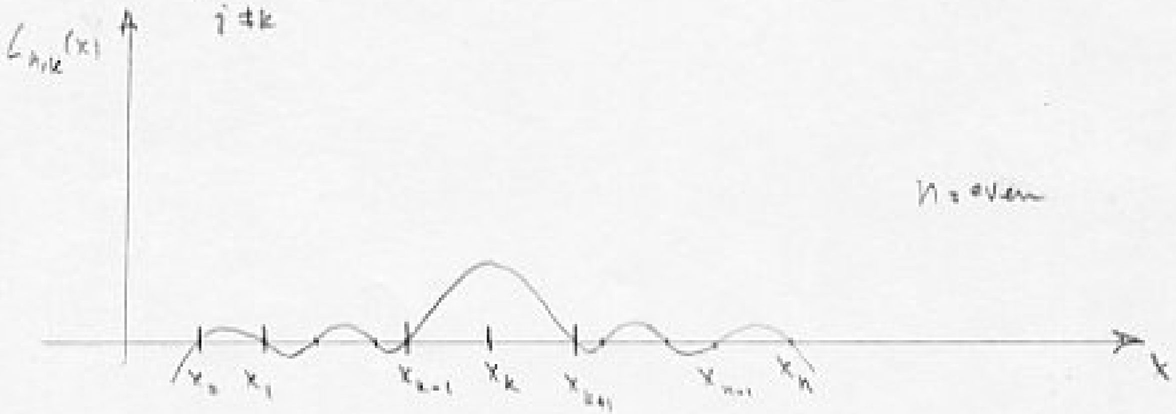
To satisfy $L_{n,k}(x_k) = 1$

B must be equal " " " " "

when $x = x_k$

Thus $\longrightarrow$ $$L_{n,k}(x) = \frac{(x-x_0)\cdots(x-x_{k-1})(x-x_{k+1})\cdots(x-x_n)}{(x_k-x_0)\cdots(x_k-x_{k-1})(x_k-x_{k+1})\cdots(x_k-x_n)}$$

$$L_{n,k}(x) = \prod_{\substack{i=0 \\ i\neq k}}^{n} \frac{(x-x_i)}{(x_k-x_i)}$$



Theo. 3.2

If $x_0, x_1, \ldots x_n$ are $(n+1)$ distinct numbers and f is a func. whose values are given at these numbers, then there exists a unique polynomial P of deg. at most n with the property that

$$f(x_k) = P(x_k) \quad \text{for each } k = 0, 1, \ldots n$$

This polynomial is given by

$$P(x) = f(x_0)\,L_{n,0}(x) + \cdots + f(x_n)\,L_{n,n}(x)$$

$$= \sum_{k=0}^{n} f(x_k)\,L_{n,k}(x)$$

where $L_{n,k}(x) = \prod_{\substack{i=0 \\ i \neq k}}^{n} \frac{(x - x_i)}{(x_k - x_i)}$ $\qquad k = 0, 1, \ldots n$

Ex.

Using the numbers, or nodes $x_0 = 2$, $x_1 = 2.5$, $x_2 = 4$, find the second interpolating polynomial $f(x) = \frac{1}{x}$.

Sol.

