Section 2. Satellite Orbits

References
- *Kidder and Vonder Haar: chapter 2*
- *Stephens: chapter 1, pp. 25-30*
- *Rees: chapter 9, pp. 174-192*

In order to understand satellites and the remote sounding data obtained by instruments located on satellites, we need to know something about orbital mechanics, especially the orbits in which satellites are constrained to move and the geometry with which they view the Earth.

2.1 Orbital Mechanics

The use of satellites as platforms for remote sounding is based on some very fundamental physics.

Newton's Laws of Motion and Gravitation (1686)
→ the basis for classical mechanics

Laws of motion:

(1) Every body continues in its state of rest or of uniform motion in a straight line unless it is compelled to change that state by a force impressed upon it.

(2) The rate of change of momentum is proportional to the impressed force and is in the same direction as that force.

Momentum = mass × velocity, so Law (2) becomes $\vec{F} = \frac{d(m\vec{v})}{dt} = m\frac{d\vec{v}}{dt} = \vec{a}$
for constant mass

(3) For every action, there is an equal and opposite reaction.

Law of gravitation:

The force of attraction between any two particles is
- proportional to their masses
- inversely proportional to the square of the distance between them

i.e. $F = \frac{Gm_1m_2}{r^2}$ (treating the masses as points)
where
G = gravitational constant = 6.673×10^{-11} Nm^2/kg^2

These laws explain how a satellite stays in orbit.

Law (1): A satellite would tend to go off in a straight line if no force were applied to it.

Law (2): An attractive force makes the satellite deviate from a straight line and orbit Earth.

Law of Gravitation:
This attractive force is the gravitational force between Earth and the satellite. Gravity provides the inward pull that keeps the satellite in orbit.

Assuming a circular orbit, the gravitational force must equal the centripetal force.

$$\frac{mv^2}{r} = \frac{Gmm_E}{r^2}$$

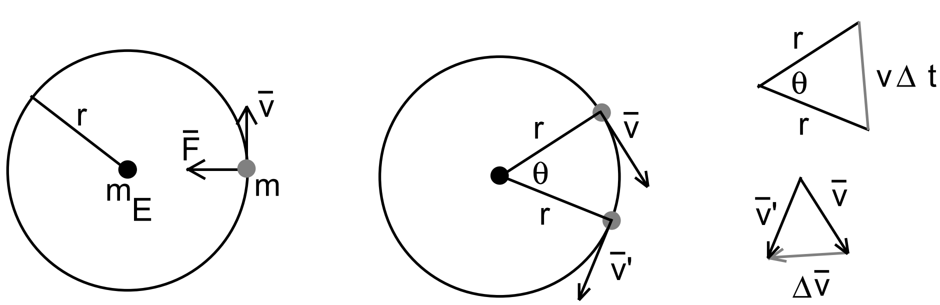
where
v = tangential velocity
r = orbit radius = R_E + h (i.e. not the altitude of the orbit)
R_E = radius of Earth
h = altitude of orbit = height above Earth's surface
m = mass of satellite
m_E = mass of Earth

$\therefore \; v = \sqrt{\frac{Gm_E}{r}}$, so v depends only on the altitude of the orbit (not on the satellite's mass).

The period of the satellite's orbit is $T = \frac{2\pi r}{v} = 2\pi r\sqrt{\frac{r}{Gm_E}} = 2\pi\sqrt{\frac{r^3}{Gm_E}}$.

Again, this is only dependent on the altitude, increasing as the orbit's altitude increases.

The acceleration of the satellite is determined using $\frac{\Delta v}{v} \approx \frac{v\Delta t}{r}$, so $a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{v^2}{r}$.



Example: The Odin satellite will orbit at ~600 km.

$\therefore$ $r = 600 + R_E = 600 + 6378$ (Earth's equatorial radius) = 6978 km

$$v = \sqrt{\frac{(6.673 \times 10^{-11} \mathrm{Nm}^2 / \mathrm{kg}^2) \times (5.974 \times 10^{24} \mathrm{kg})}{6978 \times 10^3 \mathrm{m}}} = 7558 \text{ m/s} \sim 7.6 \text{ km/s}$$

T = 5801 s = 96.7 min

Conversely, we can use these equations to calculate the altitude a satellite in geosynchronous orbit.
$\rightarrow$ the higher the satellite the longer the period of its orbit
$\rightarrow$ so moving it high enough will make its orbit match Earth's rotation rate

$$r = \sqrt[3]{G m_E \left(\frac{T}{2\pi}\right)^2} = 42{,}166 \text{ km}, \text{ so altitude above surface} = 35{,}788 \text{ km}$$

where
T = 86,164.1 s = sidereal day, the period of Earth's rotation with respect to the stars.

Kepler's Laws for Orbits

So far, we have assumed that satellites travel in circular orbits, but this is not necessarily true in practice.

Newton's Laws can be used to derive the exact form of a satellite's orbit.

However, a simpler approach is to look at Kepler's Laws, which summarize the results of the full derivation.

Kepler's Laws (1609 for 1,2; 1619 for 3) were based on <u>observations</u> of the motions of planets.

(1) All planets travel in elliptical orbits with the Sun at one focus.
$\rightarrow$ defines the shape of orbits

(2) The radius from the Sun to the planet sweeps out equal areas in equal times.
$\rightarrow$ determines how orbital position varies in time

(3) The square of the period of a planet's revolution is proportional to the cube of its semimajor axis.
$\rightarrow$ suggests that there is some systematic factor at work

For satellites, substitute "satellite" for planet, and "Earth" for Sun.

See figure ("94") – Kepler's law of equal areas

Ellipse Geometry and Definitions

See figure (K&VH 2.2) - elliptical orbit geometry

Some geometric terms:

perigee - point on the orbit where the satellite is closest to Earth

apogee - point on the orbit where the satellite is furthest from Earth

semimajor axis - distance from the centre of the ellipse to the apogee or perigee (a)

semiminor axis (b)

eccentricity - distance from the centre of the ellipse to one focus / semimajor axis (ε)

$\varepsilon = c / a$ (Note: c is a_ε in K&VH Figure 2.2)
$0 \le \varepsilon < 1$
$\varepsilon = 0$ for a circle

Can also show that $a^2 = b^2 + c^2$ or $b = a\sqrt{1-\varepsilon^2}$.

Recall the equation describing an ellipse which is centred at the origin of the x-y plane:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \text{ , with } a > b > 0$$

However, it is more convenient to move the co-ordinate system such that the origin is at the focus (i.e., the Earth), so that

$$x = x_p + c$$

$$y = y_p$$

We can show (!) that the equation for the ellipse, when converted to polar co-ordinates with the Earth at the origin becomes

$$r = \frac{a(1-\varepsilon^2)}{1+\varepsilon\cos\theta}$$

where
r = distance from the satellite to the centre of Earth
θ = the true anomaly, and is always measured counterclockwise from the perigee.

