PHY 140Y – FOUNDATIONS OF PHYSICS
2001-2002
Tutorial Questions #6 – Solutions
October 22/23

More on Newton's Laws of Motion without Friction

1. What downward force is exerted on the air by the blades of a 4300-kg helicopter when it is
   (a) hovering at constant altitude;
   (b) dropping at 21 m/s with speed decreasing at 3.2 m/s^2;
   (c) rising at 17 m/s with speed increasing at 3.2 m/s^2;
   (d) rising at a steady 15 m/s;
   (e) rising at 15 m/s with speed decreasing at 3.2 m/s^2?

Solution:

From Newton's third law, the downward force exerted on the air by the helicopter is equal and opposite to the upward force on the helicopter (the engine's thrust). If we neglect air resistance, the thrust and gravity are the only vertical forces acting, so Newton's second law for the helicopter (positive component up) is $F_{th} - mg = ma$ or $F_{th} = m(g + a)$.

(a) Hovering means a=0 (also v=0, but v doesn't enter the equation of motion if air resistance is neglected), so $F_{th} = mg = (4300\text{kg})(9.81\text{m/s}^2) = 42\text{ kN}$

(b) If v is decreasing downward, then a=3.2 m/s^2 upward, and so

$$F_{th} = m(g + a) = (4300\text{kg})(9.81 + 3.2)\text{m/s}^2 = 56\text{ kN}$$

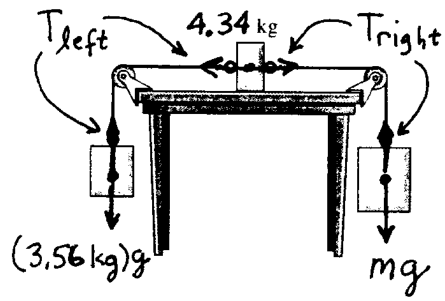
(c) Acceleration a is the same as in part (b), and so is $F_{th} = 56\text{ kN}$.

(d) If v = constant, a=0, and $F_{th} = 42\text{ kN}$ is the same as hovering.

(e) If v is decreasing upward, then a is downward, and so

$$F_{th} = m(g - a) = (4300\text{kg})(9.81 - 3.2)\text{m/s}^2 = 28\text{ kN}$$

2. In a setup like that shown in in the figure below, but with different masses, a 4.34-kg block starts from rest on the left edge of a frictionless tabletop 1.25 m wide. It accelerates to the right, and reaches the right edge in 2.84 s. If the mass of the block hanging from the left side is 3.56 kg, what is the mass hanging from the right side?



Solution:

The acceleration of the 4.34-kg block sliding horizontally to the right across the frictionless tabletop is a = $2(1.25\text{m})/(2.84)^2$=0.310 m/s^2 (from Textbook Equation 2-10 and the given conditions). This is also the magnitude of the accelerations of the other two blocks.

Since we are not interested in the tension in either string, we may use a shortcut to find the unknown mass. The net force on all three masses in the direction of motion is the difference in the right- and left-hand weights, $F_{net} = (m - 3.56\text{kg})g$, which equals the total mass times the acceleration, $(m + 3.56\text{kg} + 4.34\text{kg})a$.

Solving for m, we find

$$m = \frac{(3.56\ \text{kg} + 4.34\ \text{kg})(0.310\ \text{m/s}^2) + (3.56\ \text{kg})(9.81\ \text{m/s}^2)}{(9.81\ \text{m/s}^2 - 0.310\ \text{m/s}^2)} = 3.93\,\text{kg}$$

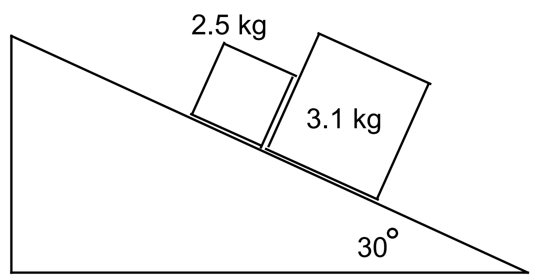
(The shortcut can be justified by adding the component of the equations of motion of the three blocks in the direction of motion:

$T_{left} - (3.56\text{kg})g = (3.56\text{kg})a$, $T_{right} - T_{left} = (4.34\text{kg})a$, and $mg - T_{right} = ma$.

In this setup, the ropes are massless, and the pulleys are massless and frictionless, so the tension in each rope is constant.)

