PHY2506
Atmospheric Data Assimilation

Lecture 11

Supplementary Slides

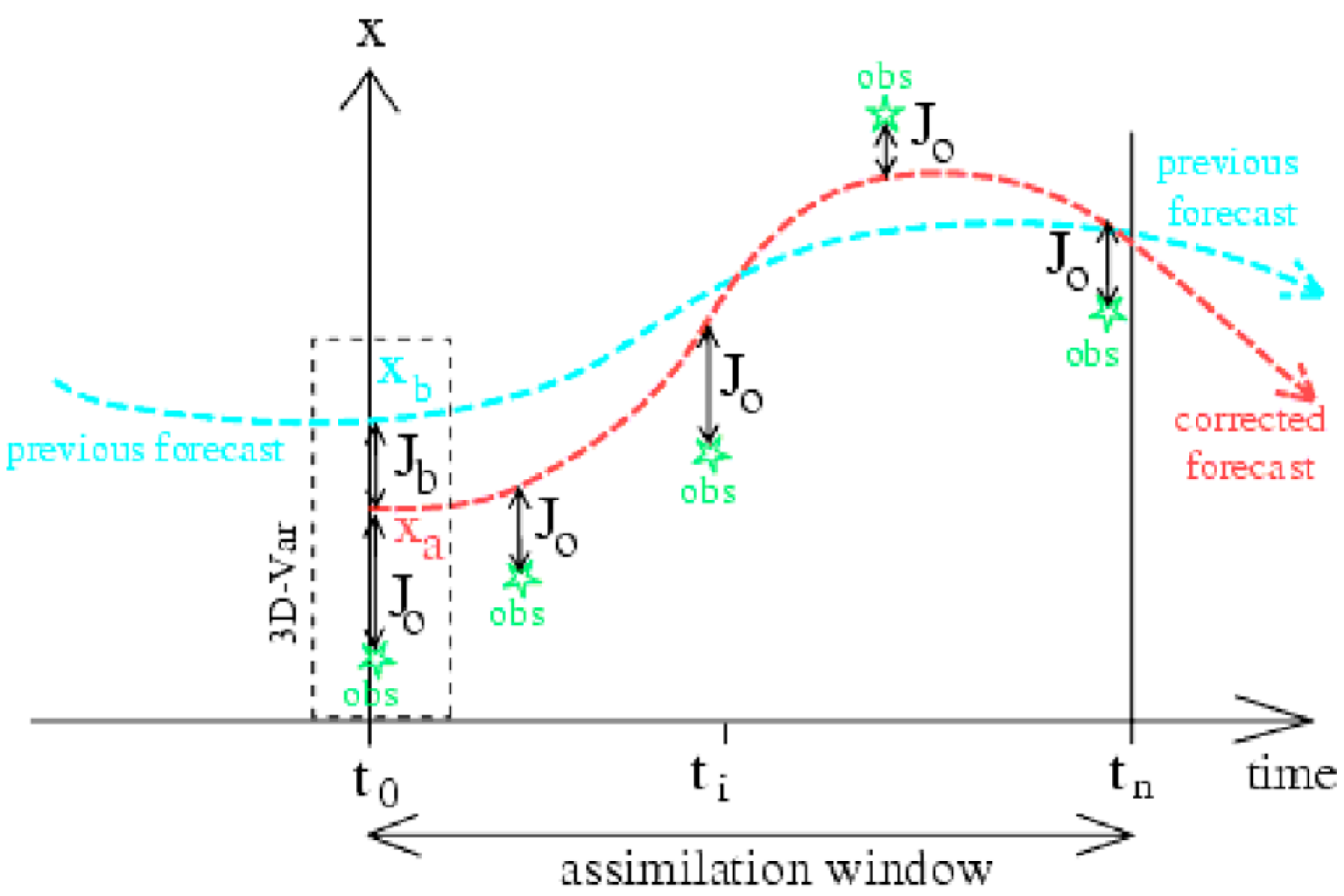


Figure 12. Example of 4D-Var intermittent assimilation in a numerical forecasting system. Every 6 hours a 4D-Var is performed to assimilate the most recent observations, using a segment of the previous forecast as background. This updates the initial model trajectory for the subsequent forecast.

[ECMWF Lecture Notes, 2003]

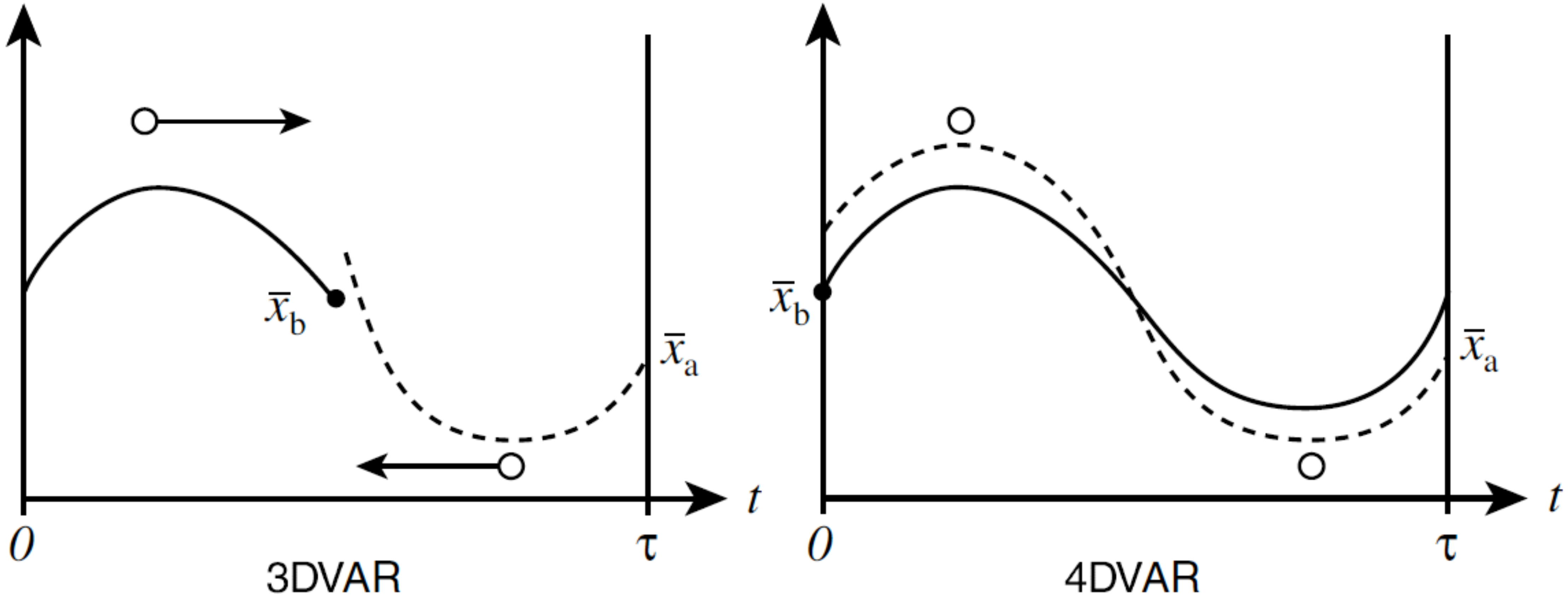


Figure 14: *The difference between 3DVAR and 4DVAR – 4DVAR is a time/model consistent interpolator of departures.*

[From ECMWF Lecture Notes by E. Holm, 2003]

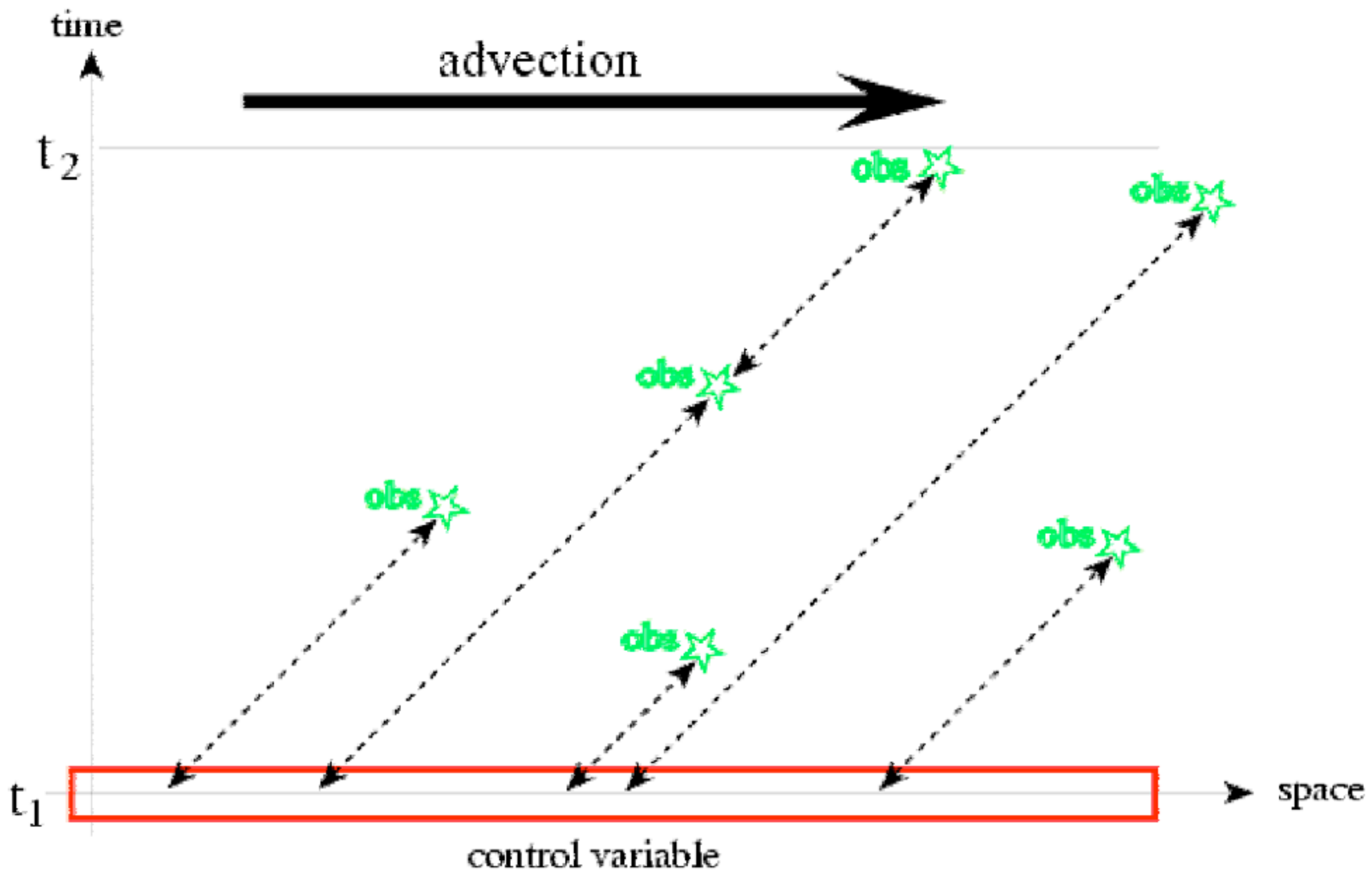


Figure 13. Example of propagation of the information by 4D-Var (or, equivalently, a Kalman filter) in a 1-D model with advection (i.e. transport) of a scalar quantity. All features observed at any point within the 4D-Var time window (t_1, t_2) will be related to the correct upstream point of the control variable by the tangent linear and adjoint model, along the characteristic lines of the flow (dashed).