At perigee, $\theta = 0$ so $r = r_{perigee} = a(1 - \varepsilon)$.
At apogee, $\theta = \pi$ so $r = r_{apogee} = a(1 + \varepsilon)$.

This equation describes the shape of the orbit, but not the dynamics of the satellite motion, i.e., we want to find $\theta(t)$.

Kepler's Time of Flight Equation

A satellite in a circular orbit has a uniform angular velocity. However, a satellite in an elliptical orbit must travel faster when it is closer to Earth.

It can be shown that a more general expression for the velocity of an orbiting satellite is

$$v = \sqrt{Gm_E\left(\frac{2}{r} - \frac{1}{a}\right)}$$

where the mass of the satellite is negligible relative to the mass of Earth.

Kepler's Second Law can be applied (non-trivial) to calculate the position of a satellite in an elliptical orbit as a function of time.

This introduces a new term, the eccentric anomaly, "e", which is defined by circumscribing the elliptical orbit inside a circle.

See figure (K&VH 2.3) - geometric relation between e and θ

Time of flight equation: $\boxed{e - \varepsilon \sin e = n(t - t_p) \equiv M}$

where
e = eccentric anomaly
ε = eccentricity
M = the mean anomaly
t = time
t_p= time of perigee passage (when $\theta = 0$)

$n = \frac{2\pi}{T} = \sqrt{\frac{Gm_E}{a^3}}$ = mean motion constant

The eccentric anomaly and the true anomaly are geometrically related by

$$\cos\theta = \frac{\cos e - \varepsilon}{1 - \varepsilon \cos e} \quad \text{and} \quad \cos e = \frac{\cos\theta + \varepsilon}{1 + \varepsilon\cos\theta}.$$

Can solve for e(t) and hence θ(t) and r(t).

2.2 Co-Ordinate Systems

Thus far, we have derived expressions for θ and r as functions of time from a consideration of Newton's and Kepler's Laws.

A more rigorous treatment would involve applying Newton's Laws to derive the equations for two-body motion. These equations could then be simplified by
- assuming $m_{satellite} < m_E$ (for which Kepler's Laws are exact)
- using a reference frame with the origin at the Earth (effectively an inertial frame).

Right Ascension-Declination System

Now we have $\theta(t)$ and r(t) which position the satellite in the plane of the orbit.
Next, we need to establish a co-ordinate system to position the orbital plane in space.

Introduce the right ascension-declination co-ordinate system (common in astronomy)

→ z axis is aligned with Earth's spin axis
→ x axis points from the centre of the Earth to the Sun at the vernal equinox position (i.e., when the Sun crosses the equatorial plane from the Southern to the Northern hemisphere)
→ y axis is chosen to make it a right-handed system

See figure (K&VH 2.4) - right ascension-declination co-ordinate system

Note: the Sun's apparent path is called the ecliptic. The obliquity of the ecliptic is 23.5°, the same angle as the tilt of the Earth's axis.

Then we can define:
declination (δ)
is the angular displacement of a point in space measured northward from the equatorial plane

right ascension (Ω)
is the angular displacement, measured counterclockwise from the x axis, of the projection of the point onto the equatorial plane

The Ω-δ system is analogous to latitude and longitude, with
δ ~ latitude, giving angular distance north or south of the celestial equator
Ω ~ longitude, giving angular distance measured eastward from a reference point on the celestial equator (i.e., from the vernal equinox)

See figure (K&VH 2.5) - definition of right ascension and declination
See figure (K&VH 2.6) - angles used to orient an orbit in space (for next section)
See figure ("95") – astronomical coordinate system

Classical Orbital Elements

In order to specify a satellite orbit or to determine the location of a satellite in space, a set of parameters are needed → the classical orbital elements, defined as follows (using the Ω-δ system):

(1) semimajor axis (a)

(2) eccentricity (ε)

(3) inclination angle (i)
is the angle between the equatorial plane and the orbital plane
$i = 0^\circ$ if these planes coincide and if the satellite revolves in the same direction as Earth's rotation
$i = 180^\circ$ if these planes coincide but the satellite revolves in the opposite direction to Earth's rotation
$i < 90^\circ$ is called a prograde orbit
$i > 90^\circ$ is called a retrograde orbit

(4) right ascension of the ascending node (Ω)
The ascending node is the point where the satellite crosses the equatorial plane going north. Ω is the right ascension of this point. In practice, it is the right ascension of the intersection of the orbital plane with the equatorial plane.

(5) argument of perigee (ω)
is the angle between the ascending node and perigee, measured in the orbital plane

(6) epoch time (t)
is the time at which the orbital elements are observed, needed because some of these elements are time-dependent. Sometimes t_p (time of perigee passage) is used.

(7) mean anomaly (M)

Note: a and ε are the "shape" elements - they define the size and shape of the orbit
i, Ω, and ω are the "orientation" elements - they position the orbit in the Ω-δ system
Orbital elements Ω, ω, t, and M depend on time, and are often subscripted with "o" to indicate their value at the epoch time.

Orbital elements for particular satellites are usually available from the agency that operates them (e.g., NASA, ESA). They can be determined by ranging instruments on a satellite or by matching surface landmarks with observations made by the satellite instruments.

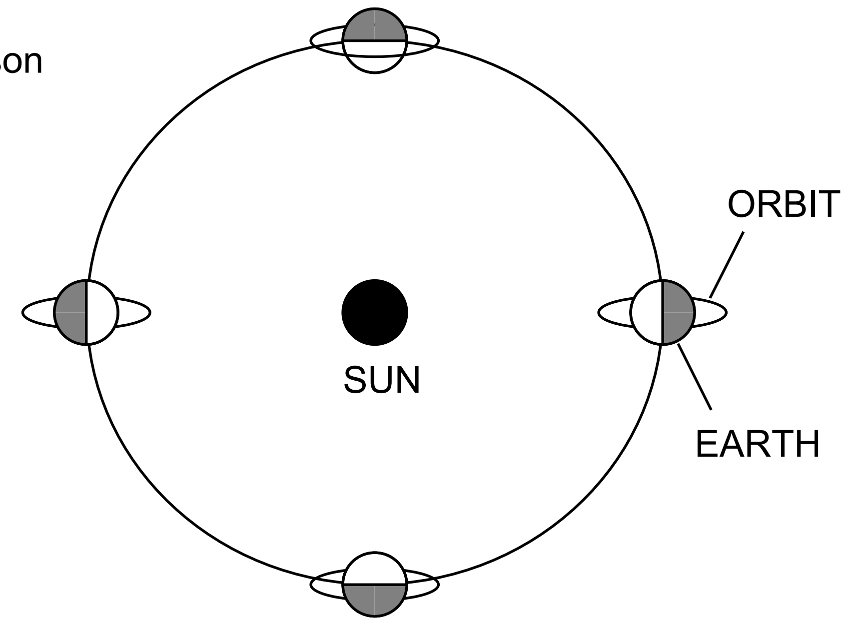
Note: orbital parameters are sometimes redefined as (1) longitude of the ascending node, (2) nodal period, (3) radius, (4) inclination, (5) time of ascending node, (6) nodal longitude increment = difference in longitude between successive ascending nodes.

