PHY 140Y – FOUNDATIONS OF PHYSICS
2001-2002
Term Test #2 (and Make-Up Version) – Solutions
Thursday, December 6, 2001
6:30 PM - 8:30 PM

Constants: $g = 9.81\ m/s^2$
$c = 3.00 \times 10^8\ m/s$

Lorentz transformations: $x' = \gamma(x - vt)$, $x = \gamma(x' + vt')$ *and* $t' = \gamma\left(t - \frac{vx}{c^2}\right)$, $t = \gamma\left(t' + \frac{vx'}{c^2}\right)$ *with* $\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$

Lorentz velocity addition: $u_x = \frac{u_x' + v}{1 + \frac{v}{c^2}u_x'}$ $u_x' = \frac{u_x - v}{1 - \frac{v}{c^2}u_x}$, $u_y = \frac{u_y'}{\gamma\left(1 + \frac{v}{c^2}u_x'\right)}$, $u_z = \frac{u_z'}{\gamma\left(1 + \frac{v}{c^2}u_x'\right)}$

QUESTIONS:

1. Describe the Michelson-Morley experiment. Include the following in your answer: ***[4 marks each for 20 total]***

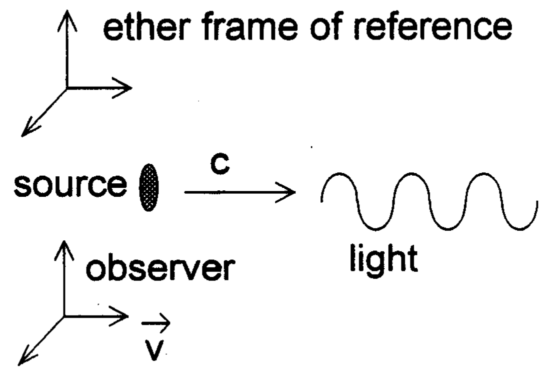
(a) a discussion of the historical context and motivation for the experiment,
(b) a sketch of the apparatus and discussion of the principle of operation of a Michelson interferometer,
(c) the derivation of the two-way travel time for each arm of the interferometer, and hence the path difference between the two arms,
(d) sketches of the expected signal for the original and rotated orientations, an explanation of why the rotation was required, and a discussion of how the phase shift is related to the path difference in (c),
(e) a summary of the results and their significance.

Solution:

(a) Maxwell's Equations (19^{th} century) assumed that EM waves propagate at speed "c" in free space regardless of the motion of the source. This created a problem: What is "c" relative to? Intuition suggested that light might behave like sound. Sound requires a medium to propagate and the speed of sound is the speed relative to this medium (e.g., air, water, etc.). By extension, light was thought to have a medium and "c" is the speed of light relative to this medium. ⇒ ether
Scientists widely believed in ether, which was thought to permeate the universe, have low viscosity, allowing it to creep everywhere and provide no resistance to motion, and to be very stiff since "c" is very large (the harder the medium, the faster sound travels, e.g., waves travel faster along a stiff spring).

Consider light emitted by a source (1D):

$v_{\text{observer}}^{\text{light}} = v_{\text{ether}}^{\text{light}} + v_{\text{observer}}^{\text{ether}} = c - v$



This implies that Maxwell's Eqns only hold in the ether frame of reference because only here do EM waves have speed "c". An observer moving wrt the ether would measure a different speed of light. This violates the principle of relativity since:

- Maxwell's Equations only hold in the ether frame
- an absolute (ether) frame could presumably be detected

There are two options:

(1) The principle of relativity does not hold for all laws. (try to detect the ether)

(2) Einstein's Special Theory of Relativity which implies that "The speed of light is the same in all inertial reference frames."