[ECMWF Lecture Notes, 2003]

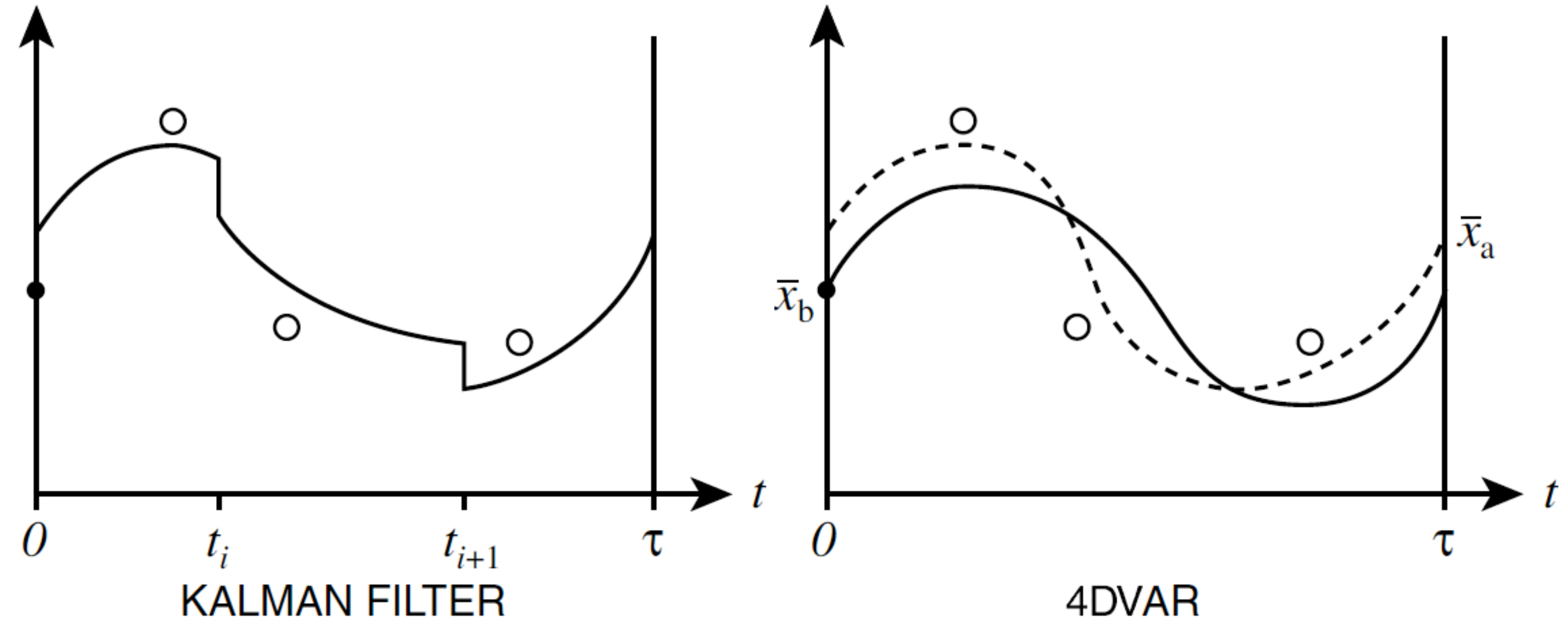
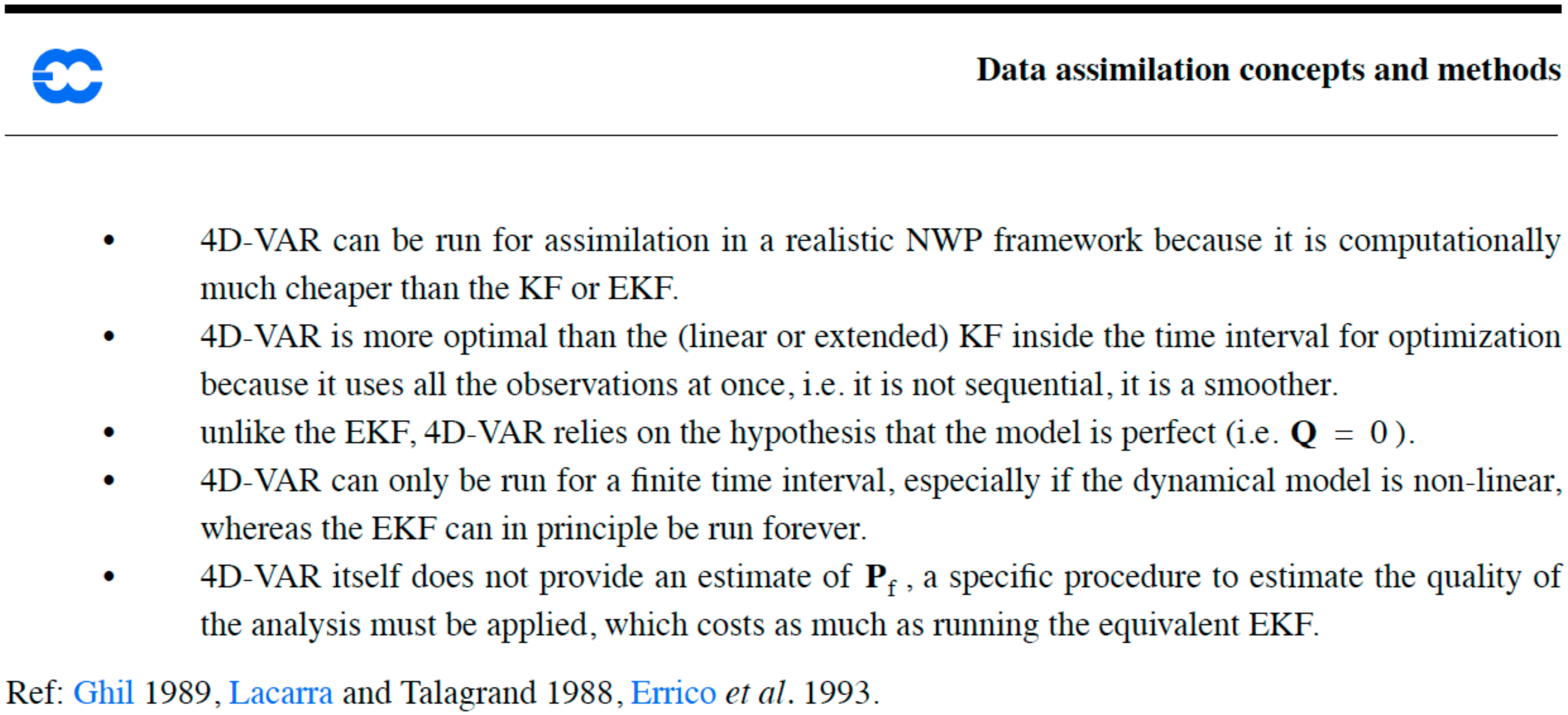


Figure 17: *The difference between the Kalman filter and 4DVAR. The Kalman filter performs an analysis at each model timestep. 4DVAR analyses all observations within a larger assimilation window simultaneously.*

[From ECMWF Lecture Notes by E. Holm, 2003]



- 4D-VAR can be run for assimilation in a realistic NWP framework because it is computationally much cheaper than the KF or EKF.
- 4D-VAR is more optimal than the (linear or extended) KF inside the time interval for optimization because it uses all the observations at once, i.e. it is not sequential, it is a smoother.
- unlike the EKF, 4D-VAR relies on the hypothesis that the model is perfect (i.e. $\mathbf{Q} = 0$).
- 4D-VAR can only be run for a finite time interval, especially if the dynamical model is non-linear, whereas the EKF can in principle be run forever.
- 4D-VAR itself does not provide an estimate of $\mathbf{P}_f$, a specific procedure to estimate the quality of the analysis must be applied, which costs as much as running the equivalent EKF.

Ref: Ghil 1989, Lacarra and Talagrand 1988, Errico *et al*. 1993.

[ECMWF Lecture Notes, 2003]

Q. J. R. Meteorol. Soc. (2000), **126**, pp. 1143–1170

The ECMWF operational implementation of four-dimensional variational assimilation. I: Experimental results with simplified physics

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(Received 17 November 1998; revised 25 May 1999)

SUMMARY

This paper presents results of a comparison between four-dimensional variational assimilation (4D-Var), using a 6-hour assimilation window and simplified physics during the minimization, and three-dimensional variational assimilation (3D-Var). Results have been obtained at 'operational' resolution T213L31/T63L31. (T defines the spectral triangular truncation and L the number of levels in the vertical, with the first parameters defining the resolution of the model trajectory, and the second the resolution of the inner-loop.) The sensitivity of the 4D-Var performance to different set-ups is investigated. In particular, the performance of 4D-Var in the Tropics revealed some sensitivity to the way the adiabatic nonlinear normal-mode initialization of the increments was performed. Going from four outer-loops to only one (as in 3D-Var), together with a change to the 1997 formulation of the background constraint and an initialization of only the small scales, helped to improve the 4D-Var performance. Tropical scores then became only marginally worse for 4D-Var than for 3D-Var. Twelve weeks of experimentation with the one outer-loop 4D-Var and the 1997 background formulation have been studied. The averaged scores show a small but consistent improvement in both hemispheres at all ranges. In the short range, each two- to three-week period has been found to be slightly positive throughout the troposphere. The better short-range performance of the 4D-Var system is also shown by the fits of the background fields to the data. More results are presented for the Atlantic Ocean area during FASTEX (the Fronts and Atlantic Storm-Track EXperiment), during which 4D-Var is found to perform better. In individual synoptic cases corresponding to interesting Intensive Observing Periods, 4D-Var has a clear advantage over 3D-Var during rapid cyclogeneses. The very short-range forecasts used as backgrounds are much closer to the data over the Atlantic for 4D-Var than for 3D-Var. The 4D-Var analyses also display more day-to-day variability. Some structure functions are illustrated in the 4D-Var case for a height observation inserted at the beginning, in the middle or at the end of the assimilation window. The dynamical processes seem to be relevant, even with a short 6-hour assimilation period, which explains the better overall performance of the 4D-Var system.