2.3 Types of Satellite Orbits

(1) Keplerian Orbits (and Effects of Earth's Non-Spherical Gravitational Field)

A Keplerian orbit is an orbit for which all the orbital elements except the mean anomaly M are constant. The ellipse thus maintains a constant size, shape, and orientation with respect to the stars.

The change with season of a Keplerian orbit.



These orbits are simple as viewed from space, but complicated when seen from Earth because Earth rotates beneath the fixed orbit. This generally results in two daily passes of the satellite above any point on Earth: one as the orbit ascends and one as the orbit descends (usually one at day and one at night) with the time changing through the year.

In practice, orbits are perturbed by a number of factors:

- Earth's non-spherical gravitational field
- the gravitational attraction of other bodies (i.e., third body interactions)
- radiation pressure from the Sun
- particle flux from the solar wind
- lift and drag forces from the atmosphere
- electromagnetic forces

All but the first of these causes random orbit perturbations which can be corrected by periodic observations of the orbital elements and adjustment of the orbit using on-board thrusters. However, the Earth's non-spherical gravitational field causes secular (linear with time) changes in the orbital elements.

The effect of the Earth's non-spherical gravitational field can be treated to first order by regarding the Earth as a sphere with a belt 21 km thick around the equator. This belt changes the speed of the orbit, exerting an equatorward force which makes the orbit precess around the z axis (rather than changing the inclination angle - think of a gyroscope). Thus Ω and ω precess. It is possible to derive expressions for $\frac{dM}{dt}$, $\frac{d\Omega}{dt}$, and $\frac{d\omega}{dt}$, given the Earth's gravitational potential.

The mean motion constant, n, of an unperturbed orbit is replaced by the anomalistic mean motion constant, $\bar{n} = \frac{dM}{dt} = n + \text{correction term}$.

Then $\frac{d\Omega}{dt}$ = rate of change of right ascension of the ascending node

$\frac{d\omega}{dt}$ = rate of change of the argument of perigee

$\bar{T} = \frac{2\pi}{\bar{n}}$ = anomalistic period, which is the time taken by the satellite to move from perigee to the next (moving) perigee.

More useful than $\bar{T}$ is the synodic or nodal period, $\tilde{T}$, which is the time taken by the satellite to move one ascending node to the next ascending node.

$$\tilde{T} = \frac{2\pi}{\bar{n} + \frac{d\omega}{dt}}$$

Thus, to first order, the non-spherical gravitational field of Earth causes a slow linear change in Ω and ω, and a small change in the mean motion constant.

(2) Sun-Synchronous Orbits

Keplerian orbits, for which the orbital plane is fixed in space while the Earth revolves around the Sun, are not very useful because the satellite passes over the same place at different time of the day throughout the year.

Think of an orbit passing over the poles. The Earth rotates under it every 24 hours so that any point on the surface will pass below the orbit every 12 hours. A satellite in this orbit will pass over the same place at the same time of day. However, because Earth orbits the Sun, this time of day will change by 24 hours during the course of the year. e.g., noon and midnight in spring, 6 AM and 6 PM in winter

Resulting problems with Keplerian orbits:

- data don't fit into operational schedules
- orientation of solar panels is difficult
- the resulting dawn and dusk images are less useful

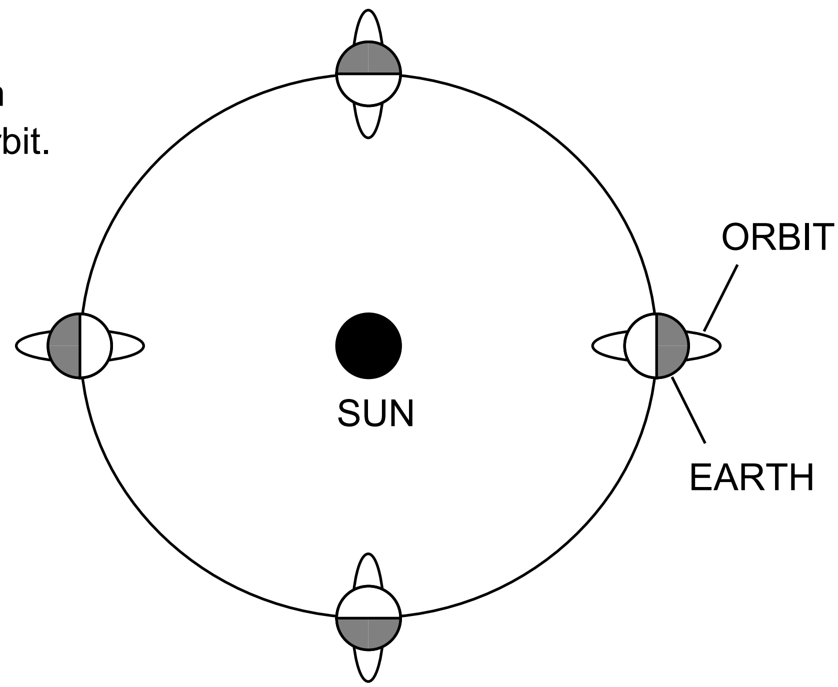
However, the orbit perturbation caused by the Earth's non-spherical gravitational field can be used to advantage. By choosing the correct inclination and altitude of the satellite orbit, the right ascension of the ascending node can be made to precess at the same rate as the Earth revolves around the Sun.

This is called a sun-synchronous orbit. It is an orbit for which the plane of the satellite orbit is always the same in relation to the Sun. It can also be defined an as orbit for which the satellite crosses the equator at the same local time each day (need to define local time).

A sun-synchronous orbit is not fixed in space. It must move 1° per day to compensate for the Earth's revolution around the Sun. Since the Earth makes one revolution (360°) around the Sun per year, we can calculate the rate of change of the Sun's right ascension: 0.9856473°/solar day.
$\rightarrow$ this is the required rate of precession of Ω.

