Dispersionless, highly superluminal propagation in a medium with a gain doublet

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In the region between two lines of a doublet of an inverted medium, there exists a point of zero group-velocity dispersion, where highly superluminal effects may be observable without significant gain, loss, distortion, or broadening. The results of this group-velocity analysis hold for sufficiently narrow-band, analytic pulses, and do not constitute a violation of causality, although the group, "signal," and energy velocities as defined by Sommerfeld and Brillouin may all exceed c or even become negative. No sharp disturbance in the pulse (a real signal) could propagate faster than light, but the scheme offers an unusual noiseless amplification scheme for the leading edge of a pulse, both at the classical and at the single-photon level.

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It is well known that in a Lorentz-model dielectric (and in real quantum-mechanical media) there exist spectral regions where the index of refraction is less than one, i.e., where the phase velocity exceeds c. It is slightly less well known that there are also regions where the group velocity exceeds c. In fact, it has recently been shown [1] that *any* causal system must possess such an anomalous group velocity at at least one frequency. In their classic papers on wave propagation, Sommerfeld and Brillouin showed [2] that despite these effects, no real signal can be transmitted faster than the vacuum velocity of light, and Einstein causality is not violated. This is frequently taken to mean that group velocity "is just not a useful concept" close to resonances, where these effects typically occur [3]. It has since been shown that superluminal (or negative) group velocities can indeed have physical meaning, accurately describing the propagation of the peak of an analytic pulse [4–6]. Such pulses better represent the signals used in typical optical systems than do the step-function envelopes of Sommerfeld and Brillouin's analysis. All the information about the shape of an analytic pulse is contained in any finite interval along, for example, its leading edge; for this reason, such propagation effects do not violate the relativistic conception of causality. Furthermore, in all the examples studied to date, the anomalous group delays occurred in media with very low transmission, either due to an optical absorption band [5] or to a tunnel barrier [6]. The transmitted pulse in all cases was sufficiently small that it fit "beneath" the leading edge of the incident pulse, extrapolated forward at the vacuum velocity of light. The process is typically described as one of "reshaping," in which the later portions of the incident pulse are preferentially absorbed (or reflected), shifting the peak of the transmitted pulse forward in time, the local velocity of energy propagation never exceeding c [7].

Recently, one of us has pointed out [8] that when the Lorentz model is modified to describe an inverted atomic system, i.e., one in which there is a gain band rather than an absorption band, the anomalously small delays occur not *within* the band, but rather *without* it, in an essentially transparent spectral region; for this reason, the transmitted pulse experiences negligible gain or loss, and the velocity of energy propagation may exceed c or even become negative, both of which cases we shall term "superluminal." (We follow the definition of electromagnetic energy velocity used by Sommerfeld and Brillouin, i.e., the ratio of the Poynting vector to the energy density, which neglects the energy flow within the atomic system.) It was shown in that paper in the dc limit (frequency much less than the resonance frequency), the group velocity is superluminal, and in a suitably chosen system, any pulse with sufficiently narrow bandwidth would undergo negligible gain or distortion. The effect is a type of coherent transient, described by a Feynman diagram in which a virtual decay of the excited atom produces a photon *before* the absorption of the incident photon; as the effect occurs off-resonance, however, the absorption is a necessary step in this diagram, ensuring energy conservation. Put another way, since the effect occurs far from resonance, little noise is contributed by spontaneous emission, and this process amounts to a virtually noiseless amplification scheme for the leading edge of a pulse, at the expense of its trailing edge; such an amplifier is faithful even at the single-photon level, and could be used to compensate for propagation delays in other optical elements of a system. In practice, however, such superluminal effects will be very small far below resonance. On the other hand, closer to resonance where the superluminality is more striking, the group-velocity dispersion (GVD) also becomes very large. In a real experiment, this would lead to a broadening and a distortion of the transmitted pulse, obscuring the effect. In this paper, we discuss the case of an inverted medium possessing a doublet line, such as the familiar doublets observed in alkali-metal atoms, split by the several-GHz ground-state hyperfine splitting. (Doublets also occur due to the isotope shift, with splittings of the same order of magnitude.) We show that there exists a point between the two gain lines where the lowest-order group-velocity dispersion vanishes, and demonstrate that if such an inversion could be maintained in an alkali vapor, it would be possible in principle