$$L_0(x) = \frac{(x-2.5)(x-4)}{(2-2.5)(2-4)} = (x - 6.5)x + 10$$

$$L_1(x) = \frac{(x-2)(x-4)}{(2.5-2)(2.5-4)} = \frac{(-4x+24)x - 32}{3}$$

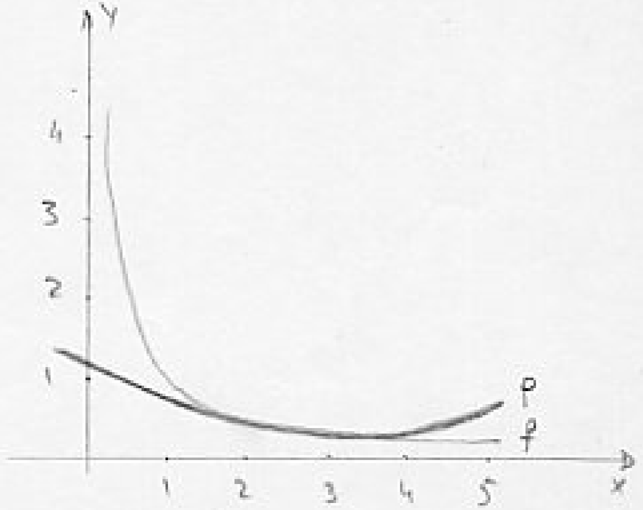
$$L_2(x) = \frac{(x-2)(x-2.5)}{(4-2)(4-2.5)} = \frac{(x-4.5)x+5}{3}$$

$f(x_0) = f(2) = 0.5 \qquad f(x_1) = f(2.5) = 0.4 \qquad f(x_2) = f(4) = 0.25$

$$P(x) = \sum_{k=0}^{2} f(x_k)\,L_k(x) = 0.5\left\{(x-6.5)x + 10\right\} + 0.4\left\{\frac{(-4x+24)x - 32}{3}\right\}$$

$$+ 0.25\left\{\frac{(x-4.5)x+5}{3}\right\} = (0.05x - 0.425)x + 1.15$$

An approx. to $f(3) \approx P(3) = 0.325$



Theo. 3.3

If $x_0, x_1, \ldots x_n$ are distinct numbers in the interval $[a,b]$ and $f \in C^{n+1}[a,b]$ $\xrightarrow{\text{then}}$ $\exists$ a number $\xi(x)$ in (a,b), $\forall x \in [a,b]$ with

$$f(x) = P(x) + \frac{f^{(n+1)}(\xi(x))}{(n+1)!} \prod_{i=0}^{n} (x - x_i)$$

where P is the interpolating polynomial

$$P(x) = \sum_{k=0}^{n} f(x_k) L_{n,k}(x)$$

Compare the error formula in this theorem with that one in Taylor polynomial:

$$\begin{cases} \dfrac{f^{(n+1)}(\xi(x))}{(n+1)!} (x - x_0)^{n+1} & \text{for Taylor polynomial} \\[2ex] \dfrac{f^{(n+1)}(\xi(x))}{(n+1)!} \displaystyle\prod_{i=0}^{n} (x - x_i) & \text{" Lagrange "} \end{cases}$$

The Taylor polynomial of deg. n about x_0 concentrates all the known information at x_0

The Lagrange polynomial of deg. n uses information at the distinct numbers $x_0, x_1, \ldots x_n$.

Ex.

Assume a table is to be prepared for the func. $f(x) = e^x$, $0 \leq x \leq 1$ (with the step size h) and the number of decimal places d.

a) If $d \geq 6$, what should h be for linear interpolation (Lagrange polynomial of deg. 1) to give an absolute error at most 10^{-6}?

b) How does the situation in part (a) change if $d < 6$?

Sol.

let $x \in [0,1]$ and $x_j \leq x \leq x_{j+1}$ $\quad \begin{pmatrix} x_j = 0 + jh \\ x_{j+1} = 0 + (j+1)h \end{pmatrix}$

Error in linear interpolation:

$$|f(x) - P(x)| = \left| \frac{f^{(2)}(\xi(x))}{2!} (x - x_j)(x - x_{j+1}) \right| = \frac{|f^{(2)}(\xi(x))|}{2} |(x-x_j)||(x-x_{j+1})|$$

$$// \quad = \frac{|f^{(2)}(\xi(x))|}{2!} |(x-jh)(x-(j+1)h)|$$

Hence

$$|f(x) - P(x)| \leq \frac{1}{2} \max_{\xi(x) \in [0,1]} |f^{(2)}(\xi(x))| \cdot \max_{x_j \leq x \leq x_{j+1}} |(x-jh)(x-(j+1)h)|$$