Can show that the precession is $\propto \cos i$ and $\propto 1/a^2$ radius.

The change with season of a sun-synchronous orbit.



See figures ("6" and Stephens 1.13b) - a sun-synchronous orbit

Need to define local time: $LT \equiv UT + \dfrac{\psi}{15°}$
where
UT = universal time (hours) and
ψ = longitude (degrees).

The equator crossing time is the local time at which a satellite crosses the equator:

$ECT \equiv UT + \frac{\psi_N}{15^\circ}$ where ψ_N = longitude of the ascending or descending node.

Now, the longitude of the Sun is $\psi_{sun} = -15^\circ \times (UT - 12)$.

Define $\Delta\psi \equiv \psi_N - \psi_{sun}$.

(e.g., at noon, UT = 12 hours and $\psi_{sun} = 0^\circ$
at midnight, UT = 0 or 24 hours and $\psi_{sun} = 180^\circ$)

Then: $ECT \equiv \left[12 - \frac{\psi_{sun}}{15^\circ}\right] + \frac{\psi_{sun} + \Delta\psi}{15^\circ} = 12 + \frac{\Delta\psi}{15^\circ}$.

This is constant for a sun-synchronous orbit because $\Delta\psi$ is constant.

ECT is used to classify sun-synchronous satellite orbits:

- noon satellites ascend near noon LT and descend near midnight LT
- morning satellites ascend between 06 and 12 hours LT and descend between 18 and 24 hours LT
- afternoon satellites ascend between 12 and 18 hours LT and descend between 00 and 06 hours LT

The oblateness of the Earth also causes the perigee to move in the orbit plane, so that the satellite altitude over a target will vary. However, this can be overcome by choosing the right location of the perigee and the right eccentricity. Thus, it is possible to obtain a constant altitude sun-synchronous orbit.

An Aside – Definitions of Time

Solar Time – is based on the observed daily motion of the Sun relative to the Earth, and thus depends on both the rotational and orbital motion of Earth.

- The hour angle (HA) is used to measure the "longitude" of a satellite westward from the meridian of an observer on Earth. 24 hours = 360° around the equator. The hour angle of the Sun is 0° at noon.
- A solar day is the length of time between two consecutive solar transits of a particular meridian. The observed or apparent solar time results in a day of variable length (e.g., due to eccentricity of Earth's orbit about the Sun).
- Mean solar time is defined by averaging the annual variations in apparent solar time, leading to a fictitious mean Sun about which Earth orbits at a constant angular velocity.

Universal Time (UT) or Greenwich Mean Time (GMT)
Is mean solar time referenced to the Greenwich meridian (0° longitude).

UT = 12 hours + HA (mean Sun at Greenwich)

where the 12 hour offset makes UT = 0 hours at midnight. At 24 hours, UT is increased by one day.

- Coordinated Universal Time (UTC) is UT corrected for variations in mean solar time: UTC $\cong$ UT

Standard Time is used in daily life and is an approximate mean hour angle of the Sun plus 12 hours, which is based on the division of Earth into time zones of approximately 15° longitude in each of which a common standard time is used.
e.g. Eastern Standard Time (EST) = UT – 5 hours
Eastern Daylight Savings Time (EDT) = EST + 1 hour

Sidereal Time – is based on the observed daily motion of the stars relative to Earth, and thus depends only on the rotational of Earth, making it more constant than solar time.
- A sidereal day is length of time between two consecutive transits of some star across a particular meridian.
- Greenwich sidereal time is the sidereal hour angle of the vernal equinox from the Greenwich meridian.
- A sidereal year is the period of the Earth's orbital motion relative to the stars.

See figure ("96") - solar vs. sidereal day

Because the vernal equinox is used to define some orbital elements, a tropical year is defined as the length of time between one vernal equinox and the next.

In order to measure time consistently, a constant time unit must be defined. Because the rotation and orbit of the Earth change with time, some single celestial event is needed. The ephemeris second is thus defined as 1/31,556,925.9747 of the tropical year 1900. The mean solar day of 1900 is then 86,400 ephemeris seconds. When time is given in seconds, it generally refers to ephemeris seconds.

EVENT	**1980**	**1900**
Mean solar day	86,400.0012 sec	86,400 sec
Sidereal day	86,164.0918 sec	86,164.09055 sec
Sidereal year	365.25636051 days	365.25636042 days
Tropical year	365.24219388 days	365.24219878 days

Atomic Time
- Atomic clocks provide an accurate and repeatable measurement of time. A standard SI second is now defined as 9,192,631,770 periods of the radiation emitted from the transition of the outer electron of cesium 133.
- The length of the SI second is chosen to be exactly equal to an ephemeris second; thus the ephemeris second can be considered as the basic time unit, with atomic clocks providing a means of monitoring the passage of time.

Julian Dates
- A Julian date (usually in seconds) is the time elapsed in mean solar days since noon at Greenwich (12:00 UT) on January 1, 4713 BC. Tables of Julian dates for the start of each year are available, and sidereal and solar time can be related to Julian dates.

(3) Polar Orbits

Orbits whose inclination is close to 90° (>60°) are called polar orbits.

Satellites in polar orbits can view polar or near polar regions, and are ideal for near global coverage because the Earth rotates beneath the satellite as it moves between the polar regions. Note: The highest latitude reached by the subsatellite point is i (or 180° - i for retrograde orbits).

See figure (Stephens 1.13a) - a polar orbit

Polar orbits are often, but not necessarily, sun-synchronous.
However, sun-synchronous orbits with their high inclination are always polar orbits.

The choice of altitude for a polar orbit is determined by several factors. A lower altitude orbit results in:

- a shorter orbital period
- poorer coverage of the surface
- stronger signal returns.
- better spatial resolution
- greater drag and a shorter lifetime

Typically, a polar-orbiting satellite has an altitude of <2000 km and a period of <2 hours.
e.g. NOAA weather satellites (850 km, i = 99°), NASA's Nimbus 7 (955 km, i = 99.5°), Landsat, SPOT

Satellite coverage depends on both the altitude and the field-of-view of the sensor.

- Landsat requires 16 days to provide full global coverage as it carries instruments having a narrow field-of-view
- meteorological satellites provide full coverage twice a day (wide field-of-view)

(4) Equatorial and Tropical Orbits

Orbits whose inclination is closer to 0° are called tropical orbits.
These provide close coverage of the tropical regions.

e.g. TRMM - Tropical Rainfall Measurement Mission (USA, Japan) launched November 1997, intended to define tropical rain cells by dense sampling (high spatial resolution): altitude = 325 km, i = 35° (nominal 3 year life despite low altitude).