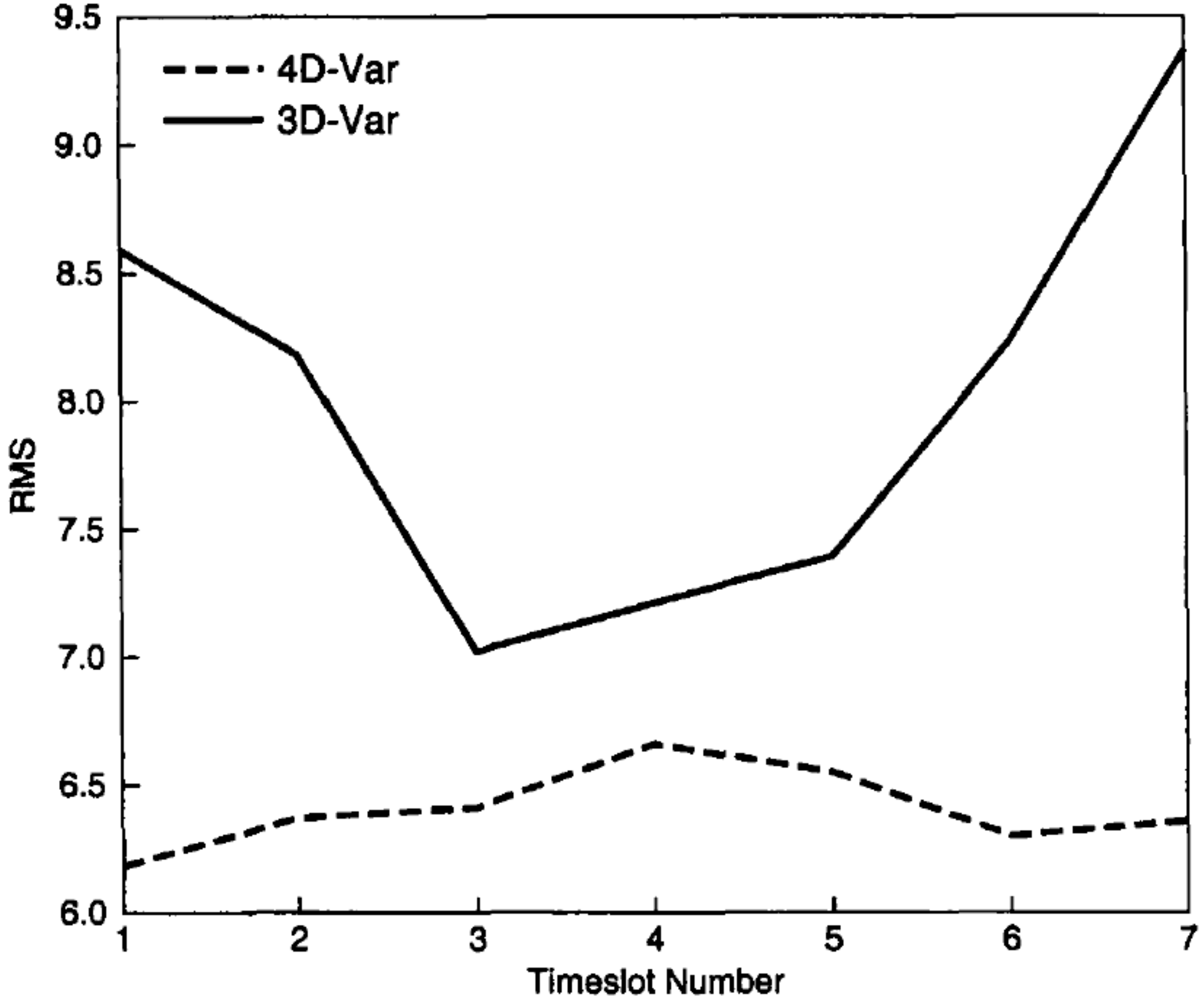
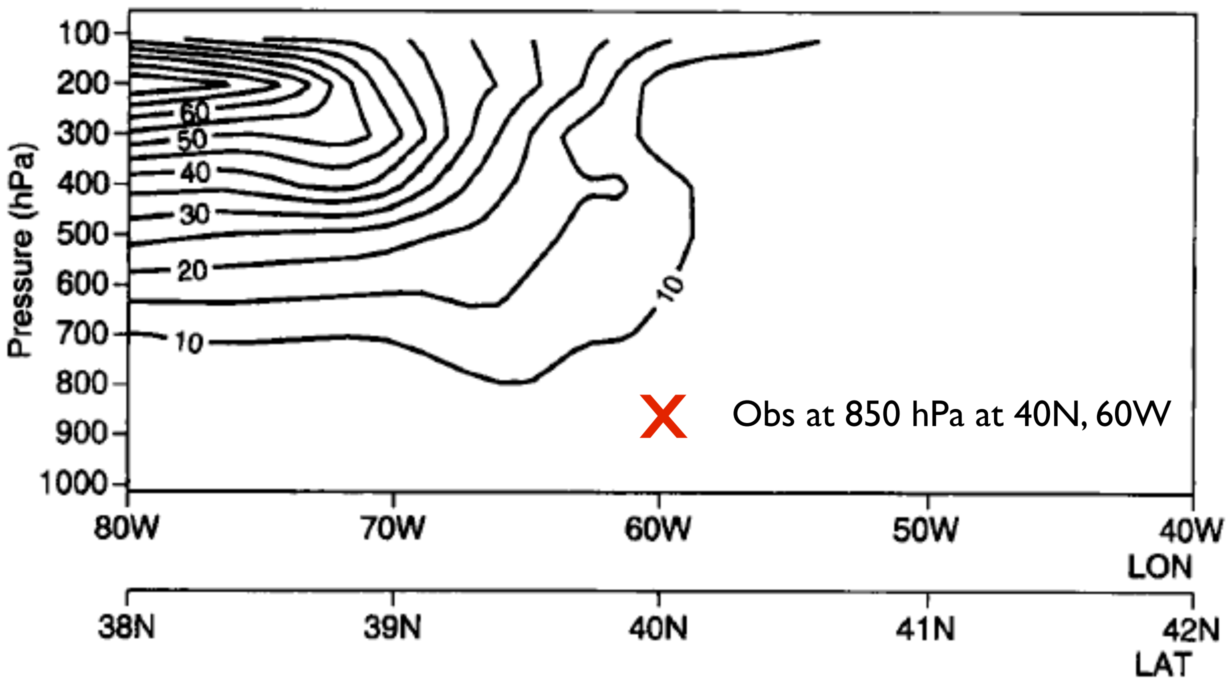


Figure 16. Root-mean-square (RMS) of the innovation vector wind for aircraft data produced over the Atlantic area, averaged over 1 to 21 February 1997, every 12 hours. 4D-Var is shown as a dashed line, 3D-Var as a solid line. The abscissa is the time slot number. The ordinate is the RMS in m s^{-1}.

[Rabier et al, QJRMS, 2000]

ECMWF OPERATIONAL 4D-VAR. I: SIMPLIFIED PHYSICS 1165



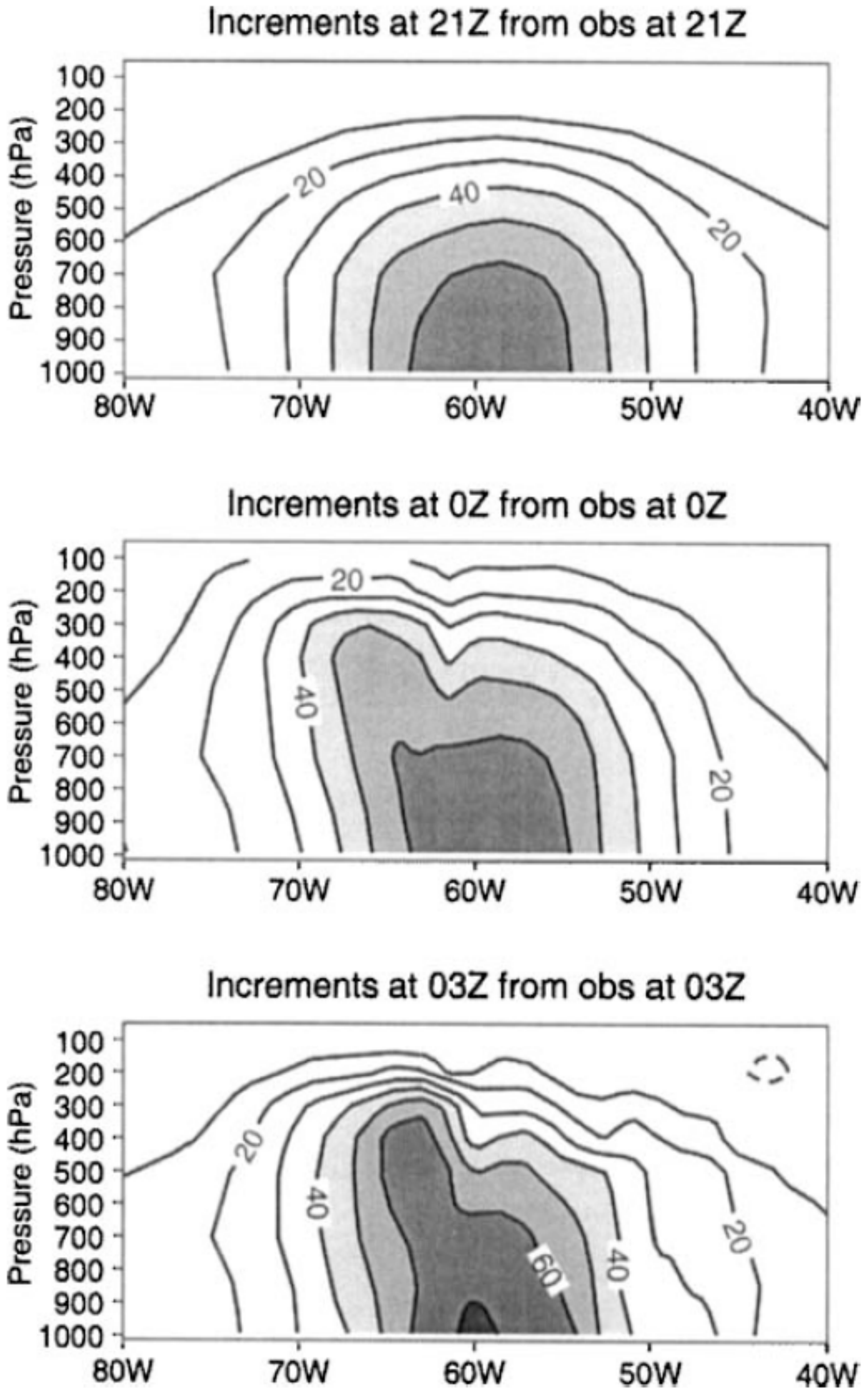
Obs at 850 hPa at 40N, 60W

Figure 17. Cross-section of the zonal wind component (m s^{-1}) of the background for 0000 UTC 5 December 1996. See text for details.