$$= \frac{1}{2} \max_{\xi \in [0,1]} e^{\xi} \cdot \max_{x_j \leq x \leq x_{j+1}} |(x-jh)(x-(j+1)h)|$$

$$\leq \frac{1}{2} e \cdot \max_{x_j \leq x \leq x_{j+1}} |(x-jh)(x-(j+1)h)|$$

Taking : $g(x) \equiv (x-jh)(x-(j+1)h)$

$$\text{Max } |g(x)| = |g((j+\tfrac{1}{2})h)| = \frac{h^2}{4} \qquad (g'(x) = 0 \to \text{max}) \quad \to x = j+\tfrac{1}{2}$$

$\longrightarrow$ The error in linear interpolation is bounded by

$$|f(x) - P(x)| \leq \frac{e h^2}{8}$$

$$\frac{e h^2}{8} \leqslant 10^{-6} \longrightarrow h < 1.72 \times 10^{-3}$$

$\longrightarrow$ $h = 0.001$ is logical choice

b) If the tables are accurate <u>only</u> to the <u>fifth</u> decimal place, it is <u>impossible</u> to obtain accurate values to the <u>sixth</u> place interpolation.

<u>Ex</u>

The following table lists the values of the Bessel func. of the first kind of order zero at various points. Compare the approximation of $f(1.5)$ obtained by various Lagrange polynomials.

<u>Sol.</u>

x	$f(x)$
1.0	0.7651977
1.3	0.6200860
1.6	0.4554022
1.9	0.2818186
2.2	0.1103623

Since 1.5 is between 1.3 and 1.6, the <u>linear</u> polynomial will use $\begin{cases} x_0 = 1.3 \\ x_1 = 1.6 \end{cases}$

$$P_1(1.5) = \frac{(1.5-1.6)}{(1.3-1.6)}(0.6200860) + \frac{(1.5-1.3)}{(1.6-1.3)}(0.4554022) = 0.5102968$$

Two polynomials of deg. two could reasonably be used:

i) $\begin{cases} x_0 = 1.3 \\ x_1 = 1.6 \\ x_2 = 1.9 \end{cases}$

$$P_2(1.5) = \frac{(1.5-1.6)(1.5-1.9)}{(1.3-1.6)(1.3-1.9)}(0.6200860) +$$
$$+ \frac{(1.5-1.3)(1.5-1.9)}{(1.6-1.3)(1.6-1.9)}(0.4554022) +$$
$$+ \frac{(1.5-1.3)(1.5-1.6)}{(1.9-1.3)(1.9-1.6)}(0.2818186) = 0.5112857$$

ii) $\begin{cases} x_0 = 1.0 \\ x_1 = 1.3 \\ x_2 = 1.6 \end{cases} \longrightarrow \hat{P}_2(1.5) = 0.5124715$

In the third deg. case there are also two choices:

i) $\begin{cases} x_0 = 1.3 \\ x_1 = 1.6 \\ x_2 = 1.9 \\ x_3 = 2.2 \end{cases} \longrightarrow P_3(1.5) = 0.5118302$

ii) $\begin{cases} x_0 = 1.0 \\ x_1 = 1.3 \\ x_2 = 1.6 \\ x_3 = 1.9 \end{cases} \longrightarrow \hat{P}_3(1.5) = 0.5118127$

The forth deg. Lagrange polynomial:

$\begin{cases} x_0 = 1.0 \\ x_1 = 1.3 \\ x_2 = 1.6 \\ x_3 = 1.9 \\ x_4 = 2.2 \end{cases} \longrightarrow P_4(1.5) = 0.5118200$

$f(1.5) = 0.5118277$ actual value

$|P_1(1.5) - f(1.5)| \approx 1.53 \times 10^{-3}$

$|P_2(1.5) - f(1.5)| \approx 5.42 \times 10^{-4}$

$|\hat{P}_2(1.5) - f(1.5)| \approx 6.44 \times 10^{-4}$

$|P_3(1.5) - f(1.5)| \approx 2.5 \times 10^{-6}$ ← most accurate

$|\hat{P}_3(1.5) - f(1.5)| \approx 1.50 \times 10^{-5}$

$|P_4(1.5) - f(1.5)| \approx 7.7 \times 10^{-6}$

However with no knowledge of the actual value of $f(1.5)$, P_4 would be accepted as the best approx.

