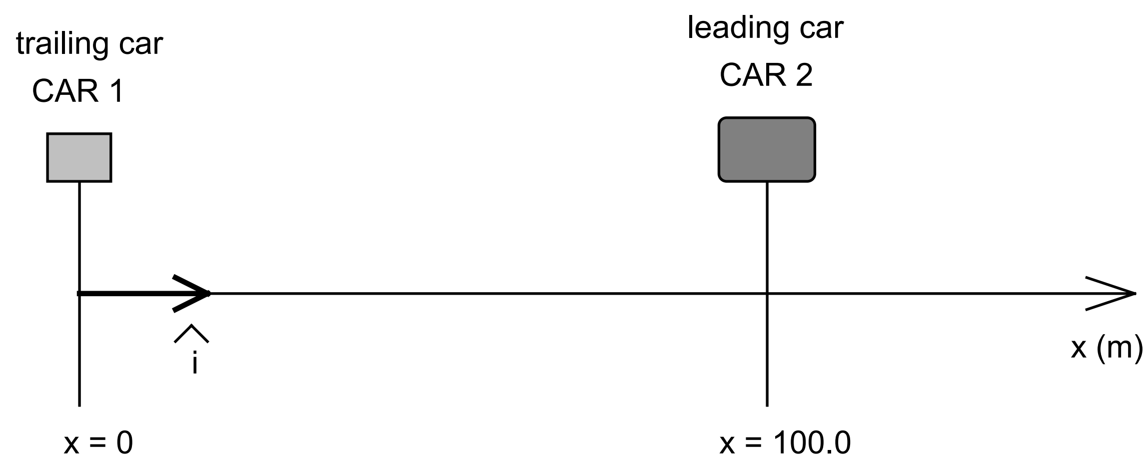
PHY 140Y - FOUNDATIONS OF PHYSICS
2001-2002
Tutorial Questions #2 - Solutions
September 24/25

Motion in a Straight Line

1. Two cars are moving with a velocity $v_o = 40.0$ m/s down a stretch of highway. They are displaced by a distance of 100.0 m. Both cars begin to accelerate - the lead car at a rate of 4.0 m/s^2 and the trailing car at a rate of 6.5 m/s^2. How long from the onset of acceleration will it take for the trailing car to catch the other car? What are their velocities at that time?

<u>Answer:</u>

First, set up the co-ordinate system (only use 1-D) and sketch the situation.



Next, write down the initial conditions.

For CAR 1 (trailing car): at $t_o = 0$, we have $x_o = 0$, $v_o = 40.0$ m/s, $a = 6.5$ m/s^2

For CAR 2 (leading car): at $t_o = 0$, we have $x_o = 100.0$, $v_o = 40.0$ m/s, $a = 4.0$ m/s^2

Set up the equations of motion. Keep all quantities in SI units (i.e., metres and seconds) and don't round off to significant figures until the end.

For CAR 1: $a_1 = 6.5 \quad m/s^2$

$$v_1(t) = v_o + a(t - t_o)$$
$$= 40.0 \quad m/s \quad + (6.5 \quad m/s^2)t$$ where t is in seconds
$$= 40.0 + 6.5t \quad m/s$$

$$x_1(t) = x_o + v_o(t - t_o) + \tfrac{1}{2}a(t - t_o)^2$$
$$= 0 + 40.0t + \tfrac{1}{2}(6.5)t^2$$ where x, v, a, and t are in SI units
$$= 40.0t + 3.25t^2 \qquad m$$

For CAR 2: $a_2 = 4.0 \quad m/s^2$

$$v_2(t) = v_o + a(t - t_o)$$
$$= 40.0 + 4.0t \quad m/s$$

$$x_2(t) = x_o + v_o(t - t_o) + \tfrac{1}{2}a(t - t_o)^2$$
$$= 100.0 + 40.0t + \tfrac{1}{2}(4.0)t^2$$
$$= 100.0 + 40.0t + 2.0t^2 \qquad m$$

When the trailing car catches up to the leading car, their displacements will be equal.

$$x_1(t) = x_2(t)$$
$$40.0t + 3.25t^2 = 100.0 + 40.0t + 2.0t^2$$
$$1.25t^2 = 100.0$$
$$t^2 = 80.0$$
$$t = 8.94 \qquad \text{sec onds}$$

So 8.94 seconds after the onset of acceleration, the trailing car will catch the leading car.