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to observe extremely superluminal propagation of a laser pulse tuned between the two gain lines, with negligible distortion or broadening. Such an experiment would in a sense be the complement of the anomalously *low* group velocities observed by Grishkowsky [9] and more recently by Harris, Field, and Kasapi [10]. In practice, the Doppler width of the gain lines will reintroduce some slight distortion; in principle, even this could be eliminated if a trap for laser-cooled atoms could maintain a high atomic density and population inversion over the entire interaction region, or if a Doppler-free gain scheme were utilized.

The particular gain mechanism is not of fundamental importance, as the Kramers-Kronig relations lead to the same dispersive effects regardless of the origin of the gain. Specifically, the identity for the real part of the susceptibility $\chi(\omega)$ in terms of its imaginary part can be written as follows:

$$\mathrm{Re}\,\chi(\omega)=\frac{2}{\pi}\mathrm{P}\int_0^\infty \frac{\omega'\mathrm{Im}\chi(\omega')}{\omega'^2-\omega^2}d\omega' . \tag{1}$$

The imaginary part of χ represents gain or loss, and if its support is restricted to one or several narrow-band regions so that it can be represented as a finite sum of delta functions, Eq. (1) leads directly to a real susceptibility with the same form as that of the undamped Lorentz model [see Eqs. (2) and (4) below]. We are currently investigating a more realistic approach for generating a gain doublet, making use of the large gain that has been observed in the stimulated Raman effect [11–13], in a Doppler-free configuration. This would obviate the need for cooling and trapping, while at the same time reducing the gain linewidth to nearly zero. The inversion in this case occurs not among the electronic states but among the hyperfine sublevels of the ground state, and can be easily achieved using optical pumping. The doublet could be produced either by using a medium with at least three ground-state sublevels, or by using a pair of pump beams detuned from one another by several gigahertz. The latter possibility, while offering less gain and thus a smaller effect than a true population inversion, also offers the possibility of tuning a great number of parameters such as the separation and the relative strength of the two gain lines. (Another possibility would be to use one of the "gain without inversion" schemes, which have been a topic of much recent attention [14].) In this paper, we focus on the possibility *in principle* of observing dispersionless propagation at highly superluminal effective velocities; for simplicity, we will use the Lorentz model as it applies to an alkali vapor. The only modification necessary for other gain schemes is the introduction of an appropriate effective oscillator strength. In the resonantly enhanced Raman scheme, for example, this oscillator strength would be approximately the square of the intrinsic atomic oscillator strength, multiplied by the ratio of the pump Rabi frequency to the pump detuning.

As discussed in [8], the extension of the Lorentz model to an inverted two-level system is simple: the oscillator strength f is replaced by $-f$ [this follows trivially from (1)]. For a relative inversion $\eta\equiv(n_e-n_g)/(n_e+n_g)$, the complex index of refraction $n(\omega)=[1+4\pi\chi(\omega)]^{1/2}$ is more generally given by

$$n(\omega)=\left[1-\eta\frac{f\omega_p^2}{\omega_0^2-\omega^2-i\gamma\omega}\right]^{1/2} , \tag{2}$$

where f is the oscillator strength defined in terms of the dipole μ as $2m|\mu|^2\omega_0/(\hbar e^2)$, ω_0 is the resonance frequency, γ is the linewidth, and ω_p is the plasma frequency defined by