(5) Geosynchronous and Geostationary Orbits

A geosynchronous orbit is one in which the satellite orbits at the same angular velocity as the Earth. Note: geosynchronous ≠ geostationary

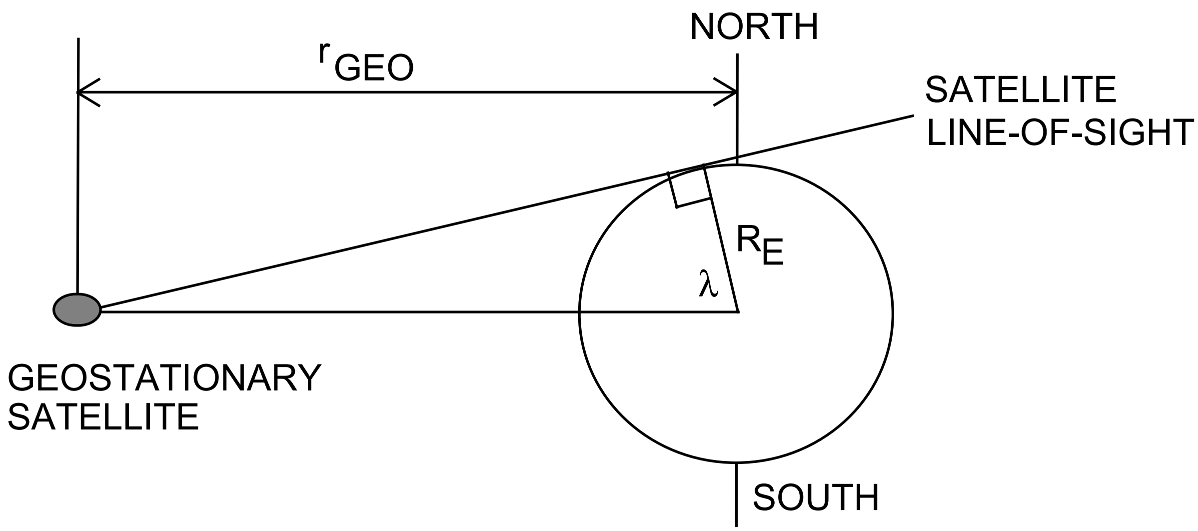
T = length of sidereal day = 86,164.1 s = 23 hours 56 minutes 4.1 seconds

$$r = \sqrt[3]{Gm_E\left(\frac{T}{2\pi}\right)^2} = 42{,}168 \text{ km}, \text{ taking correction terms into account}$$

A geostationary orbit is geosynchronous with zero inclination angle and zero eccentricity (semimajor axis =42,168 km).
$\rightarrow$ the satellite remains fixed above a point on the equator
Geostationary satellite orbits are classified by the longitude of their subsatellite point on the equator.

Because Earth rotates at a constant angular rate, a geostationary satellite must move at a constant speed in its orbit $\therefore$ a geostationary orbit must be circular

Note:
- for a circular orbit, a=r, ε=0, M=e=θ, ρ is undefined (no perigee), i, Ω, t are unchanged
- for an orbit in the equatorial plane, i=0, Ω and ρ are undefined (no line of nodes)



How high (in latitude) can a geostationary satellite view?
The maximum northern latitude is given by

$$\cos\lambda_{max} = \frac{R_E}{r_{GEO}} = \frac{6378}{42{,}164}, \text{ so } \lambda_{max} = 81.3^\circ.$$

Advantages of geostationary orbit for remote sensing:
- almost all of a hemisphere can be viewed simultaneously,
- $\therefore$ the time evolution of phenomena can be observed

Disadvantages of geostationary orbit for remote sensing:
- accurate measurements are difficult because the satellite is so far from Earth
- the polar regions are only observed at an oblique angle (good coverage only up to ~60° latitude)

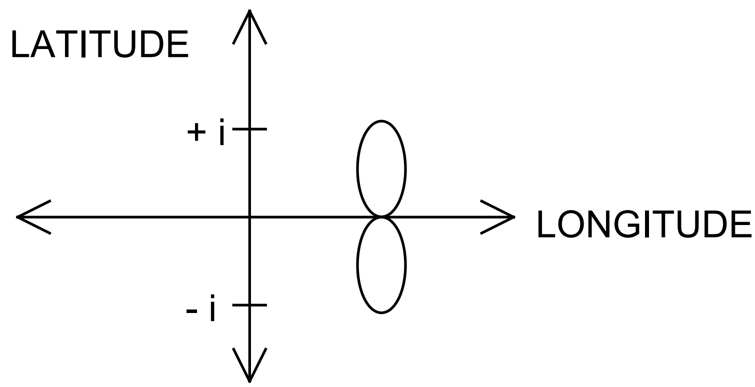
e.g. Meteosat (Europe), GOES (US), INSAT (India) - all are meteorological satellites that can observe the development and movement of storms, fronts, clouds, etc.

See figures (Stephens 1.13c and three from UCAR web site) - geostationary orbits

It would be useful to be able to put a satellite in an orbit that is fixed over any point (not just a point on the equator). This is impossible, but geosynchronous orbits provide one approximation: nodal period = Earth's rotational period, but inclination $\neq 0°$.

The sub-satellite point then traces a figure of eight that crosses a fixed point on the equator and reaches a maximum latitude of $\pm$ i.

This is less useful than a geostationary orbit because the satellite only sees one hemisphere for each half of its orbit.



(6) Molniya Orbits

A <u>Molniya orbit</u> is an elongated elliptical orbit at an inclination of 63.4° for which the argument of perigee is fixed. Because ω is fixed, the apogee stays at a given latitude.

These are used for communications satellites by the former Soviet Union because geostationary satellites provide poor coverage of the high latitudes of the FSU. They have also been suggested for meteorological observations at high latitudes.

The apogee (and slowest speed) is over the FSU, and the perigee (and fastest speed) is over the opposite side of the globe, so that the satellite spends most of its time over the FSU. For about 8 hours, centred on the apogee, the satellite is nearly stationary with respect to the Earth's surface, i.e., it behaves as a high-latitude, part-time nearly geostationary satellite.

