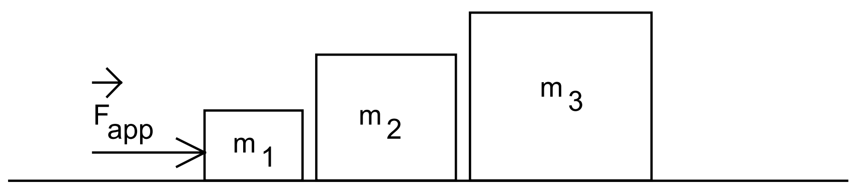
PHY 140Y – FOUNDATIONS OF PHYSICS
2001-2002
Tutorial Questions #5 – Solutions
October 15/16

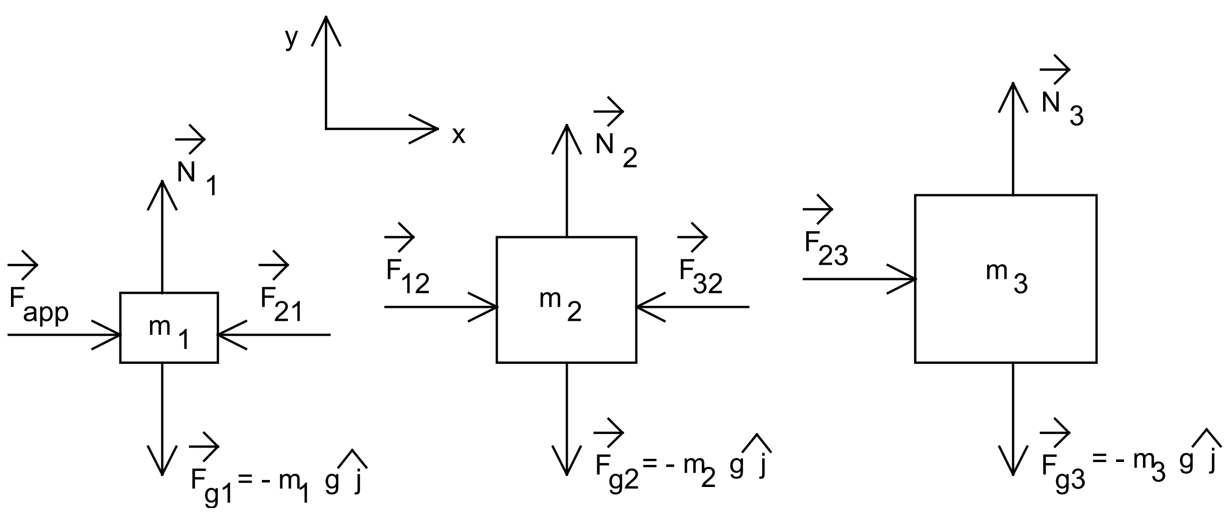
Applying Newton's Laws of Motion

1. Blocks of 1.0, 2.0, and 3.0 kg are lined up from left to right in that order on a table so that each block is touching the next one. A rightward-pointing 12-N force is applied to the leftmost block, as shown below. What is the acceleration of the blocks and what force does the middle block exert on the rightmost one?



Answer:

First draw the free-body diagram for each block. Choose the coordinate system (as shown).
Define $\vec{F}_{ij}$ as the force that block i exerts on block j.



Next, apply Newton's Second Law. We only need to consider the horizontal component as there is no motion in the vertical direction. So, applying Newton's Second Law in the x direction gives:

block 1: $F_{app} + F_{21} = m_1 a_1$ (1)

block 2: $F_{12} + F_{32} = m_2 a_2$ (2)

block 3: $F_{23} = m_3 a_3$ (3)

We appear to have three equations for seven unknowns. However, we can relate some of these unknowns. All three blocks move with the same acceleration, so $a_1 = a_2 = a_3 = a$, which is in the +x direction (positive to the right).

Also apply Newton's Third Law: $F_{12} + F_{21} = 0$, $F_{23} + F_{32} = 0$ i.e., $F_{21} = -F_{12}$, $F_{32} = -F_{23}$

So the three equations become:

block 1: $F_{app} - F_{12} = m_1 a$ (1)

block 2: $F_{12} - F_{23} = m_2 a$ (2)

block 3: $F_{23} = m_3 a$ (3)

and we now only have three unknowns.