$$\omega_p=\left[\frac{4\pi Ne^2}{m}\right]^{1/2} , \tag{3}$$

with N the number density of contributing (valence) electrons, and e and m the charge and mass of the electron. We will consider the limit of negligible linewidth γ. Typical strong transitions have natural linewidths on the order of several megahertz, much smaller than the detunings we will be considering in this paper. Even Doppler widths are generally an order of magnitude smaller than the ground-state hyperfine splittings in high-Z alkali vapors. An alkali-metal atom maintained in its first excited state can decay into either of its ground-state sublevels, with the near-unity absorption oscillator strength partitioned approximately equally (to within Clebsch-Gordon coefficients) between these two decays. Writing subscripts 1 and 2 for these two decay branches, we have for the index of refraction

$$n(\omega)=\left[1-\eta_1\frac{f_1\omega_p^2}{\omega_1^2-\omega^2}-\eta_2\frac{f_2\omega_p^2}{\omega_2^2-\omega^2}\right]^{1/2} . \tag{4}$$

This index is typically very close to one in vapors, except when ω differs negligibly from one of the resonant frequencies. (Thus in spite of the extreme change in velocity upon entering the inverted medium, the incident wave experiences essentially no Fresnel reflection and is entirely transmitted.) We therefore approximate n as

$$n(\omega)=1-\eta_1\frac{f_1}{2}\frac{\omega_p^2}{\omega_1^2-\omega^2}-\eta_2\frac{f_2}{2}\frac{\omega_p^2}{\omega_2^2-\omega^2} . \tag{5}$$

Let us now rewrite ω_2 and ω_1 as $\omega_0\pm\Omega/2$, where ω_0 is the central frequency and Ω is the hyperfine splitting. We also introduce the detuning Δ, defined as $\omega-\omega_0$. For small detunings, and $\Omega\ll\omega_0$, we expand the index of refraction to lowest order in Δ as follows:

$$n(\omega)=1+\eta_1\frac{f_1}{2}\frac{\omega_p^2}{2\omega_0(\Delta+\Omega/2)}+\eta_2\frac{f_2}{2}\frac{\omega_p^2}{2\omega_0(\Delta-\Omega/2)} . \tag{6}$$

The group velocity is defined as

$$v_g\equiv\frac{d\omega}{dk}=\frac{c}{n+\omega dn/d\omega} . \tag{7}$$

Although n is close to one, its slope may be quite large, leading to superluminal effects. The denominator, which we shall term n_{eff}, can be written

$$n_{\mathrm{eff}}(\omega)\approx 1-\eta_1\frac{f_1}{4}\frac{\omega_p^2}{(\Delta+\Omega/2)^2}-\eta_2\frac{f_2}{4}\frac{\omega_p^2}{(\Delta-\Omega/2)^2} . \tag{8}$$

Although this is less than unity everywhere for $\eta > 0$, it rapidly approaches one as the detuning increases. It diverges towards negative infinity as ω approaches resonance at either ω_1 or ω_2, but is finite everywhere between the two. It follows directly that its derivative must vanish somewhere between the two gain lines; for the simple case where $\eta_1 f_1 = \eta_2 f_2$, it vanishes at ω_0. (In fact, the two oscillator strengths will not be equal, due to the different Clebsch-Gordon coefficients for the two transitions, but we will not consider this complication here. It modifies the results we present only by numerical factors close to unity.) This derivative is the dominant source of group-velocity dispersion; the amount of broadening experienced by a pulse of bandwidth $\delta\omega$ propagating through a thickness L of dielectric is approximately

$$\Delta\tau \approx \frac{dn_{\text{eff}}}{d\omega}\frac{L}{c}\delta\omega \ . \tag{9}$$

By tuning a probe to the point of vanishing group-velocity dispersion, we can dispense with this term and hence with the dominant contribution to pulse-broadening, while still remaining within several gigahertz of the extremely strong gain region (oscillator strengths near unity, for the example, of an alkali vapor).