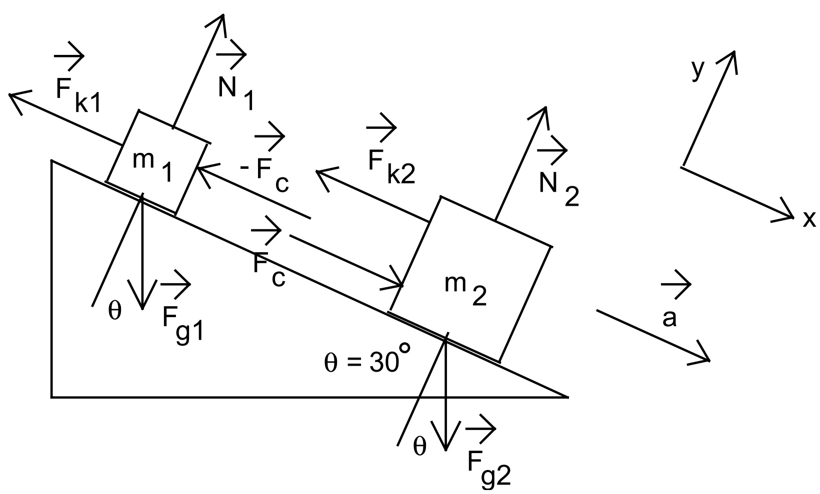
Applying Newton's Laws of Motion with Friction

3. A 2.5-kg and a 3.1-kg block slide down a 30° incline as shown below. The coefficient of kinetic friction between the 2.5-kg block and the slope is 0.23, and the coefficient of kinetic friction between the 3.1-kg block and the slope is 0.51. Determine
(a) the acceleration of the pair, and
(b) the force that the lighter block exerts on the heavier block.



Solution:

First draw the free-body diagram for each block. Choose the coordinate system (as shown).



Because $\mu_{k2} > \mu_{k1}$, there will be a contact force between the two blocks, that will have magnitude F_c and will be in opposite directions for each block.

Apply Newton's Second Law to each block:

$$\vec{F}_{net,1} = \vec{F}_{g1} + \vec{N}_1 + \vec{F}_{k1} - \vec{F}_c = m_1\vec{a}_1$$
$$\vec{F}_{net,2} = \vec{F}_{g2} + \vec{N}_2 + \vec{F}_{k2} + \vec{F}_c = m_2\vec{a}_2$$

Separate into the x and y components. Note that both blocks will have the same acceleration, which is only in the x direction.

Block 1:
$$x: \quad m_1 g \sin\theta - \mu_{k1} N_1 - F_c = m_1 a \quad (1)$$
$$y: \quad -m_1 g \cos\theta + N_1 = 0 \quad (2)$$

Block 2:
$$x: \quad m_2 g \sin\theta - \mu_{k2} N_2 + F_c = m_2 a \quad (3)$$
$$y: \quad -m_2 g \cos\theta + N_2 = 0 \quad (4)$$

(a) To find the acceleration, a, add equations (1) and (3), and then substitute in values for N_1 and N_2 from equations (2) and (4).

$$(1)+(3): \quad (m_1+m_2)g\sin\theta - \mu_{k1}N_1 - \mu_{k2}N_2 = (m_1+m_2)a$$
$$\text{use} \quad (2),(4): \quad (m_1+m_2)a = (m_1+m_2)g\sin\theta - \mu_{k1}(m_1 g\cos\theta) - \mu_{k2}(m_2 g\cos\theta)$$
$$a = \frac{(m_1+m_2)g\sin\theta - (\mu_{k1}m_1 + \mu_{k2}m_2)g\cos\theta}{m_1+m_2}$$
$$= \ldots = 1.6\,\text{m/s}^2$$

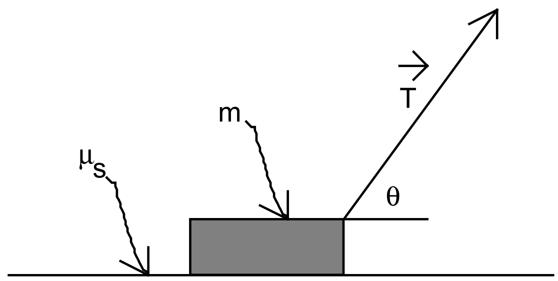
(b) To find the contact force (which is the same on both blocks), rearrange equations (1) and (3).

(1): $g\sin\theta - \dfrac{\mu_{k1}N_1}{m_1} - \dfrac{F_c}{m_1} = a$ and (3): $g\sin\theta - \dfrac{\mu_{k2}N_2}{m_2} + \dfrac{F_c}{m_2} = a$

Subtract equation (1) from equation (3), and then substitute in values for N_1 and N_2 from equations (2) and (4).