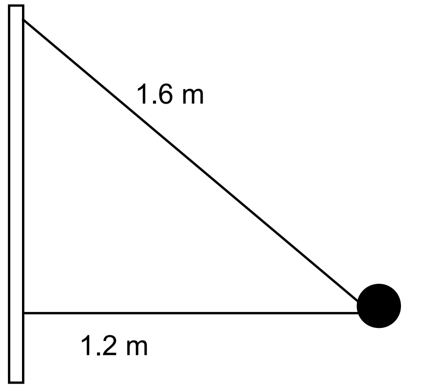
We are asked to solve for acceleration a and force F_{23}.
Substitute equation (2) and then equation (3) into equation (1):

$$F_{app} - (m_2 a + F_{23}) = m_1 a$$
$$F_{app} - (m_2 a + m_3 a) = m_1 a$$
$$(m_1 a + m_2 a + m_3 a) = F_{app}$$
$$a = \frac{F_{app}}{m_1 + m_2 + m_3} = \frac{12\text{N}}{1.0 + 2.0 + 3.0\text{kg}} = 2.0\ \text{m/s}^2$$

Then use equation (3) to solve for the force that the middle block (m_2) exerts on the rightmost one (m_3):

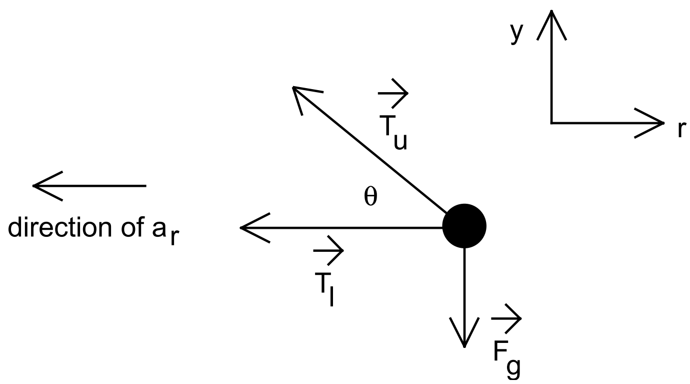
$$F_{23} = m_3 a = (3.0\text{kg}) \times (2.0\text{m/s}^2) = 6.0\ \text{N}$$

2. The figure below shows a 0.84-kg ball attached to a vertical post by strings of length 1.2 m and 1.6 m. If the ball is set whirling in a horizontal circle, find
(a) the minimum speed necessary for the lower string to be taut, and
(b) the tension in each string if the ball's speed is 5.0 m/s.



Answer:

Again, start by defining the coordinate system and drawing the free-body diagram for the ball.. Because this is a case of uniform circular motion, choose an "r-y" coordinate system.



Apply Newton's Second Law: $\vec{F}_{net} = \vec{F}_g + \vec{T}_l + \vec{T}_u = m\vec{a}$

Separate this into the r and y components.

y direction:

$$F_{net,y} = F_{g,y} + T_{l,y} + T_{u,y} = ma_y$$
$$F_{net,y} = -mg + T_u \sin\theta = 0$$
$$T_u \sin\theta = mg \qquad (1)$$

r direction:

$$F_{net,r} = F_{g,r} + T_{l,r} + T_{u,r} = ma_r$$
$$F_{net,r} = -T_l - T_u \cos\theta = -ma_r$$
$$T_l + T_u \cos\theta = m\frac{v^2}{R} \qquad (2)$$

where a_r is the radial (centripetal) acceleration and R is the length of the horizontal string (1.2 m). We can also calculate the angle θ from the lengths of the strings (valid while the strings are taut).

$$\cos\theta = \frac{1.2\text{m}}{1.6\text{m}} \quad \text{so} \quad \theta = 41.4^\circ$$

(a) The lower string will be taut if $T_l \geq 0$. To find the minimum speed that will meet this condition, eliminate T_u from equations (1) and (2).

$$T_l = m\frac{v^2}{R} - T_u \cos\theta = m\frac{v^2}{R} - \left(\frac{mg}{\sin\theta}\right)\cos\theta = m\frac{v^2}{R} - \frac{mg}{\tan\theta}$$

We want $T_l \geq 0$, so:

$$m\frac{v^2}{R} - \frac{mg}{\tan\theta} \geq 0$$

$$\frac{v^2}{R} \geq \frac{g}{\tan\theta}$$

$$v \geq \sqrt{\frac{Rg}{\tan\theta}} = \sqrt{\frac{(1.2\text{m})(9.81\text{m/s}^2)}{\tan 41.4^\circ}}$$

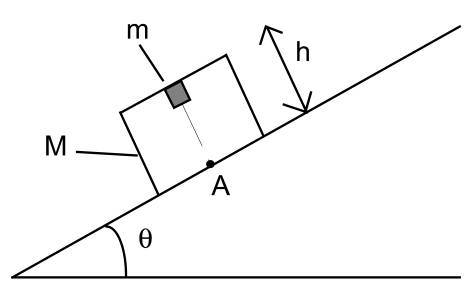
$$v \geq 3.7\,\text{m/s}$$

(b) Given that v = 5.0 m/s, we can find the two tension forces using the two equations that we have derived.