Let us examine this more closely. Setting $f_1 = f_2 = f$ and $\eta_1 = \eta_2 = \eta$ for simplicity, the first-order dispersion vanishes at $\Delta = 0$. We expand n_{eff} around this point to the lowest nonvanishing order,

$$n_{\text{eff}} = 1 - 2\eta f \left[\frac{\omega_p}{\Omega}\right]^2 \left\{1 + 12\left[\frac{\Delta}{\Omega}\right]^2 + O(\Delta^4)\right\} . \tag{10}$$

(Note that no approximation has been made that n_{eff} itself be close to unity.) For an easily achievable density of 10^{13} atoms/cm^3 in ^{87}Rb, the plasma frequency is over four times the 6.8-GHz ground-state hyperfine splitting. Thus even at its local maximum between the two gain lines, the effective index for a 100% inverted system would be of the order of -15, assuming $f \approx 0.5$ for each transition (see Fig. 1). This means that the propagation delay time for a pulse in such a medium would be on the order of $-15L/c$. For a 1-cm interaction region, this indicates that the peak of a pulse could leave the exit face of the vapor cell half of a nanosecond *earlier* than the peak of the incident pulse arrives at the entrance face. Such a shift should be readily observable with a nanosecond-pulsed laser, with a bandwidth on the order of 1 GHz, much smaller than the hyperfine splitting; the spectrum of such a laser would be nearly entirely outside of the gain band of either line, even when Doppler broadening (typically smaller than 1 GHz) is taken into account. As we shall show below, and as can be seen in Fig. 2, such a pulse also experiences very little dispersive broadening.

In the absence of gain or loss, distortion of a pulse is primarily due to group-velocity dispersion. When the first-order GVD vanishes, the main contribution comes from the first nonvanishing order,

$$\Delta\tau \approx \frac{L}{2c}\delta\omega^2 \left|\frac{d^2 n_{\text{eff}}}{d\omega^2}\right| = \frac{L}{2c}\delta\omega^2 \left\{48\eta f \frac{\omega_p^2}{\Omega^4}\right\} , \tag{11}$$

a straightforward consequence of (10). The most obvious criterion for an experiment to display a clearly superluminal effect is that this broadening be small compared

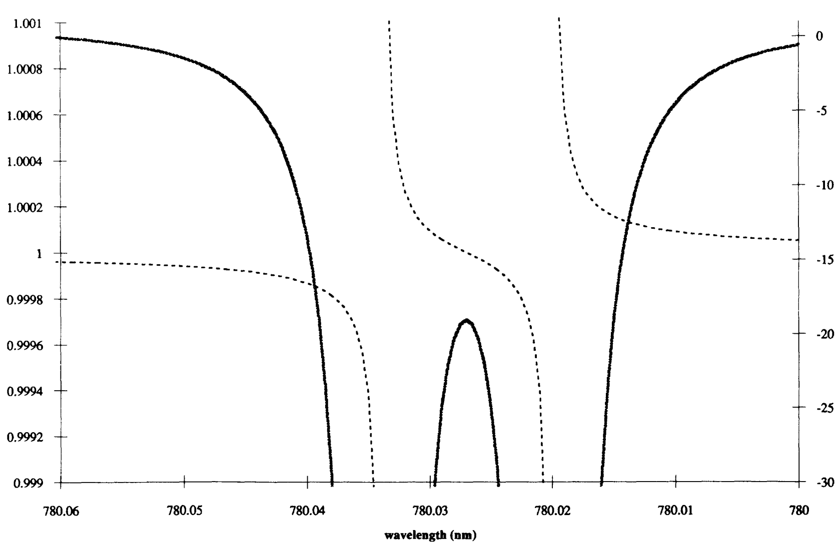

FIG. 1. Index (c/v_p) and effective index (c/v_g) for an inverted doublet split by 6.8 GHz, as in ^{87}Rb at a density of 10^{13} atoms/cm^3, with a combined oscillator strength of unity. Each horizontal division corresponds to 0.01 nm at a wavelength of 780 nm, or approximately 5 GHz. The index (dashed line, left axis) is close to 1 except at the two resonances, where it diverges. The effective index (solid line, right axis) also diverges at the resonances, but reaches a local maximum of approximately -20 halfway between them. A 1-GHz bandwidth pulse centered at this zero-group-velocity-dispersion point will travel at $-c/20$ with negligible distortion or change in amplitude, as shown in Fig. 2. (Incomplete inversion and less-than-unity oscillator strengths will in practice make these effects proportionately smaller.)

