
PHY 140Y – FOUNDATIONS OF PHYSICS
2001-2002
Tutorial Questions #4 – Solutions
October 9

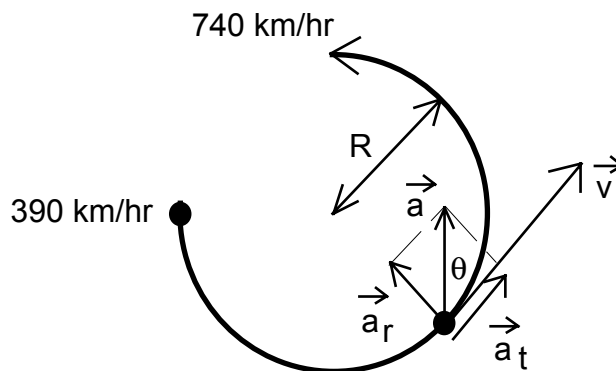
Note: October 8th is a holiday. Students from a Monday tutorial may attend one of the Tuesday October 9th tutorials, at 2-3 in UC 67, 2-3 in RW 141, 4-5 in LM 157, or 4-5 in MP 408.

Non-Uniform Circular Motion

1. After takeoff, a plane makes a three-quarter circle turn of radius 7.1 km, maintaining constant altitude but steadily increasing its speed (at a constant rate) from 390 to 740 km/hr. Midway through the turn, what are (a) the magnitude of the plane's acceleration, and (b) the angle between the plane's velocity and acceleration vectors?

Answer:

Start with a sketch. $R = 7.1$ km



- (a) In order to find the magnitude of the plane's acceleration midway through the turn, we first determine the tangential and radial components of the acceleration.

The tangential acceleration has constant magnitude, so we can regard the tangential motion as 1-D motion with constant acceleration and apply the equation for velocity derived for that case:

$$v(t)^2 = v_o^2 + 2a_t[x(t) - x(t_o)]$$

Thus:
$$a_t = \frac{v(t)^2 - v_o^2}{2[x(t) - x(t_o)]} = \text{constant}$$

However, we must replace x by s , the distance travelled along the arc of the circle. Define the initial position as $s_o = 0$.

The total distance travelled in the three-quarter circle is: $s = \frac{3}{4} \times 2\pi R = 1.5\pi R$

$$a_t = \frac{v(t)^2 - v_o^2}{2[s(t) - s(t_o)]}$$

$$= \frac{v(t)^2 - v_o^2}{2[1.5\pi R]}$$

Hence:

$$= \frac{(740 \text{ km / hr})^2 - (390 \text{ km / hr})^2}{3\pi(7.1 \text{ km})}$$

$$= 5910 \quad \text{km / hr}^2$$

$$= 0.456 \quad \text{m / s}^2$$

Because the tangential acceleration has constant magnitude, clearly this is also the tangential acceleration midway through the turn. Note that we've kept an extra significant figure to avoid rounding errors below.

The radial acceleration midway through the turn can be obtained using: $a_r(t) = \frac{v(t)^2}{R}$

Note that because $v(t)$ is changing, the magnitude of the radial acceleration is also changing. Therefore, we first need to find $v(t)$ midway through the turn. We can again use the equation applied above.

$$v(t_{\text{midway}})^2 = v_o^2 + 2a_t[s(t_{\text{midway}}) - s(t_o)]$$

$$= v_o^2 + 2a_t s(t_{\text{midway}})$$

$$= v_o^2 + 2a_t[0.5 \times 1.5\pi R]$$

$$= (390 \text{ km / hr})^2 + 2(5910 \text{ km / hr}^2)[0.5 \times 1.5\pi(7.1 \text{ km})]$$

$$= 591 \quad \text{km / hr}$$

$$= 164 \quad \text{m / s}$$

$$a_r(t_{\text{midway}}) = \frac{v(t_{\text{midway}})^2}{R} = \frac{(591 \text{ km / hr})^2}{7.1 \text{ m}}$$