$$T_u = \frac{mg}{\sin\theta} = \frac{(0.84\text{kg}) \times (9.81\text{m/s}^2)}{\sin 41.4^\circ} = 12\,\text{N}$$ (independent of v, balances gravity)

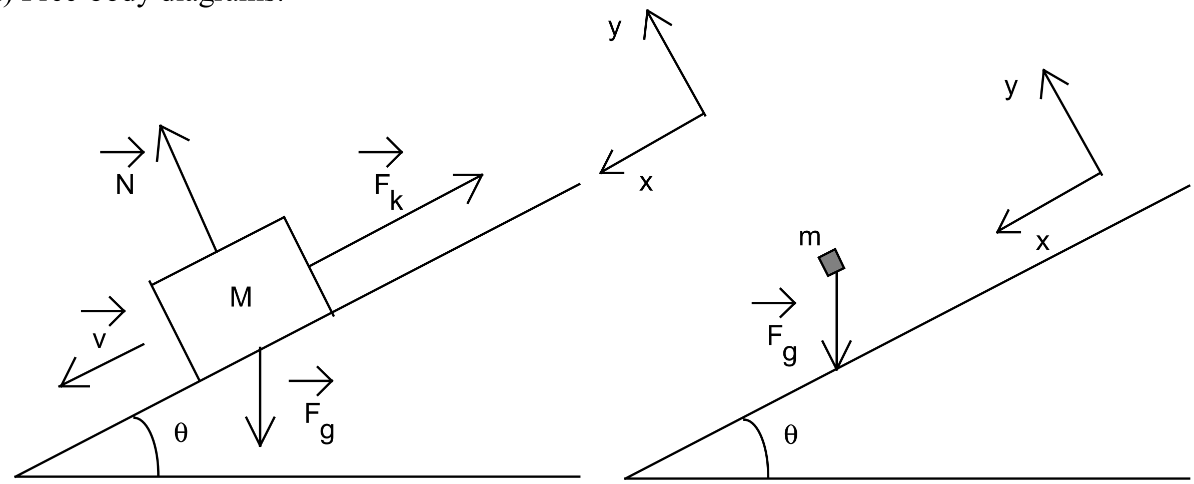
$$T_l = m\frac{v^2}{R} - \frac{mg}{\tan\theta} = \frac{(0.84\text{kg}) \times (5.0\text{m/s})^2}{1.2\text{m}} - \frac{(0.84\text{kg}) \times (9.81\text{m/s}^2)}{\tan 41.4^\circ} = 8.2\,\text{N}$$

3. A block of mass M and height h slides down a slope as shown below. The slope is inclined at angle θ from the horizontal. Connected to the top of the box is an object of mass m. The object is directly above point A on the floor of the box, as shown.
(a) Draw free-body diagrams for mass m and for mass M and identify all forces acting on each at the point when mass m has just been released from the top of the box (i.e., ignore any normal force between the two masses).
(b) If mass m drops from the top of the box, where will it fall relative to A when it reaches the floor, and how long will it take to fall, assuming that the slope is frictionless.



Answer:

(a) Free-body diagrams:



(b) Apply Newton's Second Law to each mass, ignoring friction: $\vec{F}_{net} = \vec{F}_g + \vec{N} = m\vec{a}$

mass M, x direction:
$$Mg\sin\theta = Ma_x$$
$$a_x = g\sin\theta$$

mass M, y direction:
$$N - Mg\cos\theta = Ma_y = 0$$
$$N = Mg\cos\theta$$

mass m, x direction:
$$mg\sin\theta = ma_x$$
$$a_x = g\sin\theta$$

mass m, y direction:
$$-Mg\cos\theta = Ma_y = 0$$
$$a_y = -g\cos\theta$$

Since the acceleration in the x direction is the same for both masses, mass m will fall on point A when it reaches the floor.

To find the time that mass m takes to fall, start with the acceleration in the y direction.

$$a_y = -g\cos\theta$$
$$v_y(t) = v_{oy} + a_y t = v_{oy} - gt\cos\theta$$
$$y(t) = y_o + v_{oy}t + \tfrac{1}{2}a_y t^2 = y_o + v_{oy}t - \tfrac{1}{2}gt^2\cos\theta$$

$$y(t) - y_o = v_{oy}t - \tfrac{1}{2}gt^2\cos\theta$$

Thus: $-h = (0)t - \tfrac{1}{2}gt^2\cos\theta$

$$t = \sqrt{\frac{2h}{g\cos\theta}}$$