Note that the error (or remainder terms) can not be applied here, since no knowledge of the forth derivative of f is available.

Unfortunately, this is generally the case.

3.3 Iterated Interpolation:

The deg. of the polynomial needed for the desired accuracy, using the method in 3.2, is generally not known until computations are determined (Comparing the result of P_n with P_{n-1}). (Since error term is difficult to work with)

Another difficulty;

One can not get information for P_n from P_{n-1}, and each one must be calculated separately.

Def. 3.4

Let f be a func. defined at $x_0, x_1 \cdots x_n$ and suppose that $m_1, m_2 \cdots m_k$ are k distinct integers with $0 \le m_i \le n$ $\forall i$.

The Lagrange polynomial that agrees with f at the k points $x_{m_1}, x_{m_2} \cdots x_{m_k}$ is denoted $P_{m_1, m_2, \cdots m_k}$

Ex. If $x_0 = 1$, $x_1 = 2$, $x_2 = 3$, $x_3 = 4$, $x_4 = 6$ and $f(x) = x^3$ then $P_{1,2,4}$ is the polynomial that agrees with f at $x_1 = 2$, $x_2 = 3$, and $x_4 = 6$;

$$P_{1,2,4}(x) = \frac{(x-3)(x-6)}{(2-3)(2-6)}(8) + \frac{(x-2)(x-6)}{(3-2)(3-6)}(27) + \frac{(x-2)(x-3)}{(6-2)(6-3)}(216)$$

Theo. 3.5

Let f be defined at $x_0, x_1, \dots x_k$ and x_j, x_i be two distinct numbers in this set. If

$$P(x) = \frac{(x-x_j)\, P_{0,1,\dots,j-1,j+1,\dots,k}(x) - (x-x_i)\, P_{0,1,\dots,i-1,i+1,\dots,k}(x)}{(x_i - x_j)}$$

then P is the kth Lagrange polynomial that interpolates f at the $k+1$ points $x_0, x_1, \dots x_k$.

Ex.

We obtained the values of various Lagrange polynomials at $x = 1.5$ using the following table in the last example.

Now we calculate the approx. of $f(1.5)$ using Theo. 3.5.

x	$f(x)$
1.0	0.7651977
1.3	0.6200860
1.6	0.4554020
1.9	0.2818186
2.2	0.1103623

If $\begin{cases} x_0 = 1.0 \\ x_1 = 1.3 \\ x_2 = 1.6 \\ x_3 = 1.9 \\ x_4 = 2.2 \end{cases} \longrightarrow \begin{cases} f(1.0) = P_0 \\ f(1.3) = P_1 \\ f(1.6) = P_2 \\ f(1.9) = P_3 \\ f(2.2) = P_4 \end{cases}$ (deg. zero consts.)

$$P_{0,1}(1.5) = \frac{(1.5-1.0)P_1 - (1.5-1.3)P_0}{(1.3-1.0)} = \frac{0.5(0.6200860) - 0.2(0.7651977)}{0.3}$$

$= 0.5233449$

$P_{1,2}(1.5) = 0.5102968$ $\quad P_{2,3}(1.5) = 0.5132634$ $\quad P_{3,4}(1.5) = 0.5104270$

These are first deg. polynomials.