Background zonal wind

[Rabier et al, QJRMS, 2000]

Increments at same time as obs


Increments at 21Z from obs at 21Z

Increments at 0Z from obs at 0Z

Increments at 03Z from obs at 03Z

Figure 18. Structure functions for a height observation at 850 hPa, 40°N, 60°W. Isolines show the resulting increment, in geopotential units. The top panel corresponds to an observation at 2100 UTC, the middle panel to an observation at 0000 UTC, and the bottom panel to an observation at 0300 UTC.

6 hr window from 21Z-03Z

Increments at middle of assim. window (0Z)

ECMWF OPERATIONAL 4D-VAR. I: SIMPLIFIED PHYSICS 1165

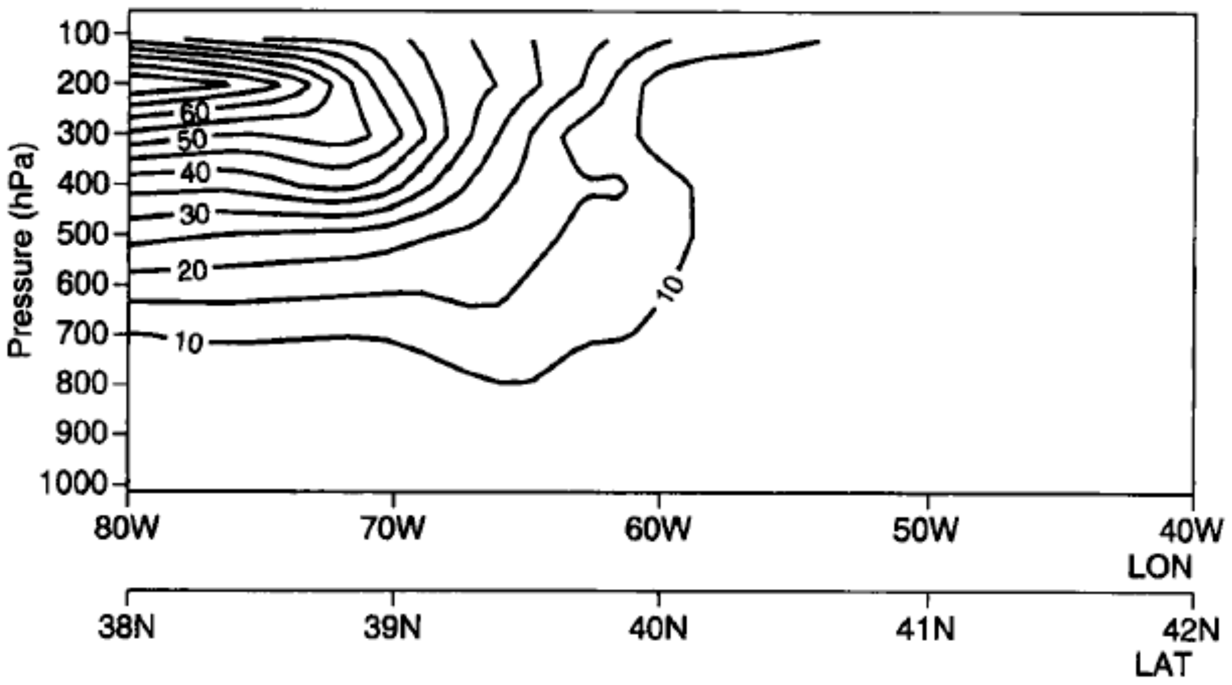
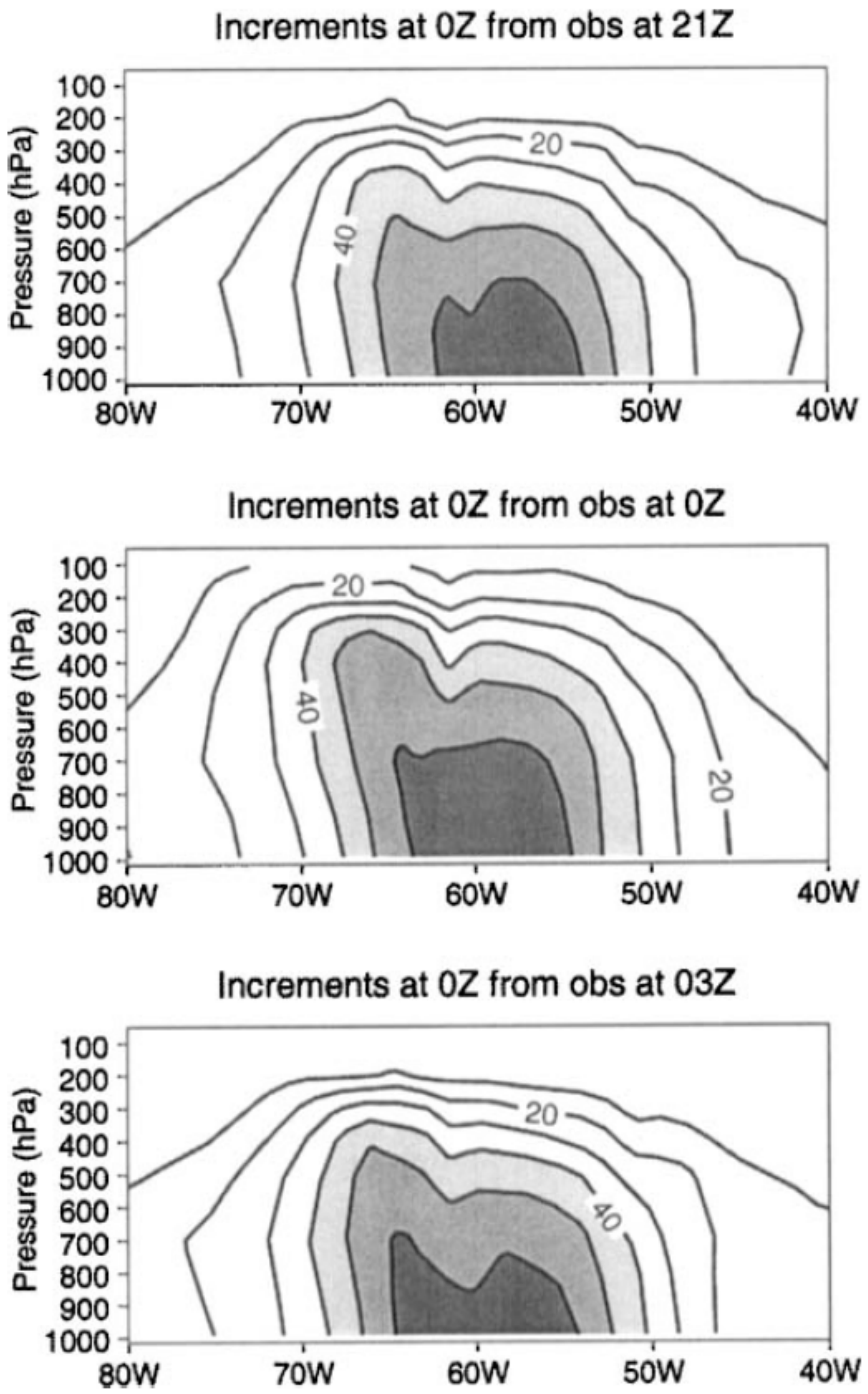


Figure 17. Cross-section of the zonal wind component (m s^{-1}) of the background for 0000 UTC 5 December 1996. See text for details.

Background zonal wind


Increments at 0Z from obs at 21Z

Increments at 0Z from obs at 0Z

Increments at 0Z from obs at 03Z

Figure 19. Increments using 4D-Var at 0000 UTC for a height observation at 850 hPa, 40°N, 60°W. Isolines show the resulting increment, in geopotential units. The top panel corresponds to an observation at 2100 UTC, the middle panel to an observation at 0000 UTC, and the bottom panel to an observation at 0300 UTC.

[Rabier et al, QJRMS, 2000]

6 hr window from 21Z-03Z

Q. J. R. Meteorol. Soc. (1996), **122**, pp. 535–561

Dynamical structure functions in a four-dimensional variational assimilation: A case study

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(Received 7 March 1995; revised 21 August 1995)

SUMMARY

This paper contributes to the understanding of the structure functions used implicitly in the four-dimensional variational assimilation (4D-Var) developed at the European Centre for Medium-Range Weather Forecasts in the last few years. The theoretical equivalence between 4D-Var and the Kalman filter allows us to interpret (after normalization by the error standard deviations) the analysis increments produced by one single observation as the structure functions used implicitly in 4D-Var. The shape of the analysis increments provides a three-dimensional picture of the covariances of the background errors, modified by the dynamics.

We study a baroclinic situation and observations have been regularly distributed along a latitude circle crossing the baroclinic wave. Eight standard pressure levels have been considered to sample the vertical.

The forecast error standard deviations and the structure functions implied in 4D-Var may differ considerably from those used in the 3D-Var analysis. Unlike 3D-Var, the structure functions are flow dependent: the effective background error standard deviation can be four times larger and the correlation length scale twice as short in the vicinity of a low. A meridional extension of the experimentation at the surface shows that the effective background error standard deviations at 1000 hPa are largest in the areas of strong pressure gradient.

We quantify the link between the analysis increments produced by 4D-Var and the fastest growing perturbations over the same time interval. In the depression, the explained variance of the analysis increments by the first 13 singular vectors reaches 30%.