The radial acceleration is then:

$$= 49,275 \quad \text{km / hr}^2$$

$$= 3.80 \quad \text{m / s}^2$$

Finally, we can calculate the magnitude of the acceleration at the midpoint of the turn, rounding to two significant figures at the end.

$$a(t_{\text{midway}}) = \sqrt{a_t^2 + a_r(t_{\text{midway}})^2}$$

$$= \sqrt{5910^2 + 49,275^2} \quad \text{km / hr}^2 = 49,628 \quad \text{km / hr}^2 = 5.0 \times 10^4 \quad \text{km / hr}^2$$

$$\text{OR} = \sqrt{0.456^2 + 3.80^2} \quad \text{m / s}^2 = 3.83 \quad \text{m / s}^2 = 3.8 \quad \text{m / s}^2$$

(b) Midway through the turn, the angle between the plane's velocity and acceleration vectors is:

$$\theta(t_{\text{midway}}) = \tan^{-1} \left[\frac{a_r(t_{\text{midway}})}{a_t} \right] = \tan^{-1} \left[\frac{3.80}{0.456} \right] = 83^\circ$$

Relative Motion

2. An airplane with an air speed of 370 km/hr flies perpendicular across the jet stream. To achieve this flight, the airplane must be pointed into the jet stream at an angle of 35° from the perpendicular direction of its flight. What is the speed of the jet stream?

Answer:

First, define the velocities. Let:

$\vec{v}_{\text{ground}}^{\text{air}}$ = velocity of the jet stream, i.e., of air relative to the ground (“wind speed”)

$\vec{v}_{\text{ground}}^{\text{plane}}$ = velocity of the airplane relative to the ground (“ground speed” of the airplane)

$\vec{v}_{\text{air}}^{\text{plane}}$ = velocity of the airplane relative to the air (“air speed” of the airplane)

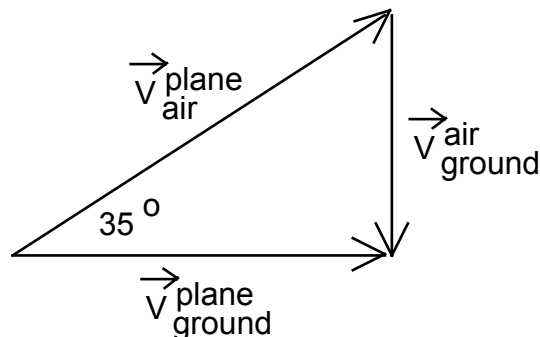
We are given that:

- the ground speed \perp jet stream, i.e., $\vec{v}_{\text{ground}}^{\text{plane}} \perp \vec{v}_{\text{ground}}^{\text{air}}$
- the angle between $\vec{v}_{\text{ground}}^{\text{plane}}$ and $\vec{v}_{\text{air}}^{\text{plane}}$ is 35°
- the magnitude of $\vec{v}_{\text{air}}^{\text{plane}}$, i.e., the air speed, is 370 km/hr

Apply the Chain Rule:

$$\vec{v}_{\text{ground}}^{\text{plane}} = \vec{v}_{\text{air}}^{\text{plane}} + \vec{v}_{\text{ground}}^{\text{air}}$$

Clearly, the three vectors form the sides of a triangle. Sketch the geometry.



$\vec{v}_{\text{air}}^{\text{plane}}$ is the hypotenuse of a right angle triangle, so

$$\sin(35^\circ) = \frac{\vec{v}_{\text{ground}}^{\text{air}}}{\vec{v}_{\text{air}}^{\text{plane}}}$$

The speed of the jet stream is thus:

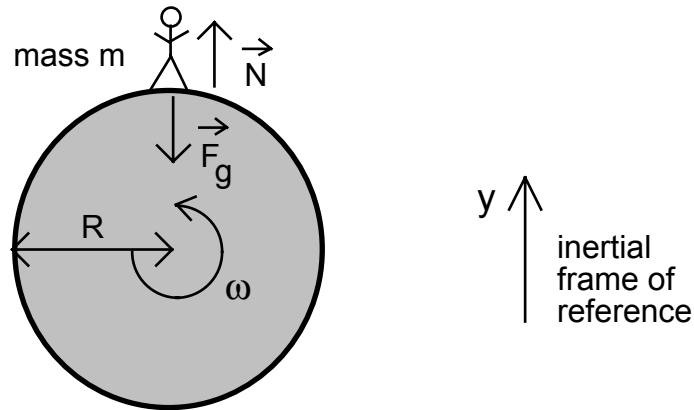
$$\vec{v}_{\text{ground}}^{\text{air}} = \vec{v}_{\text{air}}^{\text{plane}} \sin(35^\circ) = (370 \text{ km/hr}) \sin(35^\circ) = 210 \quad \text{km/hr}$$

Newton's Laws of Motion

3. (a) What is the normal force felt by a person of mass 60.0 kg standing on the Earth's equator? By what percentage does the Earth's rotation reduce the normal force from that due to gravity? (b) What would happen to this force if the Earth's rotation rate increased (or, alternatively, if the period elapsed in a "day" decreased)? (c) How long would the Earth's day have to be in order to make the normal force zero. In this case, what would the person feel? The equatorial radius of the Earth is 6378 km.

Answer:

Consider the view looking down on the North Pole with the equator as the circumference of the circle. Sketch the force diagram and define the inertial frame of reference.



- (a) There are two forces acting on the person: a normal force and the force of gravity, so

$$\vec{F}_{\text{net}} = \vec{N} + \vec{F}_g = N\hat{j} - mg\hat{j}$$

$$\vec{F}_{\text{net}} = m\vec{a} = m\vec{a}_y = \text{centripetal force}$$

Apply Newton's Second Law:

$$= -m \frac{v^2}{R} \hat{j}$$

All of these vectors are along the y axis, i.e., in 1-D, so we can drop the vector notation and equate the two expressions for the net force.

$$F_{\text{net}} = N - mg = -m \frac{v^2}{R}$$

Thus:

$$N = mg - m \frac{v^2}{R} = m \left(g - \frac{v^2}{R} \right)$$

In order to solve for N , we need to find v . This requires the angular speed of the rotating Earth, which is just 2π radians per day, i.e., one full rotation.

$$\begin{aligned}
 v &= \omega R = \left(2\pi \frac{\text{rad}}{\text{day}} \times \frac{1\text{day}}{24\text{hr}} \times \frac{1\text{hr}}{3600\text{s}} \right) \times \left(6378\text{km} \times \frac{1000\text{m}}{\text{km}} \right) \\
 &= (7.272 \times 10^{-5} \text{rad/s}) \times (6.378 \times 10^6 \text{m}) \\
 &= 463.8 \quad \text{m/s}
 \end{aligned}$$

So the normal force felt by a person of mass $m = 60.0 \text{ kg}$ on the Earth's equator is:

$$\begin{aligned}
 N &= m \left(g - \frac{v^2}{R} \right) \\
 &= (60.0\text{kg}) \times \left(9.81\text{m/s}^2 - \frac{(463.8\text{m/s})^2}{6.378 \times 10^6 \text{m}} \right) \\
 &= (60.0\text{kg}) \times (9.81 - 0.03373)\text{m/s}^2 \\
 &= 587 \quad \text{Newtons}
 \end{aligned}$$

The Earth's rotation reduces the normal force by $\frac{0.03373}{9.81} \times 100\% = 0.344\%$.

(b) If the Earth's rotation rate increased (or if the period elapsed in a "day" decreased), then ω would increase and $v = \omega R$ would also increase. Thus, the centripetal force, mv^2/R , would increase and the normal force, $N = mg - mv^2/R$, would decrease.