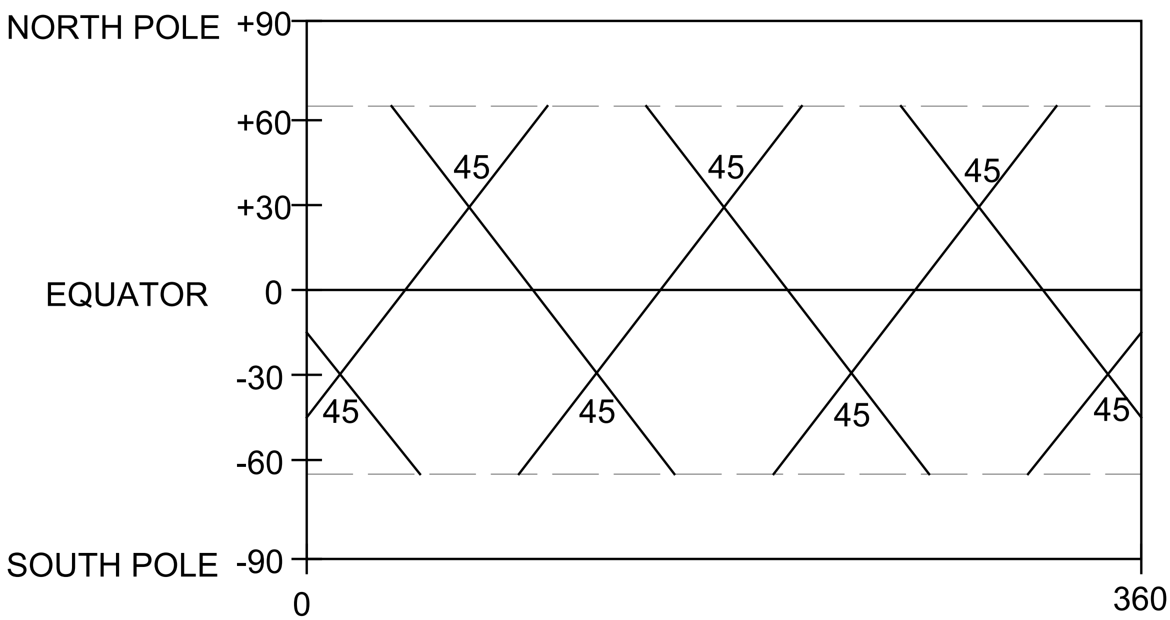
e.g., semimajor axis = 26,554 km
eccentricity = 0.72
perigee = 7378 km (altitude = 1000 km wrt equatorial radius)
apogee = 45,730 km (altitude = 39,352 km wrt equatorial radius)
period = 717.8 minutes = 11.96 hours

(7) Some Specialized Orbits

GEO - geostationary orbit
LEO - low Earth orbit (includes most non GEOs such as polar and tropical orbits)
GEO or sun-synchronous LEO are not always required for Earth observation satellites, especially when different Sun conditions are needed.

Example 1: TOPEX/POSEIDON (USA, France, launched in August 1992)

- designed to measure sea surface height to 13 cm accuracy
- because the Sun is a major driver of tides, a sun-synchronous orbit would cause the Sun's tidal effect to be measured as a constant sea surface elevation (a false signal) ∴ it was essential that the orbit NOT be sun-synchronous
- wanted evenly spaced grid of tracks over the ocean
- wanted satellite tracks that cross at 45° so that the slope of the sea surface could be measured in the East-West and North-South directions (so polar and tropical orbits were unsuitable)
- also wanted to observe to high latitudes
- resulting orbit has altitude = 1334 km, inclination = 66°, which provides crossing angles of 45° for the ascending and descending orbits at 30° latitude



Example 2: ERBS - Earth Radiation Budget Satellite

- designed to measure incoming and outgoing radiation from Earth
- also uses a non-sun-synchronous orbit
- orbits at 600 km with i = 57°
- precesses wrt Sun in order to sample all local times at a location over each month

Example 3: satellites that measure the Earth's gravity

- Earth's gravity depends only on its internal structure, so a sun-synchronous orbit is not necessary
- preferable to have the satellite as close to Earth as possible in order to detect small changes in the gravity field
- optimal orbit is ~160 km with i = 90° (about as low an altitude as possible without excessive drag and risk of burning up)

2.4 Launching, Positioning, Tracking, Navigation

Satellite Launching

For greater detail, see Kidder & Vonder Haar, Section 2.5.

To place a satellite in a stable orbit, the Earth's gravitational attraction and the atmospheric resistance must be overcome.

This is achieved with a rocket, a vehicle that carries all its own fuel and derives forward thrust from the backward expulsion of the combustion products.

Rocket equation (ignoring gravity and friction):

$$\Delta V = U \ln\left(\frac{M_i}{M_f}\right)$$

where
ΔV = change in velocity of the rocket
U = velocity of exhaust gases relative to rocket
M_i = initial mass of rocket and fuel
M_f = final mass of rocket

For a satellite in LEO, $\Delta V \approx 7$ km/s, while $U \approx 2.4$ km /s, typically.
$\therefore$ $\Delta M = 0.95\ M_i$, or the fuel should be 95% of the initial mass.

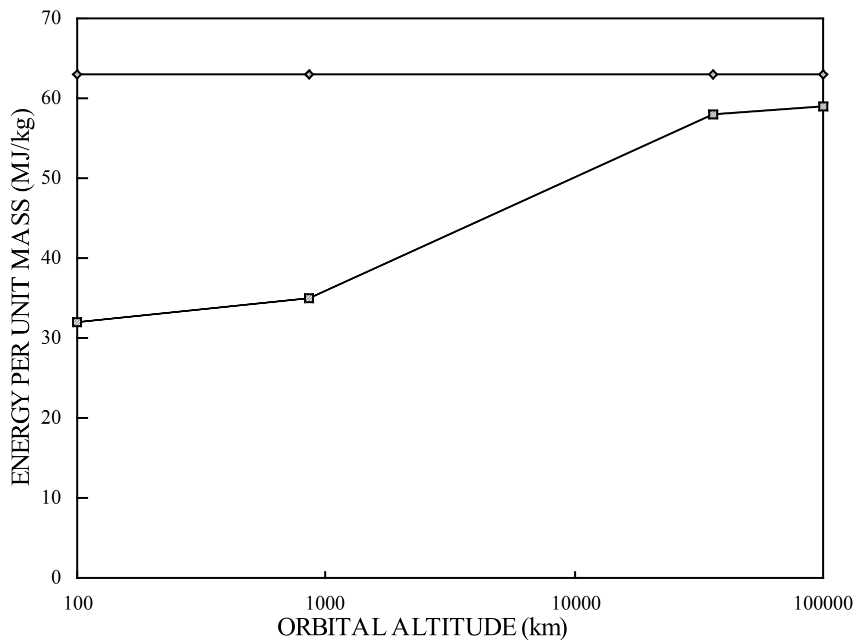
Taking gravity into account, this increases to 97%, so only 3% of the total mass is available for the rocket and satellite payload. Therefore, single stage rockets can only put small masses into orbit.