The velocities at this time are:

$$v_1(8.94) = 40.0 + 6.5(8.94) \quad m/s$$
$$= 98.1 \qquad m/s$$

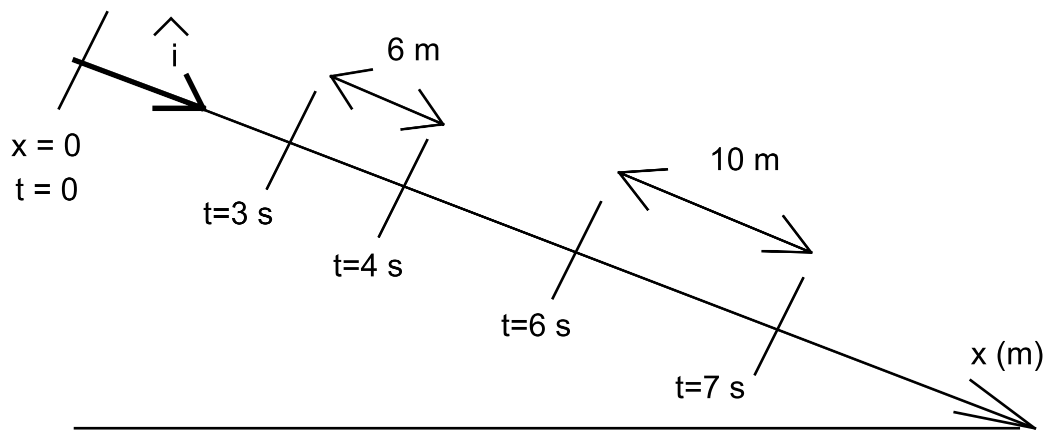
and

$$v_2(8.94) = 40.0 + 4.0(8.94) \quad m/s$$
$$= 75.8 \qquad m/s$$

2. A sled is sliding down a hill and is being timed at one second intervals for 10.0 seconds. In the time interval t = 3.0 s to t = 4.0 s, it is observed to travel 6.0 m. In the time interval from t = 6.0 s to t = 7.0 s, it is observed to travel 10.0 m. What is the total distance that the sled travels in the 10..0 seconds of timing?

Answer:

Again, first set up the co-ordinate system and sketch the situation.



Define the co-ordinate system such that at $t_o = 0$, we have $x_o = 0$.

The sled is accelerating at constant unknown value: $a(t) = a$

The velocity is given by: $v(t) = v_o + a(t - t_o) = v_o + at$

The displacement is: $x(t) = x_o + v_o(t - t_o) + \frac{1}{2}a(t - t_o)^2 = v_o t + \frac{1}{2}at^2$

We have only been given information about the displacement over two time intervals, but we can derive two useful equations from these.

$$\begin{aligned} x(4.0s) - x(3.0s) &= \left[v_o(4.0) + \tfrac{1}{2}a(4.0)^2\right] - \left[v_o(3.0) + \tfrac{1}{2}a(3.0)^2\right] \\ &= 4.0v_o + 8.0a - 3.0v_o - 4.5a \\ &= v_o + 3.5a \\ &= 6.0\text{m} \quad \text{(given)} \end{aligned}$$

$$\Rightarrow \; v_o + 3.5a = 6.0 \quad \text{Eqn. (1)}$$

$$\begin{aligned} x(7.0s) - x(6.0s) &= \left[v_o(7.0) + \tfrac{1}{2}a(7.0)^2\right] - \left[v_o(6.0) + \tfrac{1}{2}a(6.0)^2\right] \\ &= 7.0v_o + 24.5a - 6.0v_o - 18a \\ &= v_o + 6.5a \\ &= 10.0\text{m} \quad \text{(given)} \end{aligned}$$

$$\Rightarrow \; v_o + 6.5a = 10.0 \quad \text{Eqn. (2)}$$

Now we have two simultaneous equations which we can solve for unknowns v_o and a.

Eqn. (2) – Eqn. (1) gives:

$$3.0a = 4.0$$
$$a = \frac{4}{3} = 1.3 \qquad m/s^2$$

Substitute a back in Eqn. (1):

$$v_o = 6.0 - 3.5(0.75) = \frac{4}{3} = 1.3 \qquad m/s$$

Now we can finally solve for the total distance travelled during the 10.0 seconds of timing:

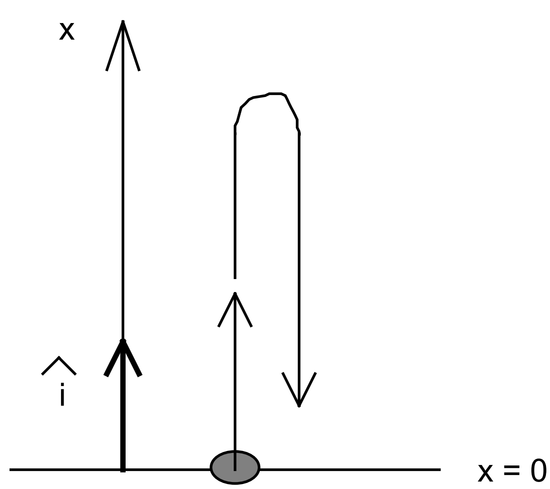
$$\begin{aligned} x(10.0s) - x(0s) &= \left[\frac{4}{3}(10.0) + \frac{1}{2}(\frac{4}{3})(10.0)^2\right] - \left[\frac{4}{3}(0) + \frac{1}{2}(\frac{4}{3})(0)^2\right] \\ &= \frac{40}{3} + \frac{200}{3} \\ &= 80. \qquad m \end{aligned}$$

The Acceleration of Gravity

3. A rock on the Moon is thrown straight up with a velocity of 5.0 m/s. After 10.0 seconds, it has a downward velocity of 11 m/s. What is the acceleration due to gravity on the Moon? How high above the starting point did the rock go before it began to fall?

Answer:

Once again, start by setting up the co-ordinate system and sketching the situation.
Define +x as upwards.



Initial conditions: at $t_o = 0$, we have $x_o = 0$, $v_o = 5.0$ m/s

Final conditions: at $t_o = 10.0$ s, we have $v = 11$ m/s downwards

$a(t) = -a_g$ downwards in the co-ordinate system chosen above

$$\begin{aligned} v(t) &= v_o + a(t - t_o) \\ &= v_o - a_g t \qquad \text{where t is in seconds} \\ &= 5.0 - a_g t \quad \text{m / s} \end{aligned}$$

$$\begin{aligned} x(t) &= x_o + v_o(t - t_o) + \tfrac{1}{2} a(t - t_o)^2 \\ &= v_o t - \tfrac{1}{2} a_g t^2 \qquad \text{where x, } a_g\text{, and t are in SI units} \\ &= 5.0t - \tfrac{1}{2} a_g t^2 \qquad \text{m} \end{aligned}$$

At t = 10 seconds:

$$\begin{aligned} 5.0 - a_g t &= 5.0 - 10.0 a_g = -11 \\ a_g &= 1.6 \qquad \text{m / s}^2 \end{aligned}$$

So the acceleration due to gravity on the Moon is $a(t) = -1.6$ m/s^2.

Now, at the maximum height, the velocity must be 0.

Thus:

$$\begin{aligned} 5.0 - a_g t &= 0 \\ t &= \frac{5.0}{a_g} = \frac{5.0}{1.6} = 3.1 \qquad \text{s} \end{aligned}$$

So the maximum height at this time is:

$$\begin{aligned} x(t) &= 5.0t - \tfrac{1}{2} a_g t^2 \\ &= 5.0\left(\frac{5.0}{1.6}\right) - \tfrac{1}{2}(1.6)\left(\frac{5.0}{1.6}\right)^2 \qquad \text{m} \\ &= 7.8 \qquad \text{m} \end{aligned}$$

4. Lisa challenges Bart to catch a five-dollar bill as follows. She holds the five-dollar bill vertically from the top, with the centre of the bill between Bart's index finger and thumb. Bart must catch the bill after Lisa releases it without moving his hand downwards. Would you bet on Lisa or Bart?

Answer:

The five-dollar bill is in free fall, starting from rest and undergoing a downward acceleration of g = 9.8 m/s^2. The initial conditions are $x_o = 0$ and $v_o = 0$ at at $t_o = 0$.

Thus, in time t, the bill will fall a distance

$$x(t) = x_o + v_o(t - t_o) + \tfrac{1}{2}a(t - t_o)^2 = \tfrac{1}{2}gt^2$$

There is a time delay between the instant that Lisa releases the five-dollar bill and the time that Bart reacts and closes his fingers. The reaction time of most people is at best about 0.2 seconds.

In 0.2 seconds, the five-dollar bill falls a distance

$$x(0.2s) = \tfrac{1}{2}gt^2 = \tfrac{1}{2}(9.8m / s^2)(0.2s)^2 = 0.2m = 20cm$$

This distance is about twice the distance between the centre of the five-dollar bill and its top edge (≈ 8 cm). Therefore, Bart will not be able to catch the bill without moving his hand downwards (or anticipating and starting to close his fingers before Lisa lets go).

Try this!