PHY 305F – ELECTRONICS LABORATORY I
Fall Semester 2003

EXPERIMENT 2
RESONANT SYSTEMS AND THE "Q" FACTOR
Lab notebook is due at 1 PM in MP238 on October 6

LEARNING OBJECTIVES:

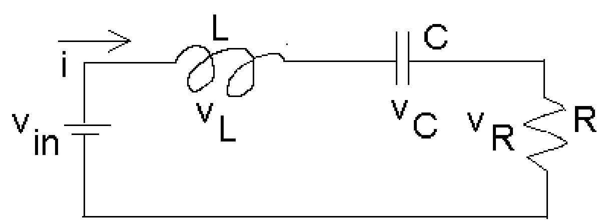
(1) To become familiar with resonant systems, particularly electrical ones.

(2) To understand what the quality factor, Q, is.

INTRODUCTION:

Many resonant systems result from dynamical equations which are linear and second order. In the electrical case of the inductor (L), resistor (R), and capacitor (C) in series as shown in the figure below, the current (i) and the input voltage (v_{in}) are related by the differential equation:

$$v_{in} = QR\left(\frac{1}{\omega_o}\frac{di}{dt} + \frac{1}{Q}i + \omega_o \int i\,dt\right) \quad \text{where } \omega_o = \frac{1}{\sqrt{LC}} \quad \text{and} \quad Q = \frac{1}{R}\sqrt{\frac{L}{C}}.$$



If v_{in} is sinusoidal, then the relation between v_{in} and i is given by $v_{in} = R\left[1 + jQ\left(\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega}\right)\right]i$

and i / v_{in} as a function of ω displays resonant behaviour with a central frequency ω_o and resonance width $\Delta\omega = \omega_o / Q$. This is the steady state solution to the resonance equation.

If, on the other hand, v_{in} is a constant which is "provoked" by a transient change (as in the case of a square wave), then there are three solutions:

For $Q > \frac{1}{2}$, $i = A\sin(\omega t + \phi)e^{-at}$ $\quad \omega = \omega_o\sqrt{1 - \frac{1}{4Q^2}}$ $\quad a = \frac{\omega_o}{2Q}$ (underdamped)

For $Q = \frac{1}{2}$, $i = (A + Bt)e^{-at}$ $\quad a = \frac{\omega_o}{2Q}$ (critically damped)

For $Q < \frac{1}{2}$, $i = \left[Ae^{+bt} + Be^{-bt}\right]e^{-at}$ $\quad a = \frac{\omega_o}{2Q}$ $\quad b = \omega_o\sqrt{\frac{1}{4Q^2} - 1}$ (overdamped)

WHAT TO DO:

(1) Using the inductance meter available, find the value of inductance (L_s) for the inductor that you are provided with. Also find the equivalent AC series resistance (R_s) of that inductor. Measure the resistance of the inductor with an ohmmeter. Are the two values the same? Why or why not?

(2) Choose a capacitor such that, in series with the inductor, it will provide a value of ω_o around $2\pi \times 5$ kHz.

(3) Connect a series LCR circuit to the signal generator and connect the CRO so that you can measure the applied voltage v_{in} simultaneously with the voltage across the resistor v_R. Note that v_R gives you a measure of the current in the circuit, i, because $i = v_R / R$.

(4) Measure and plot (as a function of frequency) the ratio of v_R/v_{in} for sine wave AC, for R set to give two values of Q, one around 7 (or as high a value as you can achieve) and the other around 2. Does the value of Q that you calculate from your resonance curves match your calculated value from the known values of your components?

(5) Observe the curves on the CRO of either v_R or v_C with a square wave of sufficiently low frequency applied to the input. Choose values of R to produce both the underdamped case and overdamped case. Do the values of Q that you calculate from your CRO traces match your calculated values from the known values of your components?