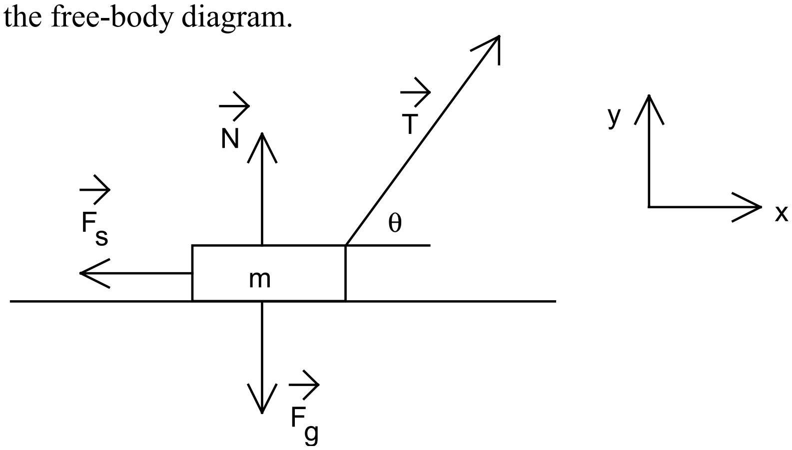
$$\frac{F_c}{m_2} + \frac{F_c}{m_1} - \frac{\mu_{k2}N_2}{m_2} + \frac{\mu_{k1}N_1}{m_1} = 0$$
$$m_1F_c + m_2F_c - m_1\mu_{k2}N_2 + m_2\mu_{k1}N_1 = 0$$
$$F_c = \frac{m_1\mu_{k2}N_2 - m_2\mu_{k1}N_1}{m_1+m_2}$$
$$= \frac{m_1\mu_{k2}(m_2 g\cos\theta) - m_2\mu_{k1}(m_1 g\cos\theta)}{m_1+m_2}$$
$$= \frac{m_1 m_2 g\cos\theta(\mu_{k2} - \mu_{k1})}{m_1+m_2}$$
$$= \ldots = 3.3\,\text{N}$$

4. A box of mass m sits on a rough horizontal surface. The coefficient of static friction is μ_s. The box is pulled by a massless rope as shown below.
(a) What is the magnitude of the force T so that the box will just start moving horizontally?
(b) At what angle θ_{min} is the force required to move the box a minimum value?
(c) Why, physically, is there a minimum value?



Solution:

Again, start with the free-body diagram.



Apply Newton's Second Law: $\vec{F}_{net} = \vec{F}_g + \vec{N} + \vec{T} + \vec{F}_s = m\vec{a}$

where

F_s is the force of static friction, and $F_s = \mu_s N$ as the box starts to move.

Separate this into the r and y components.

x direction: $F_{net,x} = T\cos\theta - F_s = ma_x = 0$ as the box just begins to move

$$T\cos\theta - \mu_s N = 0 \quad (1)$$

y direction: $F_{net,y} = -mg + N + T\sin\theta = ma_y = 0$

$$N = mg - T\sin\theta \quad (2)$$

(a) Substitute equation (2) into equation (1):

$$T\cos\theta - \mu_s(mg - T\sin\theta) = 0$$

$$T = \frac{\mu_s mg}{\cos\theta + \mu_s\sin\theta}$$

This is the force that must be applied such that the box will just start to move.

(b) In order to find the angle for which the force is a minimum, take the derivative of T with respect to this angle, i.e., we want $\frac{dT(\theta)}{d\theta} = 0$ for $\theta = \theta_{min}$.

$$\frac{dT(\theta)}{d\theta} = \frac{d}{d\theta}\left(\frac{\mu_s mg}{\cos\theta + \mu_s\sin\theta}\right)$$

$$= -\frac{\mu_s mg(\sin\theta - \mu_s\cos\theta)}{(\cos\theta + \mu_s\sin\theta)^2} = 0$$

Thus:

$$\sin\theta_{min} = \mu_s\cos\theta_{min}$$
$$\tan\theta_{min} = \mu_s$$
$$\theta_{min} = \tan^{-1}(\mu_s)$$

(c) As θ decreases ($\theta < \theta_{min}$), i.e., as $\theta \to 0°$, then $\begin{array}{l}\sin\theta \to 0 \\ N = mg - T\sin\theta \to mg\end{array}$.

This means that N increases to its maximum possible value, and so $T\cos\theta = \mu_s N$, and hence the tension force, must also increase.

As θ increases ($\theta > \theta_{min}$), i.e., as $\theta \to 90°$, then $\cos\theta \to 0$ and so $T = \mu_s N / \cos\theta$ must increase to offset the decrease in $\cos\theta$.

The tension force, T, is doing two things:

- pulling the box off the ground, which reduces the normal force N, and hence reduces the friction force
  ∴ need less force to pull the box sideways, as there is less friction
- pulling the box sideways.

This is why it is easier to pull an object (e.g., a cart or trolley) than to push it! Pulling decreases the friction by reducing the normal force. Pushing increases friction by increasing the normal force.