Three or four stage rockets can put
- several tons into LEO
- smaller payloads into GEO

Examples:
- Saturn 5 rocket – 100 tons into LEO
- Space Shuttle – 30 tons into 400 km orbit, 6 tons into GEO
- Ariane 5 rocket – 6800 kg into GEO

Launch into GEO requires more energy than a launch into LEO, but calculation of the energy per unit mass required to place a satellite in orbit as a function of orbital altitude shows that the first step into space is the most energy-consuming stage.

e.g., ~35 MJ/kg are needed to reach 850 km
~23 MJ/kg more are needed to reach GEO (42 times further from the surface)



Three methods for orbit insertion:

(1) "power-all-the-way" ascent – rocket burns until orbit is reached

- more costly but less risky (no restart)
- used for manned flights

(2) ballistic ascent – a large first stage rocket propels the payload to high velocity, then it coasts to the location of the desired orbit, where a second stage rocket is fired to adjust the trajectory as needed

(3) elliptical ascent

i) payload is placed in LEO "parking orbit" by method (1) or (2)
ii) a rocket is fired to move the payload into an elliptical transfer orbit whose perigee is the parking orbit and whose apogee is at the desired orbit altitude
iii) when apogee is reached, another rocket is used to modify the orbit to the desired shape

- used for geostationary satellites

Launch locations are limited by the fact that satellites are usually launched in the plane of the orbit, to reduce the fuel requirements.
∴ In the USA, Florida is not useful for polar orbits because of the populated areas to the north of the launch site.

Launch costs are also reduced by launching in the direction of Earth's rotation, hence from Florida and from Kourou out over the Atlantic Ocean.

Once a satellite has been launched into orbit, need to to able to: (1) determine its position in space, (2) track it from Earth, and (3) know where its instruments are pointing.

Satellite Positioning

Satellite positioning involves locating the position of a satellite in its orbit.

If the orbital elements a, ε, i, M(t), Ω(t), and ω(t) are known, then the following can be calculated:

- location of the satellite in the plane of its orbit, i.e., θ(t), r(t)
- position (x,y,z) in the Ω-δ co-ordinate system
- then position (r,δ,Ω) of the satellite
- finally the latitude and longitude of the subsatellite point

subsatellite point = the point on the Earth's surface directly between the satellite and the centre of the Earth

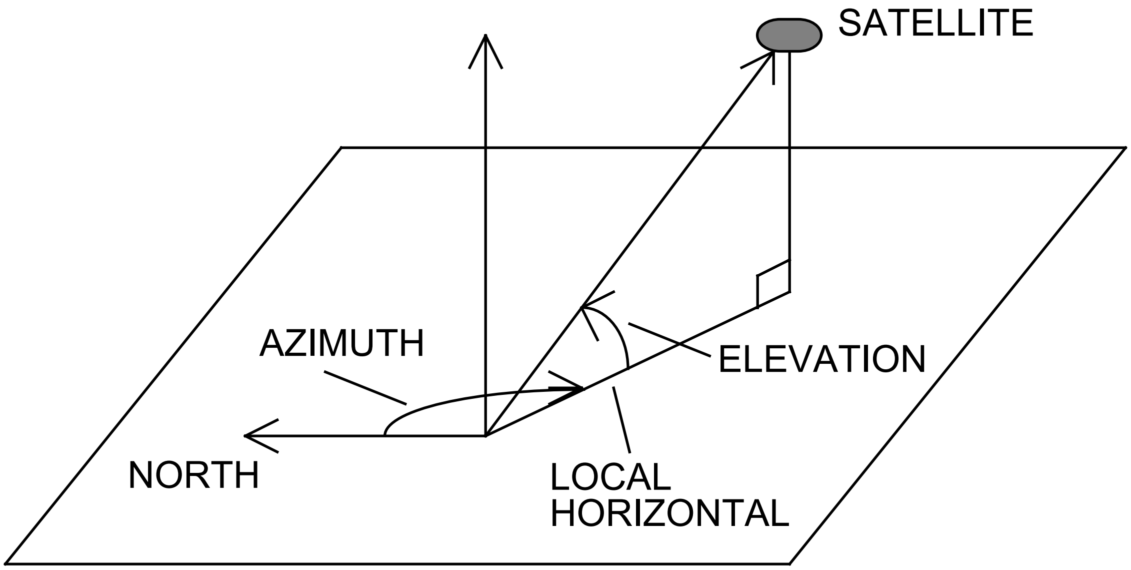
ground track = the path of the subsatellite point on the Earth's surface

The ephemeris of a satellite is a list of its position versus time, i.e., latitude, longitude, altitude versus time.

Satellite Tracking

Satellite tracking involves pointing an antenna (located on the ground) at a satellite and following its position in its orbit.

Given the ephemeris data and the location (latitude, longitude, altitude) of the antenna, it is possible to calculate the elevation angle (measured from the local horizontal) and the azimuth angle (measured clockwise from North) of the satellite relative to the antenna. Knowledge of these angles allow the antenna to track the satellite.
In practice, calculated and observed positions of the satellite are compared to improve the knowledge of the forces acting on the satellite. Errors range from 10 cm to 1-2 m.



Satellite Navigation (or Pointing)

Satellite navigation (or pointing) involves determining the Earth co-ordinates (latitude, longitude) of the area viewed by a satellite instrument.

Note: Satellite images are usually obtained by scanning instruments. Data are in <u>scan lines</u>, each composed of <u>pixels</u> or <u>scan spots</u>. Satellite navigation provides the latitude and longitude of each pixel.