The impact of the temporal dimension is assessed. A period of 24 hours seems a minimum for the increments to develop fully baroclinic structures.

Background Fields

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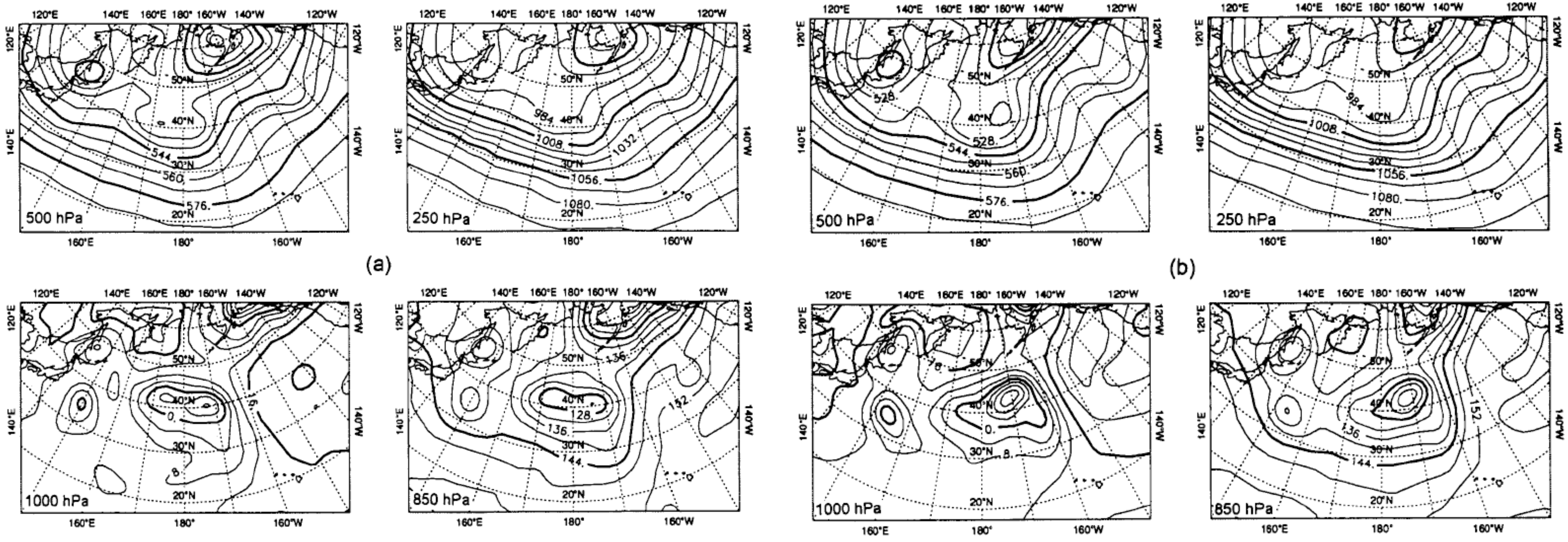
Geopotential Heights



1200 UTC, 4 March 1994

0000 UTC, 5 March 1994

[Thépaut et al, QJRMS, 1996]

Geopotential Heights

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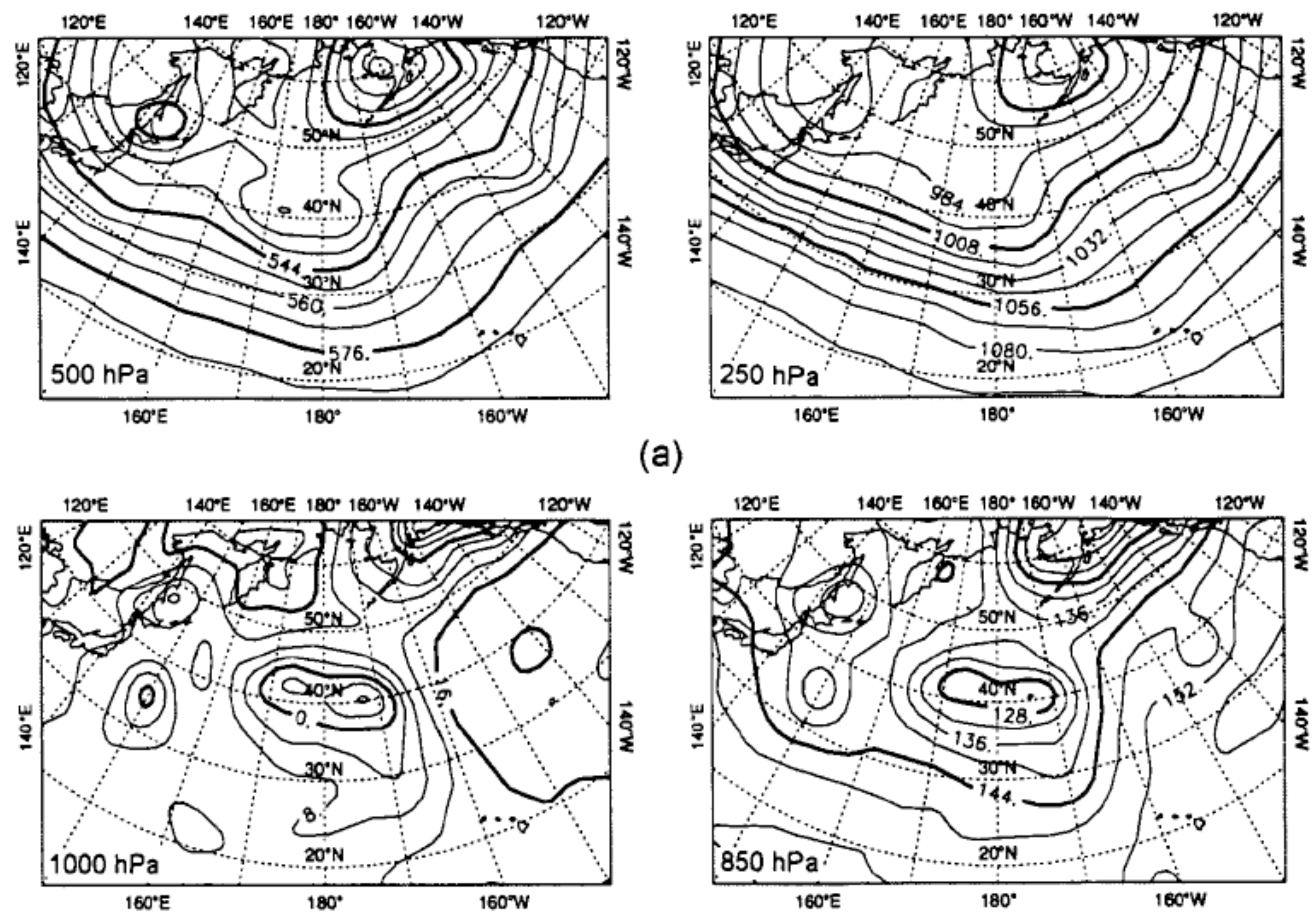
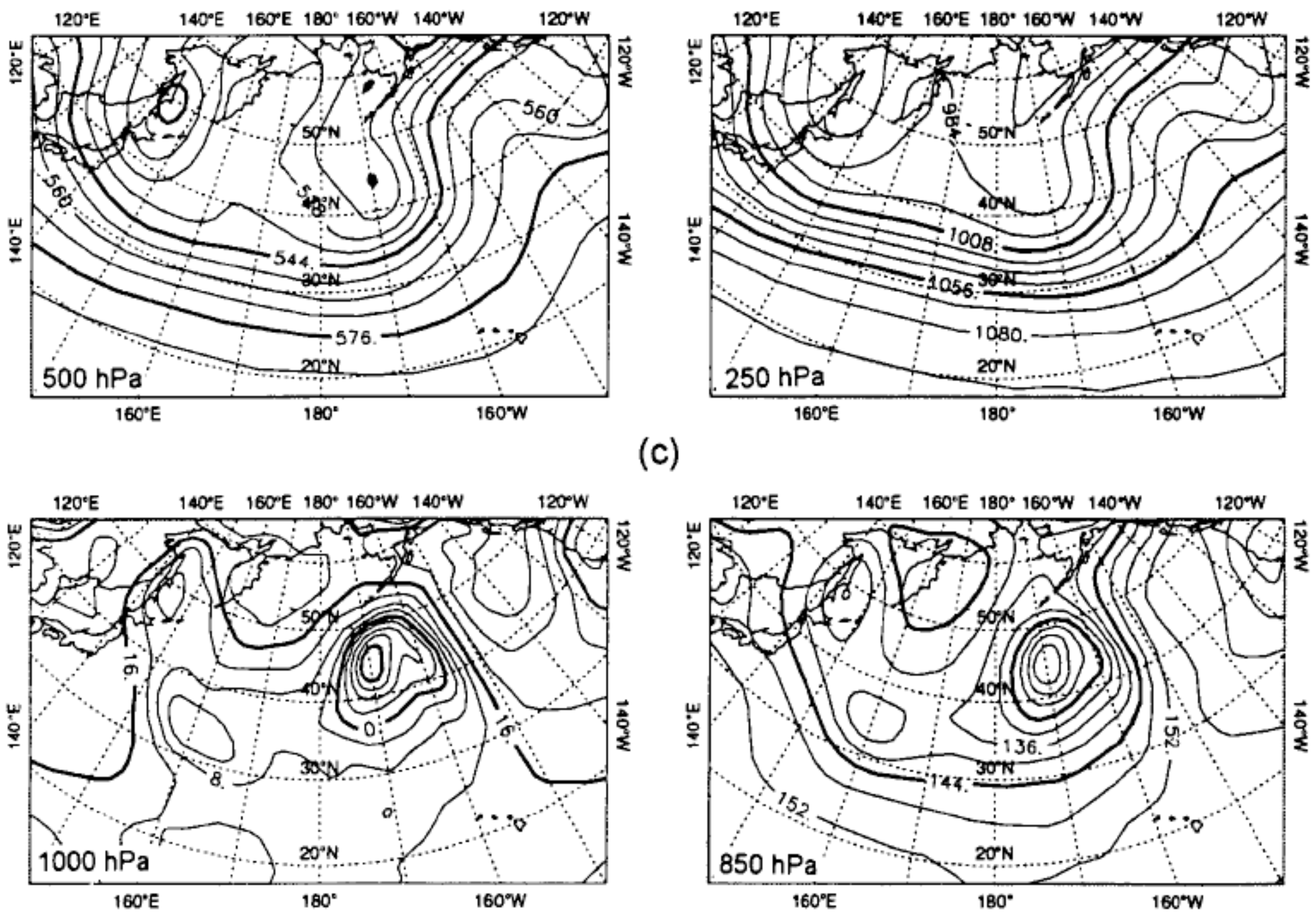