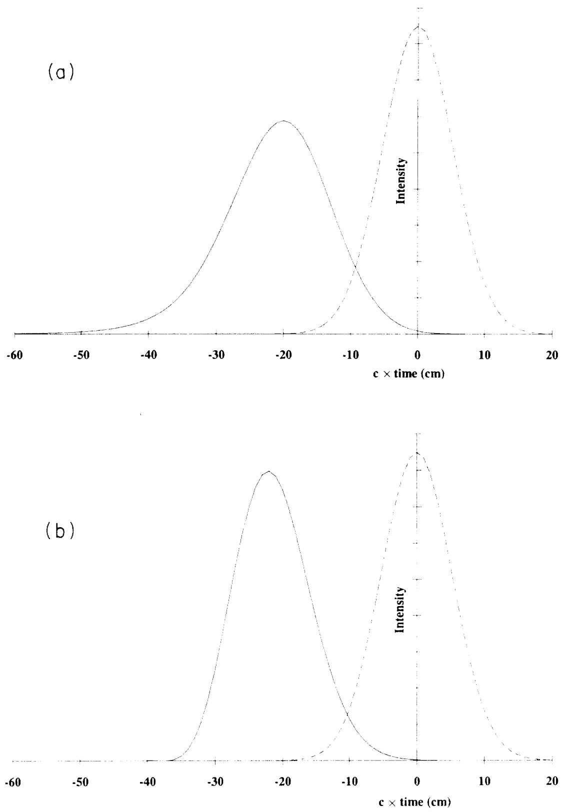

FIG. 2. 1-GHz bandwidth (full width at half maximum) Gaussian pulse before and after traversing 1 cm of an inverted medium. Note that the transmitted pulse (solid line) arrives at negative times with respect to the incident pulse (dashed line), centered at approximately −20 cm divided by c (i.e., −0.7 ns). The width of the incident pulse is about 12 cm or 0.4 ns. The curves are based on an undamped Lorentz model for the index of refraction of the inverted medium. In (a), the hyperfine splitting is suppressed and the result is for a pulse detuned by 3.4 GHz from a unit-oscillator-strength gain line; the effects of group-velocity dispersion on the transmitted pulse are evident. In (b), the pulse is tuned at the midpoint of a doublet split by 6.8 GHz, such as that of Fig. 1. Dispersion is essentially eliminated in the latter case.

to the shift itself, that is, to $|n_{\text{eff}}-1|L/c$. Since both the shift and the broadening are proportional to the sample length and to $\eta f \omega_p^2$, no restrictions are placed on these quantities by this requirement. The only necessary condition is that

$$\delta\omega^2 < \Omega^2/12 \ , \tag{12}$$

again following trivially from (10). This is a simple refinement of the natural stipulation that the bandwidth be smaller than the separation between the two resonances where the group delay diverges.

A more stringent condition arises when we introduce two more criteria. First of all, for the distortion of the pulse to be negligible, not only should $\Delta\tau$ be small compared to the group-delay shift, but it should also be small compared to the initial pulse width, of the order of $1/\delta\omega$. Secondly, for the shift to be easily resolvable, it should be at least of the same order as the initial pulse width. (These two conditions combine transitively to yield the first criterion discussed above.) The first of these two conditions can be written

$$\frac{24\eta f L}{c}\delta\omega^2\left[\frac{\omega_p^2}{\Omega^4}\right] < \frac{1}{\delta\omega} \tag{13}$$