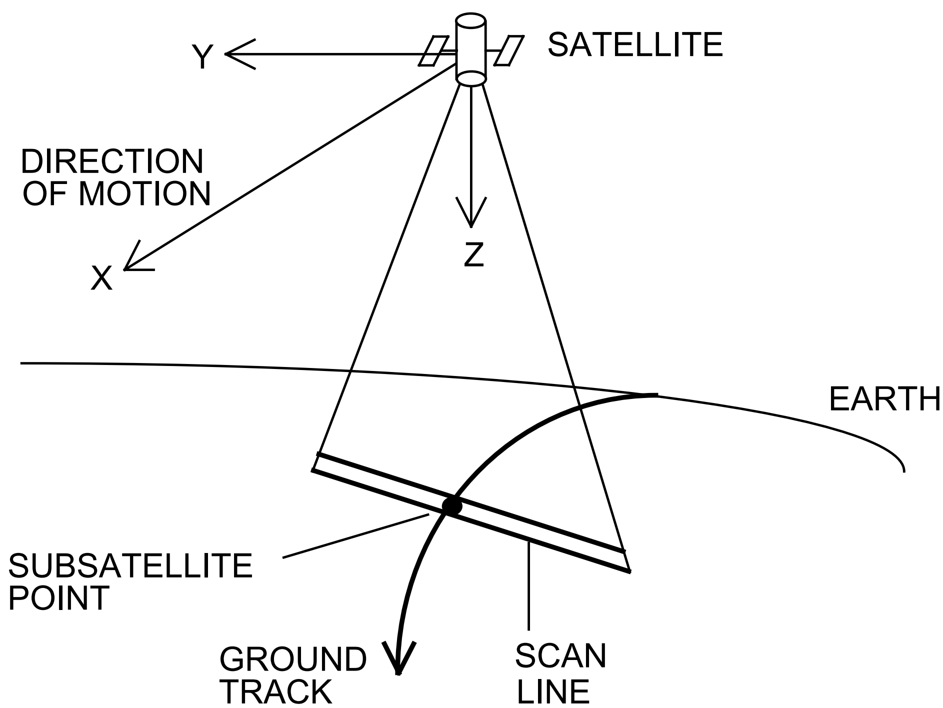
This is a complex geometry problem that requires knowledge of
- where the satellite is in its orbit
- the orientation (or <u>attitude</u>) of the satellite
- the scanning geometry of the instrument

See Kidder and Vonder Haar, Section 2.5.3 for more details of this calculation.
See figure (K&VH 2.9) - ground track of a typical sun-synchronous satellite

2.5 Observational Geometries

The observational geometry is related to satellite navigation.

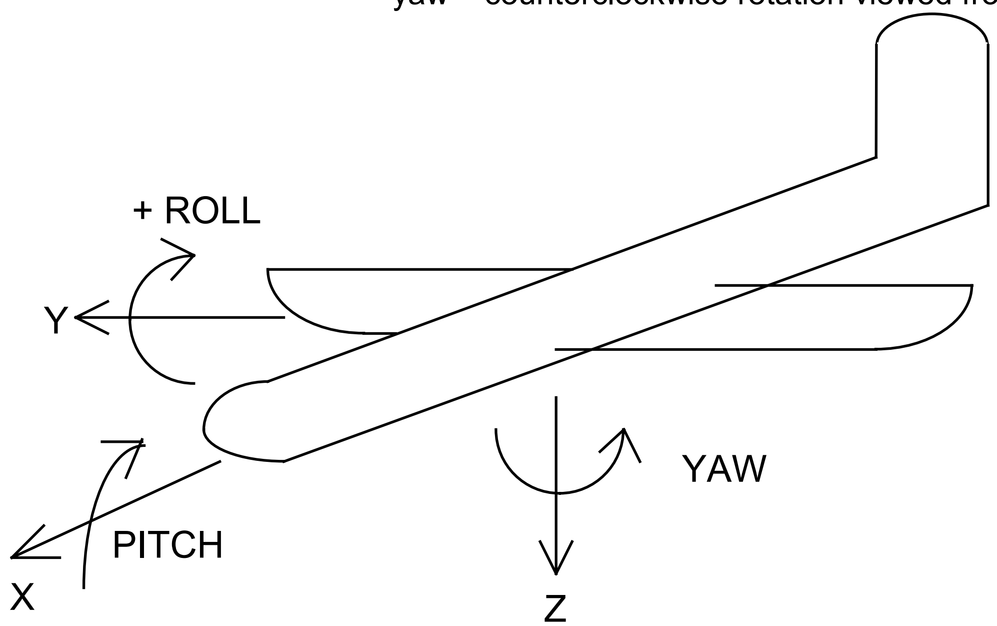
First, need to define a co-ordinate system for the satellite attitude.
- z axis points from the satellite to the centre of the Earth
- x axis points in the direction of the satellite motion (in its orbit)
- y axis makes a right-handed system



Three angles define the satellite orientation in this system:
<u>roll</u> = rotation about the x axis
<u>pitch</u> = rotation about the y axis
<u>yaw</u> = rotation about the z axis

A combination of these three angles can be used to describe nearly any scan geometry (usually applied to LEO rather than GEO satellites).

e.g. airplane – for positive angles: roll – right wing points upwards
pitch – nose points upwards
yaw – counterclockwise rotation viewed from above



Examples of satellite viewing geometries and scanning patterns:
Refer to figure – "scanning systems for acquiring remote sensing images".
Note: these use scanners and detector arrays rather than just satellite motion.

(1) simple nadir viewing
- no scanning
- looks vertically downwards
- limited coverage
- good horizontal resolution

(2) cross track scanning
- simple scanning, achieved by changing the roll or using a scanning mirror
- rotate through pixels

(3) circular scanning
- achieved by changing the yaw or using a scanning mirror
- sweeps out an arc

(4) along track scanning (or line imaging)
- achieved by changing the pitch
- or using a 2-D detector array, with the forward motion of the satellite giving the second dimension

(5) side scanning
- achieved by changing the roll about a non-zero angle or using a scanning mirror
- used in radar observations with an antenna on satellite
- loses horizontal resolution

(6) limb scanning
- used in atmospheric remote sounding
- worst horizontal resolution, but good vertical resolution

Space-Time Sampling

This is determined by both the satellite orbit and the instrument scan pattern.

See figures (three from UCAR web site) - orbital tracks and coverage of North polar region

GEO satellites
- stationary over a point on the equator
- can view ~42% of the globe (fixed area) all the time
- so instruments can view a point at any local time, but at only one elevation angle and one azimuth angle

LEO satellites
- sampling is highly dependent on the orbit

e.g., for meteorological LEO satellites
- area viewed on one orbit overlaps area viewed on the previous and succeeding orbits
- usually view every point on the Earth twice a day
- view each point a small number of local times but at varying elevation and azimuth angles

Sun-synchronous satellites
- repeat the ground track if they make an integral number of orbits in an integral number of days

e.g., Landsat 1, 2, 3 have nodal period $\tilde{T}$ = 103.27 min

$\therefore$ number of orbits/day = 1440 min/day / 103.27 min/orbit = 13.94403 orbits/day
= 13 and 17/18 orbits/day = 251/18 orbits/day

The orbit track repeats every 18 days, with 251 orbits made during this time.

i.e., $T_{repeat} = n_1 T_{orbit} = n_2 T_{day}$ where n_1 and n_2 are integers

Alternatively, if a sun-synchronous satellite makes N + k/m orbits per day (N, k, m integers), then the orbit track repeats every m days after making mN + k orbits.