1200 UTC, 4 March 1994



1200 UTC, 5 March 1994

[Thépaut et al, QJRMS, 1996]

3Dvar Analysis Increments

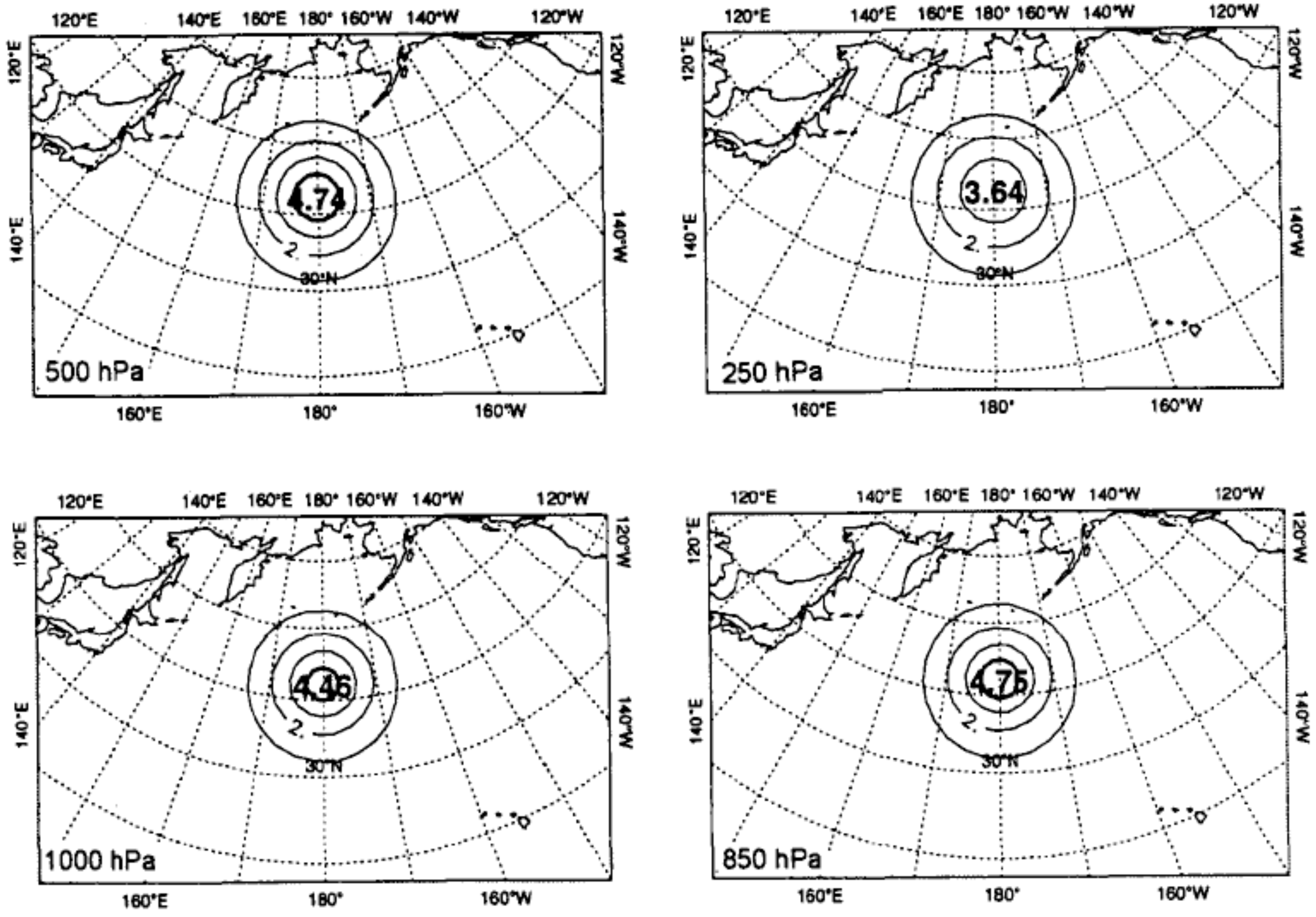


Figure 2. The 3D-Var analysis increments (analysis–background), at the end of the 24-hour assimilation period, for geopotential height and at 1000, 850, 500 and 250 hPa, corresponding to a single height observation at 500 hPa at the location (42°N, 180°E). Contour interval: 0.1 m.

[Thépaut et al, QJRMS, 1996]

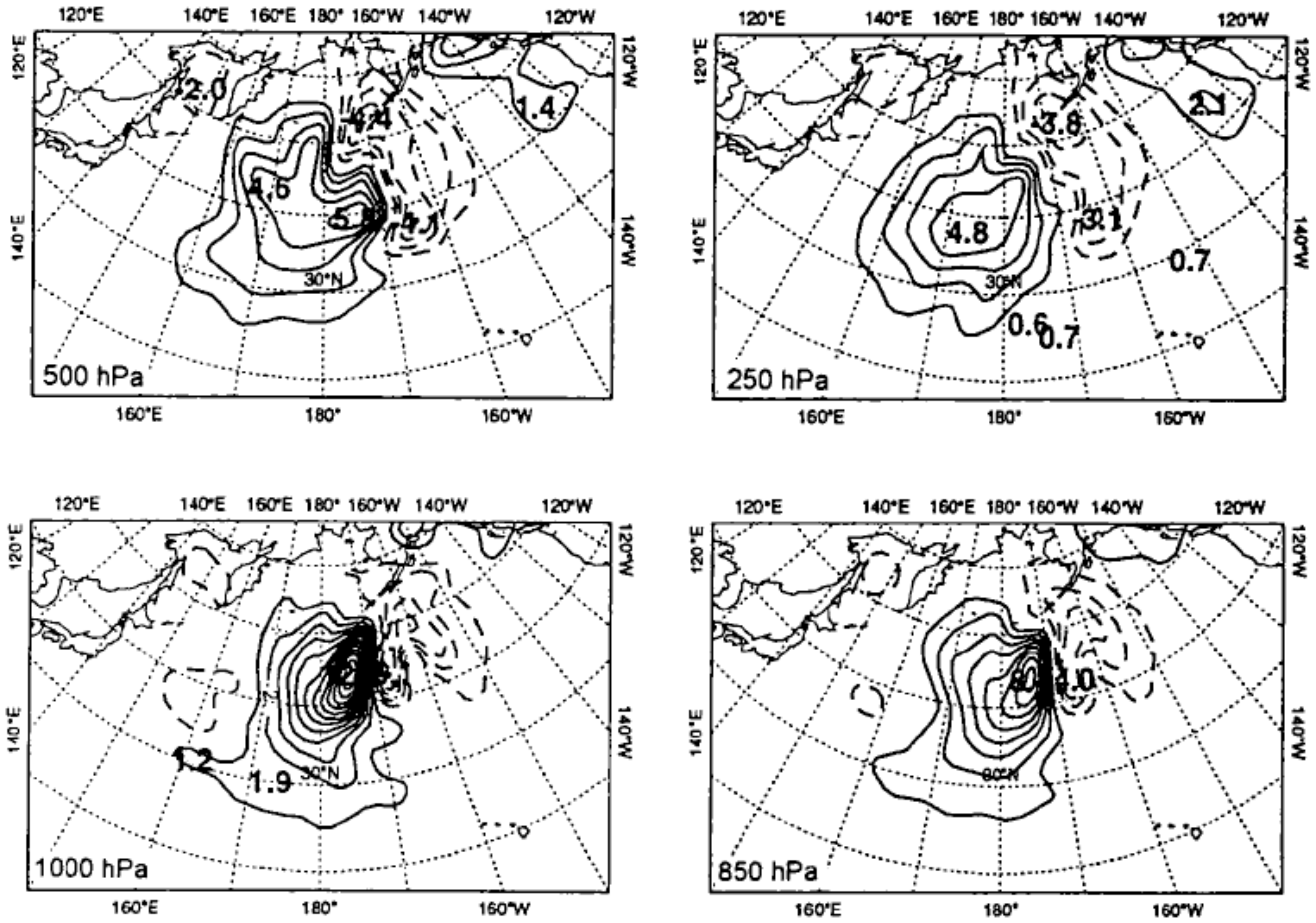


Figure 13. Same as Fig. 7 but for a single height observation at 1000 hPa at the location (42°N, 180°W). Contour interval: 0.1 m.

[Thépaut et al, QJRMS, 1996]

