PHY 140Y – FOUNDATIONS OF PHYSICS
2001-2002
Tutorial Questions #10
November 19/20

Note: Answers are given in brackets. Try getting full solutions before the tutorials!

Damped and Driven Harmonic Motion, Resonance

1. A mass of 220 g is connected to a light spring having force constant 5.4 N/m. It is free to oscillate on a horizontal frictionless surface.
   (a) If the mass is displaced 7.0 cm from the equilibrium position and released from rest, what are the natural frequency and the period of its motion? [T =1.3 s]
   (b) What are the maximum speed and acceleration of the mass? [0.35 m/s, 1.7 m/s^2]
   (c) What is the total energy of the system? What are the kinetic and potential energies of the system when the mass has a displacement of 2.0 cm from equilibrium? [0.013 J, 0.012 J]
   (d) If the surface is no longer frictionless, but instead gives rise to a damping force with damping constant b = 1.7 kg/s, is the motion of the mass underdamped, critically damped, or overdamped? Justify your answer. [underdamped]

2. A 250-g mass is mounted on a spring with a spring constant of k = 3.3 N/m. The damping constant for this system is b = 8.4×10^{-3} kg/s. How many oscillations will the system undergo during the time it takes the amplitude to decay to 1/e of its original value? [34]

3. A pendulum of length 1.00 m is released from an initial angle of 15.0°
   (a) What is the resonant frequency of this pendulum? [3.13 rad/s]
   (b) After 1000. seconds, its amplitude is reduced by friction to 5.50°. What is the value of b/2m? [1.00 x 10^{-3} s^{-1}]

4. A mass-spring system has b/m = $\omega_o/5$, where b is the damping constant and ω_o is the natural frequency. When this system is driven at frequencies 10% above and below ω_o, how does its amplitude compare with its amplitude at ω_o? [A/A_{res} = 66% and 76%]

5. A 2.00 kg mass attached to a spring is driven by an external force $F(t) = (3.00\text{N})\cos(2\pi t)$. If the force constant of the spring is 20.0 N/m and damping is negligible, determine:
   (a) the period, and [1.00 s]
   (b) the amplitude of the motion. [0.0509 m]