or

$$L\,\delta\omega^3 < \frac{c\,\Omega^4}{24\eta f \omega_p^2} \ , \tag{14}$$

which offers an upper bound for the sample length and/or bandwidth, holding the parameters of the atomic system constant. The second condition gives

$$\frac{2\eta f L}{c}\left[\frac{\omega_p^2}{\Omega^2}\right] > \frac{1}{\delta\omega} \ , \tag{15}$$

i.e.,

$$L\,\delta\omega > \frac{c\,\Omega^2}{2\eta f \omega_p^2} \ . \tag{16}$$

This offers a *lower* bound on the length-bandwidth product; for a shorter sample or a narrower bandwidth (and hence longer) incident pulse, the superluminal effect is not resolved. Since the bandwidth appears in different powers in the two conditions (14) and (16), we can rewrite them as separate conditions on the bandwidth and on the sample length. For the bandwidth, we divide (14) by (16), and immediately regain the condition we already saw in (12); this reflects the syllogism mentioned earlier that joins our two new constraints to imply that the broadening be smaller than the shift, the requirement that yielded (12). To find a condition for the length L, on the other hand, we cube (16) and then divide by (14), obtaining the following lower bound on the sample length:

$$L > \sqrt{3}\frac{c\,\Omega}{\eta f \omega_p^2} \ . \tag{17}$$

For the example quoted earlier of a rubidium vapor at a density of 10^{13} atoms per cubic centimeter, this works out to a length of approximately $0.07/\eta f$ centimeters. For example, with an inversion of 20% (i.e., 60% of the atoms in the excited state) and $f \approx 0.5$, a 1-cm cell and a laser bandwidth on the order of 1 GHz will simultaneously satisfy the conditions of an easily resolvable superluminal effect and negligible pulse distortion.

In order for such an experiment to be successfully carried out, there remain several hurdles, not the least of which is the preparation of such an atomic inversion, or the design of some other system that would provide two strong, closely spaced gain lines with narrow linewidths. While the stimulated Raman effect is known to offer extremely high gain [11–13], it is still generally smaller than that of an inverted alkali-metal atom. There is, however, no reason in principle that such a gain doublet be unobtainable. That issue aside, there remains the fact that there will be spontaneous emission noise at the two gain lines (as well as stimulated emission if the tails of the probe spectrum overlap the gain lines at all), and this will have to be filtered out in order for the signal to be observable; care will also need to be taken to prevent lasing from occurring on these lines. The extremely high gain makes these systems reminiscent of those used in the study of superradiance [15,16], a fact that led a referee to point out that it might be impossible to sustain the necessary population inversion. However, superradiance involves atomic coherences that develop after an inversion is prepared at one initial time; this differs from the present case, where incoherent pumping would be used to maintain a steady-state inversion. Calculations by Bolda, Garrison, and Chiao suggest that under continuous pumping, an initial transient superradiant pulse may be followed by a steady-state inversion [17]; this question certainly calls for more careful examination before an experiment is attempted. Finally, for large effects to be observed, the tails of the input pulse need to be very clean, since this propagation process relies on a transient effect, that is, on the analytic tails of the pulse. For "dirty" probe signals, the transmitted wave packet may be greatly distorted.

In conclusion, it should be feasible to observe superluminal propagation of Gaussian laser pulses in a transparent medium, near but not within a region of high gain: by situating the probe frequency between a *pair* of gain lines, one can essentially eliminate any distortion or broadening that would otherwise arise from the large group-velocity dispersion. This proposal extends previous discussions of anomalous group velocities to a regime in which the transmitted pulse is unchanged in intensity and in form, and yet experiences extremely superluminal propagation. This underscores the fact that although Einstein causality rules out the propagation of any *signal* faster than the speed of light, it does not limit the velocity of electromagnetic energy propagation (in the sense of Sommerfeld and Brillouin) to *c*. Causality is saved not by the smallness of the transmitted pulse, but by the smooth nature of its long analytic tails. The amplification of the pulse's leading edge at the expense of its trailing edge should be faithful even at the single-photon level.

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