4Dvar Analysis Increments

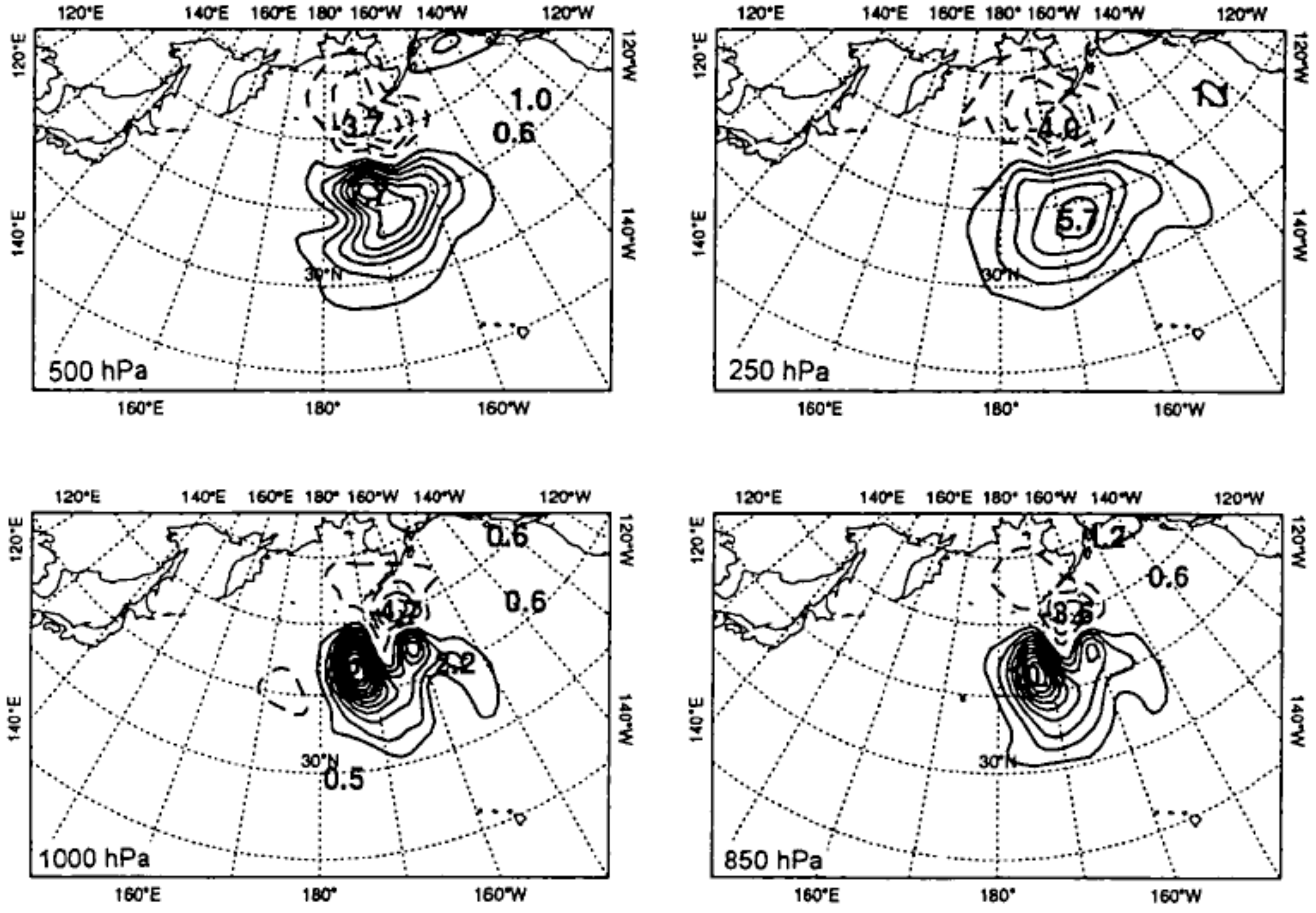


Figure 5. The 4D-Var analysis increments (analysis–background) for geopotential height and at 1000, 850, 500 and 250 hPa, corresponding to a single height observation at 850 hPa at the location (42°N, 170.625°W). Contour interval: 0.1 m.

Observation at 42°N, 170.625°W

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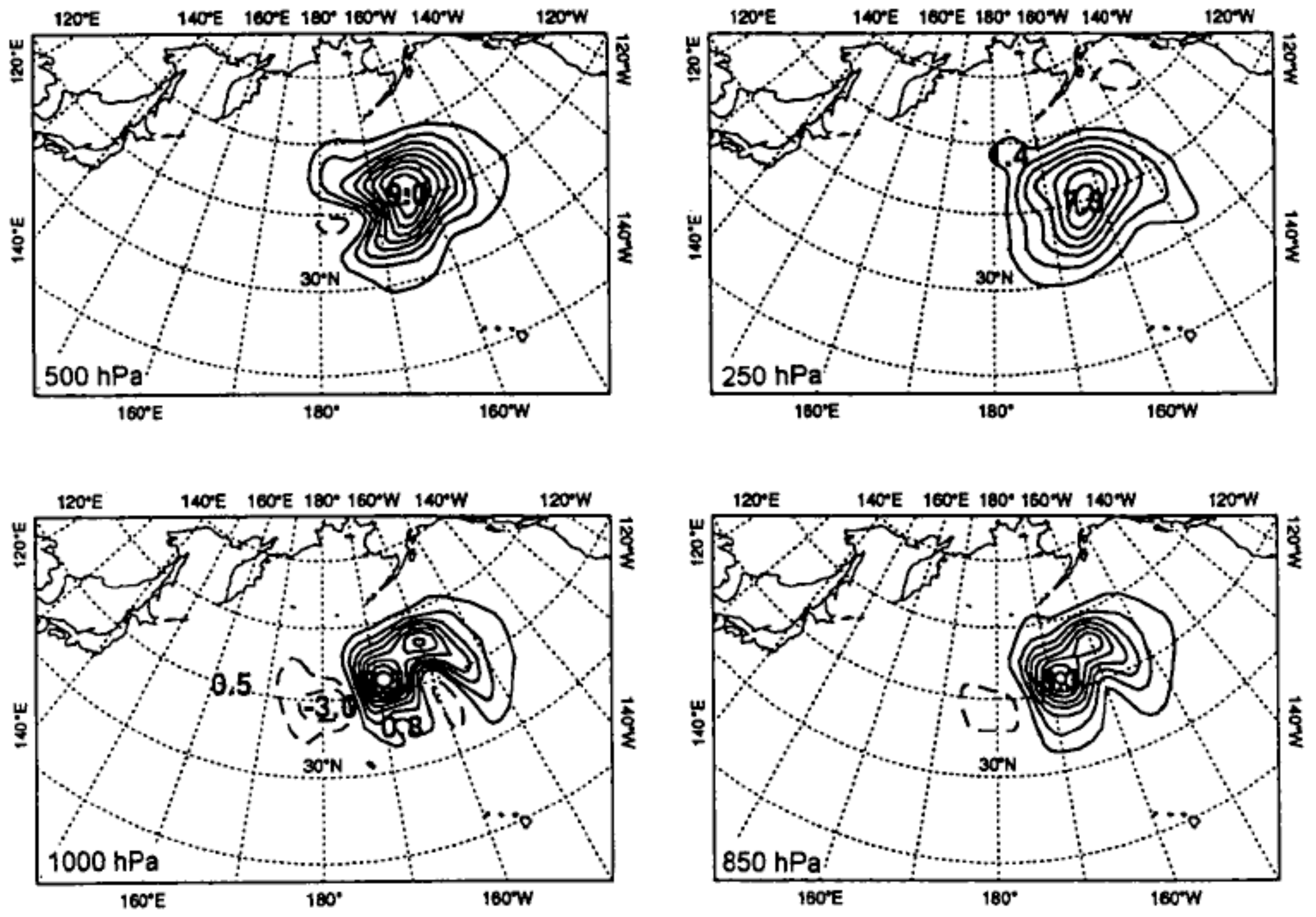

Figure 6. The 4D-Var analysis increments (analysis–background) for geopotential height and at 1000, 850, 500 and 250 hPa, corresponding to a single height observation at 850 hPa at the location (42°N, 168.75°W). Contour interval: 0.1 m.

Observation at 42°N, 168.75°W

[Thépaut et al, QJRMS, 1996]

Vertical Structure of Analysis Increment

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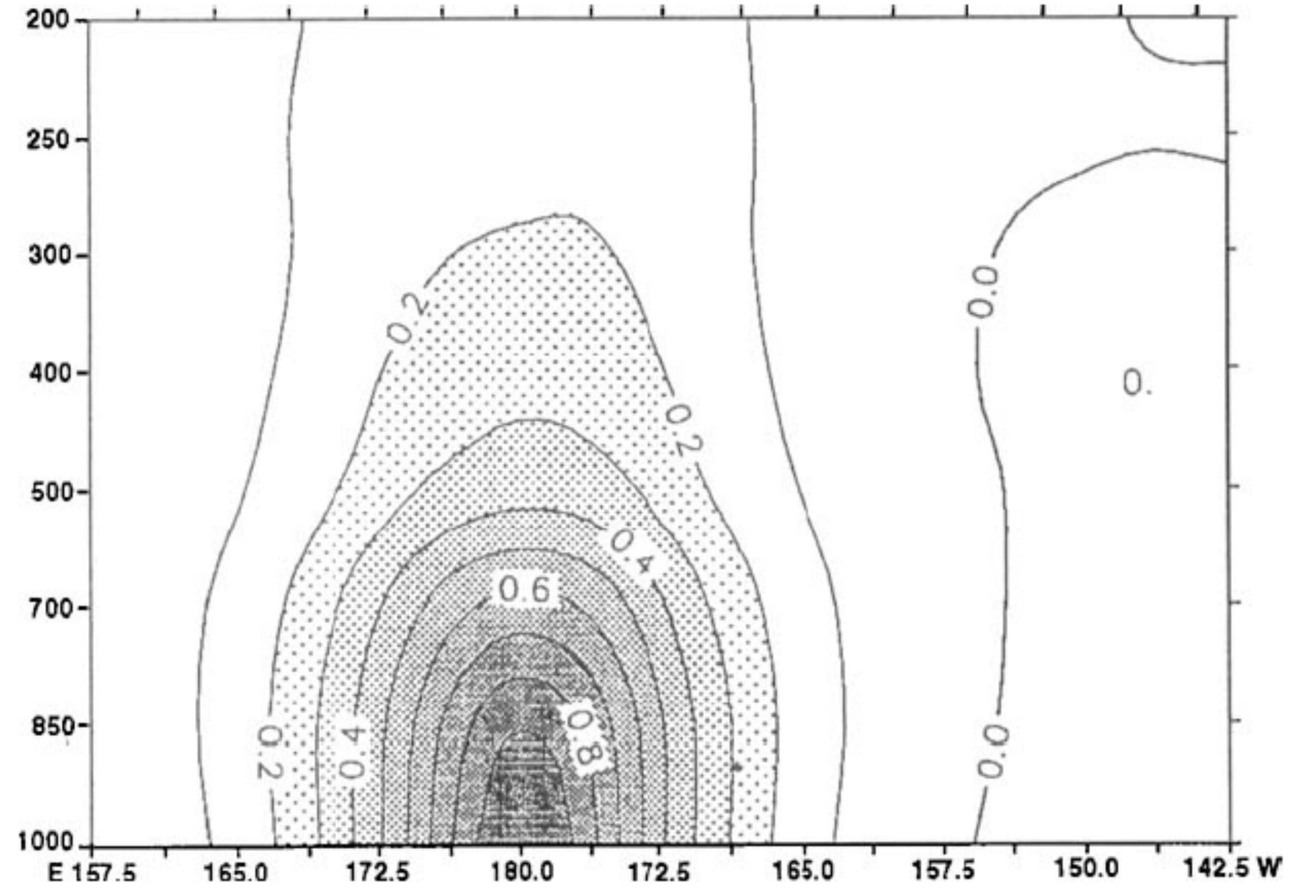


Figure 3. Cross-section of the 3D-Var structure function for an observation at location (42°N, 168.75°W, 1000 hPa). Contour interval: 0.1 m.