$P_{1,2}$ is expected to the best approximation, since $x_1 = 1.3 < 1.5 < x_2 = 1.6$

The approx. using second deg.:

$$P_{0,1,2}(1.5) = \frac{(1.5-1.0)\,P_{1,2} - (1.5-1.6)\,P_{01}}{(1.6-1.0)} = 0.5124715$$

$P_{1,2,3}(1.5) = 0.5112857 \qquad P_{2,3,4}(1.5) = 0.5137361$

The higher deg. approx. are generated in a similar manner:

x_0	P_0				
x_1	P_1	$P_{0,1}$			
x_2	P_2	$P_{1,2}$	$P_{0,1,2}$		
x_3	P_3	$P_{2,3}$	$P_{1,2,3}$	$P_{0,1,2,3}$	
x_4	P_4	$P_{3,4}$	$P_{2,3,4}$	$P_{1,2,3,4}$	$P_{0,1,2,3,4}$
1.0	0.7651977				
1.3	0.6200860	0.5233449			
1.6	0.4554022	0.5102968	0.5124715		
1.9	0.2818186	0.5132634	0.5112857	0.5118127	
2.2	0.1103623	0.5104270	0.5137361	0.5118302	0.5118200

Suppose it is decided that the latest approximation $P_{0,1,2,3,4}$ is not accurate as desired. Another node x_5 can be selected, another row added to the table;

$$x_5;\ P_5,\ P_{4,5},\ P_{3,4,5},\ P_{2,3,4,5},\ P_{1,2,3,4,5},\ P_{0,1,2,3,4,5}$$

and $P_{0,1,2,3,4}$, $P_{1,2,3,4,5}$ and $P_{0,1,2,3,4,5}$ can be compared to determine further accuracy.

In this example, the value of the Bessel func. of the first kind of order zero at $x=2.5$ is -0.0483838, and the new row;

2.5 -0.0483838 0.4807699 0.5301486 0.5114070 0.5118630 0.5118277

The new entry is correct to six decimal places.

The procedure just outlined is called Neville's Method.

Let us use a practical notation;

$Q_{i,j}$ $i \geq j$: interpolating polynomial of deg. j on $(j+1)$ numbers $x_{i-j}, x_{i-j+1}, \dots x_{i-1}, x_i$

To compute; $Q_{i,j} = P_{i-j,\, i-j+1, \dots i-1,\, i}$

by Neville's Method, we use;

$$Q_{i,j-1} = P_{i-(j-1), \dots\, i-1,\, i} \qquad Q_{i-1,j-1} = P_{i-j,\, i-j+1, \dots,\, i-1}$$

$$Q_{i,j}(x) = \frac{(x-x_{i-j})\,Q_{i,j-1}(x) - (x-x_i)\,Q_{i-1,j-1}(x)}{x_i - x_{i-j}} \qquad \begin{cases} j = 1,2,3 \dots \\ i = j, j+1, \dots \\ Q_{i0} = f(x_i) \end{cases}$$

x_0	$Q_{0,0}$				
x_1	$Q_{1,0}$	$Q_{1,1}$			
x_2	$Q_{2,0}$	$Q_{2,1}$	$Q_{2,2}$		
x_3	$Q_{3,0}$	$Q_{3,1}$	$Q_{3,2}$	$Q_{3,3}$	
x_4	$Q_{4,0}$	$Q_{4,1}$	$Q_{4,2}$	$Q_{4,3}$	$Q_{4,4}$

Nevilles Iteration Interpolation Algorithm 3.1

Evaluation of the interpolating polynomial P on the $(n+1)$ distinct numbers $x_0, \dots x_n$ at the number x for the func. f.

Input; $x,\ x_0, \dots x_n\ ,\ f(x_0) \dots f(x_n)$

Output; the table Q with $P(x) = Q_{n,n}$

Step 1 Do $i = 1$ to n

Do $j = 1$ to i

$$Q_{i,j} = \frac{(x - x_{i-j})\, Q_{i,j-1} - (x - x_i)\, Q_{i-1,j-1}}{x_i - x_{i-j}}$$

S2 Output Q

stop.

Stopping creterion $|Q_{i,i} - Q_{i-1,i-1}| < \epsilon$ may be used