(c) If the normal force were zero, then

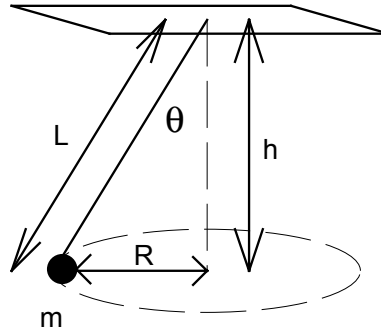
$$\begin{aligned}
 N &= mg - m \frac{v^2}{R} = 0 \\
 \text{so} \quad v &= \sqrt{gR}
 \end{aligned}$$

We want to determine the length of the day, i.e., the time for one full rotation = the period of rotation.

$$\begin{aligned}
 T &= \frac{2\pi}{\omega} = \frac{2\pi R}{v} = \frac{2\pi R}{\sqrt{gR}} = 2\pi \sqrt{\frac{R}{g}} \\
 &= 2\pi \sqrt{\frac{6.378 \times 10^6 \text{m}}{9.81\text{m/s}^2}} \\
 &= 5070 \quad \text{s} \\
 &= 84.4 \quad \text{min} \\
 &= 1.41 \quad \text{hr}
 \end{aligned}$$

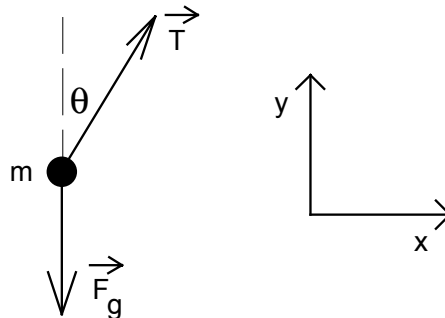
This should look familiar ... compare with question #5 from Tutorial #3.

4. Consider the conical pendulum shown in the figure. The pendulum consists of a mass m on the end of a massless string of length L . The other end of the string is fixed to a ceiling. Given the proper push, this pendulum can swing in a circle at a given angle θ , maintaining the same height h throughout its swing. (a) What is the force diagram for such a pendulum? (b) If the mass of the pendulum is $m = 0.2$ kg, the length of the pendulum is $L = 0.5$ m, and the angle at which it swings is $\theta = 10^\circ$, what is the speed of the mass as it swings? (c) Returning to general expressions, what is the angular velocity ω of the mass in terms of the string angle θ and string length L ?



Answer:

- (a) Sketch the force diagram. There are two forces acting: gravity and tension.



- (b) Given: $m = 0.2$ kg, $L = 0.5$ m, and $\theta = 10^\circ$.

Balance the forces and apply Newton's Second Law to the x and y components separately.

$$\vec{F}_{\text{net}} = \vec{T} + \vec{F}_g = m\vec{a}$$

y direction (no acceleration):

$$T_y + F_g = ma_y$$

$$T \cos \theta - mg = 0$$

$$T = \frac{mg}{\cos \theta}$$

$$T_x = ma_x$$

x direction (centripetal acceleration):

$$T \sin \theta = m \frac{v^2}{R}$$

$$T = \frac{mv^2}{R \sin \theta}$$

Equate these two expressions for T and solve for v:

$$\frac{mv^2}{R \sin \theta} = \frac{mg}{\cos \theta}$$

$$v = \sqrt{gR \tan \theta}$$

Lastly, we need to use $R = L \sin \theta$

$$v = \sqrt{gL \sin \theta \tan \theta}$$

to get

$$= \sqrt{(9.81 \text{ m/s}^2)(0.5 \text{ m}) \sin(10^\circ) \tan(10^\circ)}$$

$$= 0.4 \quad \text{m/s}$$

(c) The angular velocity ω of the mass in terms of the string angle θ and string length L can be simply obtained from the equation that we derived for v.

$$\omega = \frac{v}{R} = \frac{\sqrt{gR \tan \theta}}{R} = \sqrt{\frac{g \tan \theta}{R}} = \sqrt{\frac{g \tan \theta}{L \sin \theta}}$$

$$= \sqrt{\frac{g}{L \cos \theta}}$$