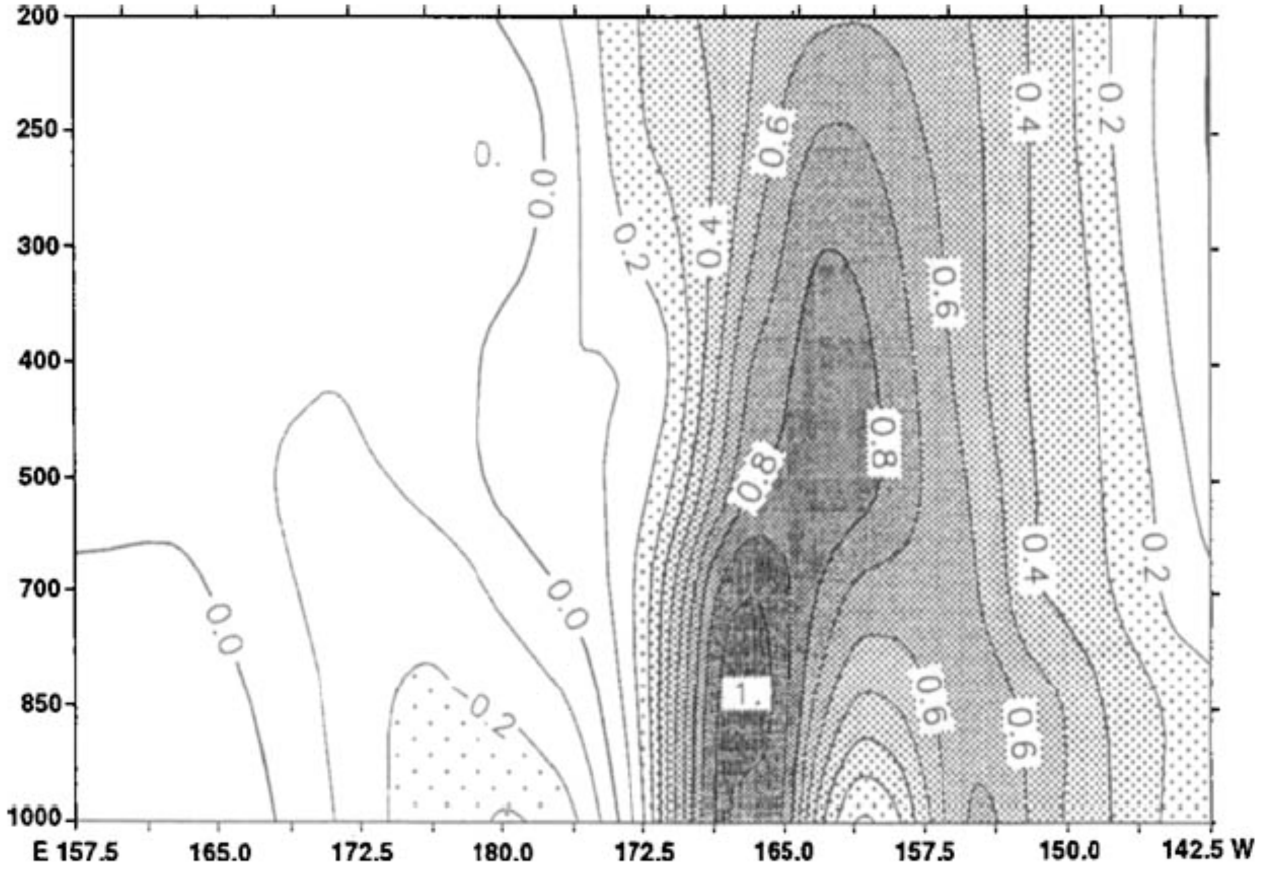
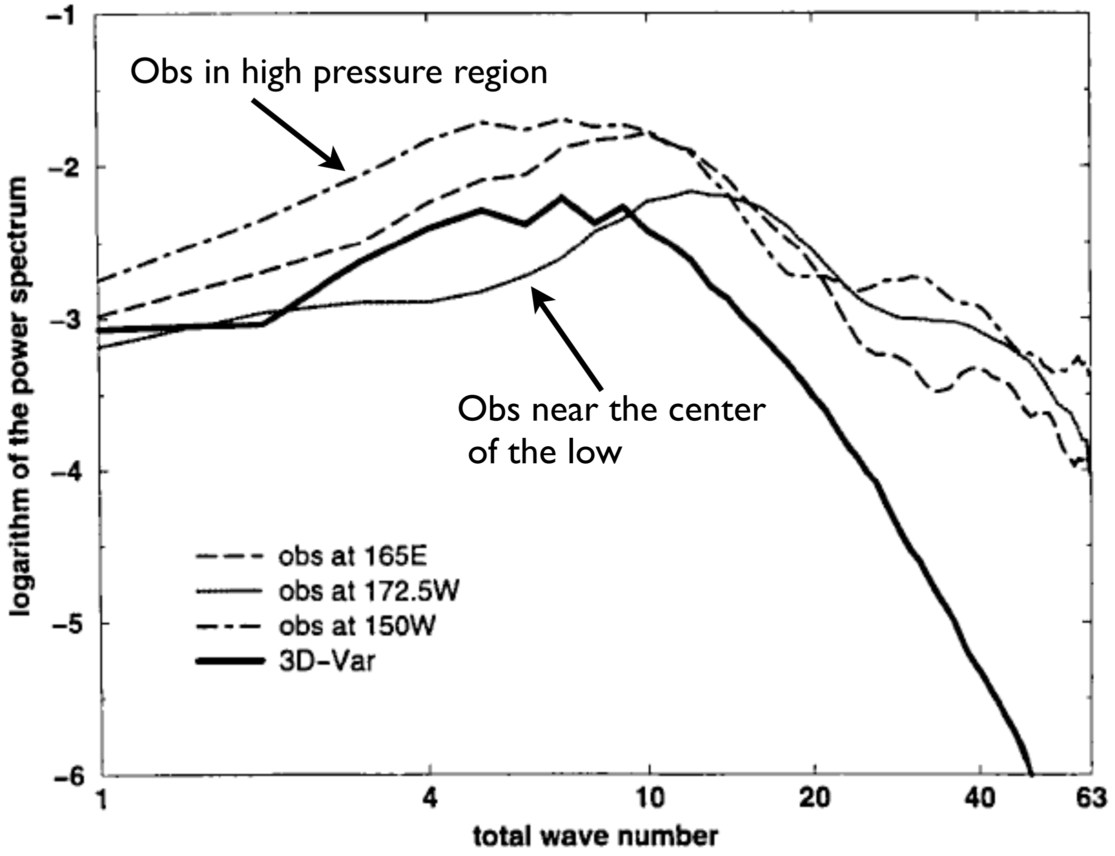


Figure 12. Cross-section of the 4D-Var structure function, obtained with an observation at location (42°N, 168.75°W, 1000 hPa). Contour interval: 0.1 m.

[Thépaut et al, QJRMS, 1996]

Power Spectrum

Obs at 1000 hPa



Obs in high pressure region

Obs near the center of the low

Figure 15. 4D-Var increments power spectra at 1000 hPa produced by single observations located between longitude 165°E and 150°W at the same level. And for reference: 3D-Var increments spectrum for one observation also located at 1000 hPa.

Obs at 500 hPa

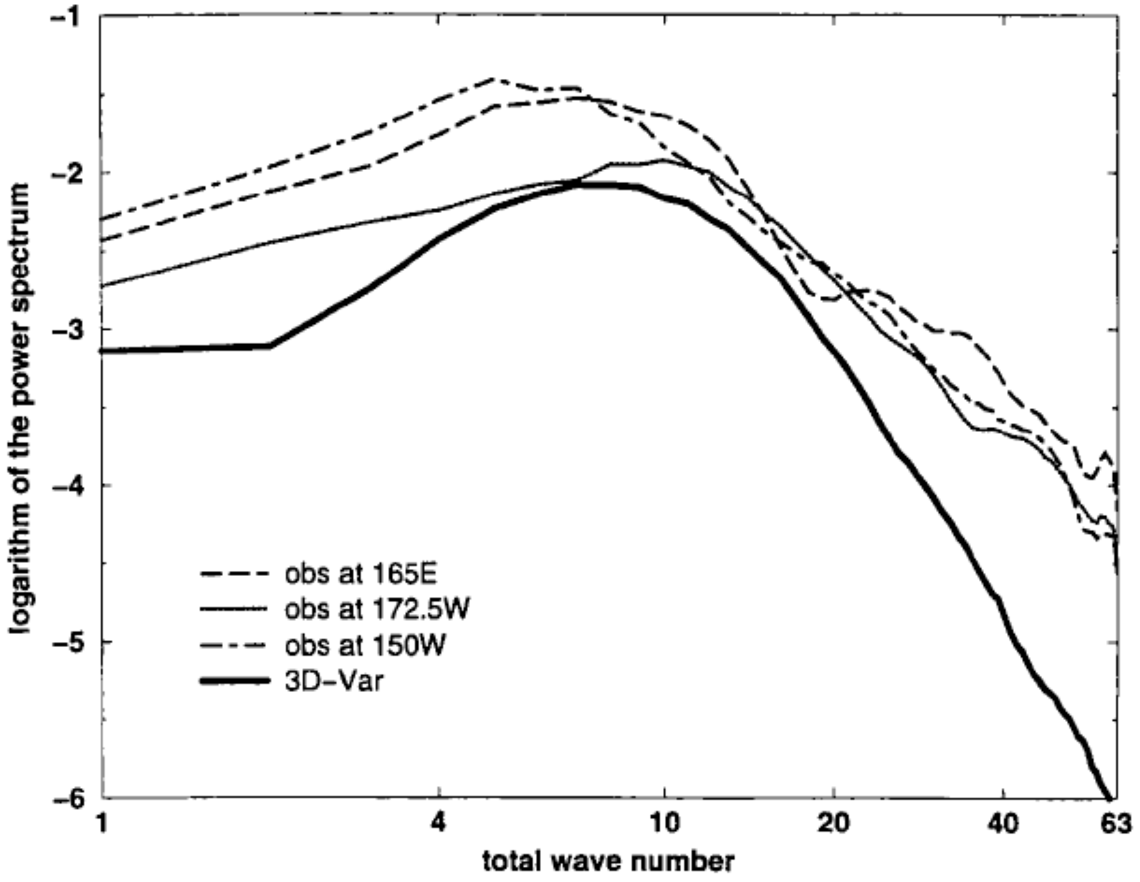


Figure 16. 4D-Var increments power spectra at 500 hPa produced by single observations located between longitude 165°E and 150°W at the same level. And for reference: 3D-Var increments spectrum for one observation also located at 500 hPa.

[Thépaut et al, QJRMS, 1996]