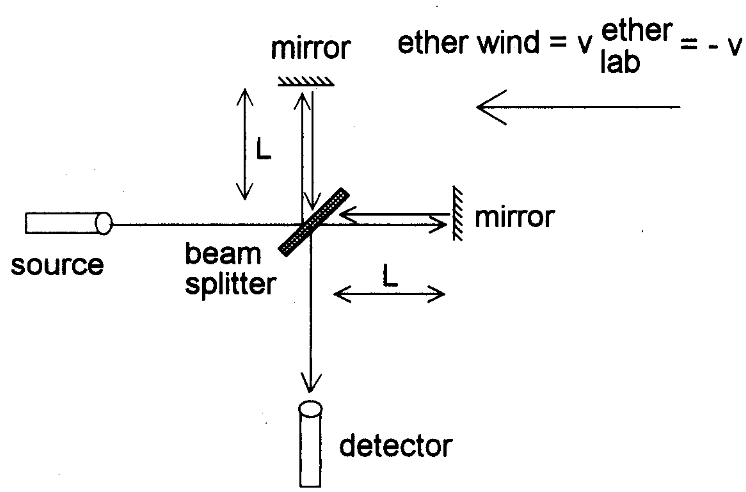
→ only in this way that Maxwell's Eqns could hold in all inertial reference frames

→ ether does not exist

Tests to detect the ether in order to support option (1) can be seen as an important test of STR. Let's consider approaches to detecting the ether. Aberration of Starlight was used to demonstrate that the ether is not dragged by the Earth, which therefore must move through it.

Can we detect Earth's motion through the ether? The Michelson-Morley Experiment (1881-1887 - before STR) was devised in order to answer this question. This experiment had sufficient sensitivity to detect the ether wind if it existed.

(b) The Michelson interferometer was invented for this experiment (Nobel prize). Discuss diagram below.



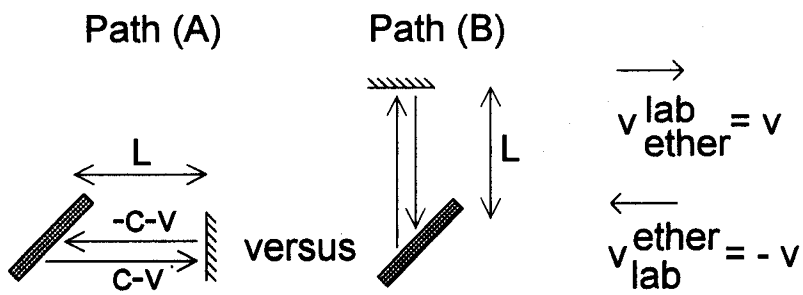
(c) Since the Earth revolves around the Sun, the Earth must be moving relative to the ether. Let's say that the ether wind is from right to left (i.e., Earth moves through the ether from left to right). What is the difference in travel time between the two arms of the interferometer?

Path (A) First consider the round trip parallel to the motion (ether wind) and back.

Define: $v_{\text{ether}}^{\text{lab}} = v \quad (= v_{\text{ether}}^{\text{Earth}})$

So:
$$u = v_{\text{lab}}^{\text{light}} = v_{\text{ether}}^{\text{light}} + v_{\text{lab}}^{\text{ether}} = \begin{cases} c - v & (\text{left} \rightarrow \text{right}) \\ -c - v & (\text{right} \rightarrow \text{left}) \end{cases}$$

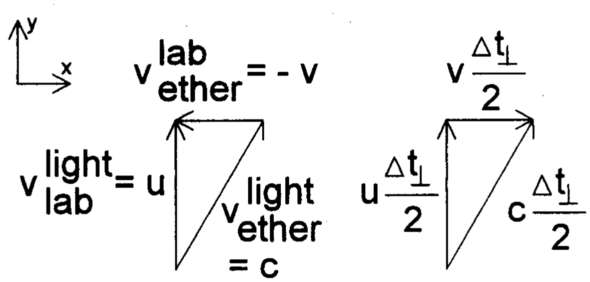
Therefore, the round-trip travel time along path (A) is:
$$\Delta t_{\parallel} = \frac{L}{v_{\text{ether}}^{\text{light}} + v_{\text{lab}}^{\text{ether}}} (\text{left} \rightarrow \text{right}) + \frac{L}{v_{\text{ether}}^{\text{light}} + v_{\text{lab}}^{\text{ether}}} (\text{right} \rightarrow \text{left}) = \frac{L}{|c - v|} + \frac{L}{|-c - v|} = \frac{2L/c}{1 - v^2/c^2}$$



Path (B) Now consider the round trip perpendicular to the motion (ether wind) and back.

$$\vec{v}_{\text{lab}}^{\text{light}} = \vec{v}_{\text{ether}}^{\text{light}} + \vec{v}_{\text{lab}}^{\text{ether}} \quad \text{so} \quad c^2 = u^2 + |-v|^2$$

$$\vec{u} = \vec{c} + \vec{v} \qquad \therefore u = \sqrt{c^2 - v^2}$$



The round-trip travel time along path (B) is:
$$\Delta t_{\perp} = \frac{2L}{u} = \frac{2L}{\sqrt{c^2 - v^2}} = \frac{2L/c}{\sqrt{1 - v^2/c^2}} \neq \Delta t_{\parallel}$$

The difference in the round-trip travel times is thus:
$$\Delta t_{\parallel} - \Delta t_{\perp} = \frac{2L/c}{1 - v^2/c^2} - \frac{2L/c}{\sqrt{1 - v^2/c^2}} = \frac{2L}{c}\left[\frac{1}{1 - v^2/c^2} - \frac{1}{\sqrt{1 - v^2/c^2}}\right]$$

Now, we need to use the approximations:
$$\frac{1}{1 - \Delta x} \cong 1 + \Delta x \quad (\Delta x << 1) \qquad \frac{1}{\sqrt{1 - \Delta x}} \cong 1 + \frac{\Delta x}{2} \quad (\Delta x << 1)$$

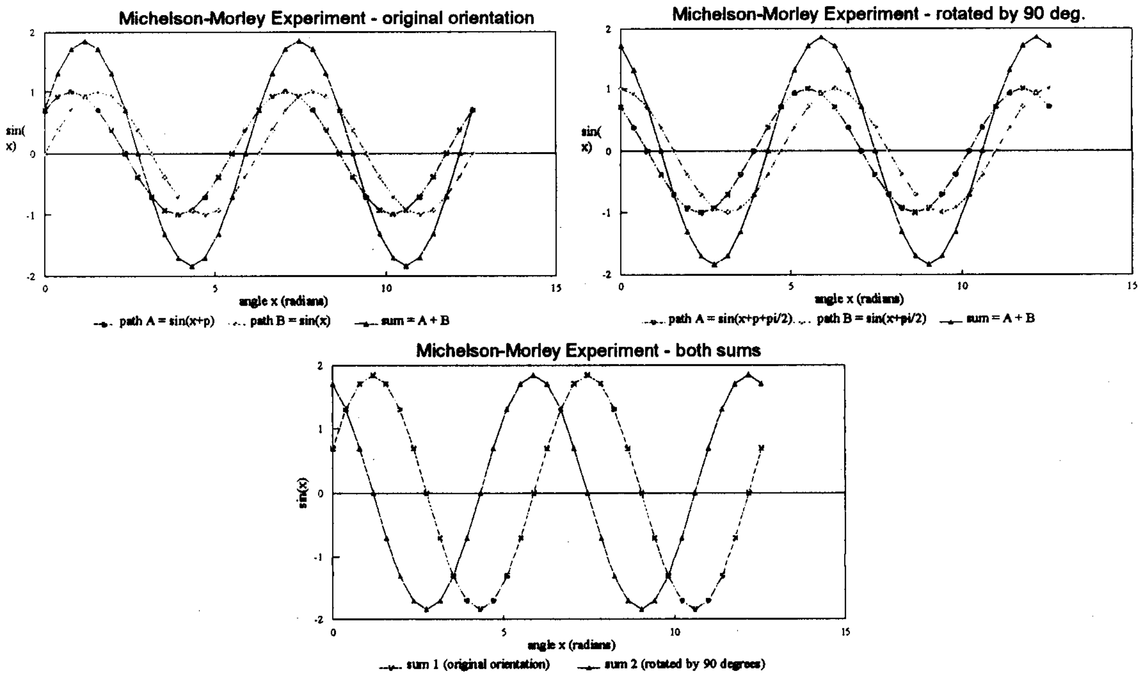
So
$$\Delta t_{\parallel} - \Delta t_{\perp} = \frac{2L}{c}\left[\left(1 + \frac{v^2}{c^2}\right) - \left(1 + \frac{v^2}{2c^2}\right)\right] = \frac{2L}{c}\frac{v^2}{2c^2} = \frac{L}{c}\frac{v^2}{c^2}$$

Path difference between (A) and (B): $c(\Delta t_{\parallel} - \Delta t_{\perp}) = \dfrac{Lv^2}{c^2}$

(d) Now, we don't know which way the ether is directed. Also, we can only observe the combination of the two light beams (waves) - we can't distinguish them individually. The combination generates interference fringes of dark and bright bands. Michelson and Morley rotated their interferometer by 90°, interchanging the role of the two light paths. They observed the combination of the two beams in the original orientation and their combination after rotation by 90°.

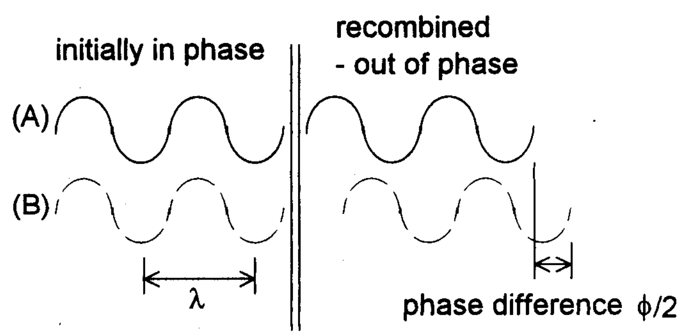
Rotation should shift the interference fringes. Total path difference is $\Delta d = 2\frac{Lv^2}{c^2}$.

The *change in the time difference* between the orientations is $2(\Delta t_{\parallel} - \Delta t_{\perp})$.



The two beams are waves - let's say that they start in phase. A change in pathlength of one λ corresponds to a shift of one fringe. Let ϕ = the phase shift between the two combined beams:

$$\phi = \frac{\Delta d}{\lambda} = 2\frac{Lv^2}{\lambda c^2}$$



(e) Michelson and Morley predicted $\phi = 0.4$ using their interferometer, with L = 11 m, $\lambda = 5.90 \times 10^{-7}$ m (590 nm), and v = orbital speed of Earth, so that $v/c = 10^{-4}$.
They were capable of observing a shift of 0.01. However, they detected no phase shift! Thus, there was no experimental evidence for the ether wind. The results are consistent with the later STR.
Note: This experiment does not prove STR. Einstein said that no number of experiments could prove STR correct, but a single experiment could prove STR wrong. Such an experiment has yet to be found.

<u>Constancy of the speed of light:</u> The speed of light in a vacuum has the same value in all inertial frames of reference (regardless of the velocity of the observer or the velocity of the source emitting the light).

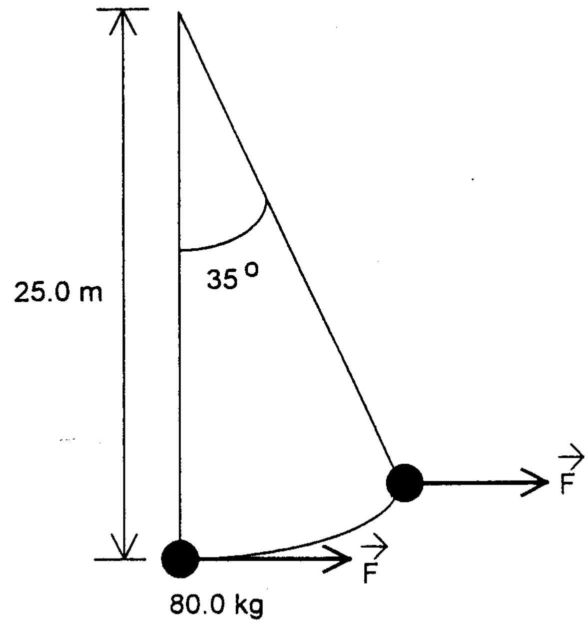
[Each of the following five questions is worth 16 marks.]

2. An 80.0 kg sphere is suspended by a wire of length 25.0 m from the ceiling of a science museum as indicated in the figure. A horizontal force of magnitude F is applied to the ball, moving it very slowly at constant speed until the wire makes an angle with the vertical direction equal to 35°.

(a) Sketch the force diagram for the sphere, indicating all forces acting on it at any point along the path.

(b) Is the force needed to accomplish the task constant in magnitude along the path followed by the ball? Clearly derive your answer.

(c) From a consideration of work and energy, find the work done by the force $\vec{F}$.



Solution:

a) Take the mass to be the system. The forces acting on the mass are:

1. its weight $\vec{w}$, of magnitude mg and directed straight down;
2. the force $\vec{T}$ of the cord on the mass, directed parallel to the cord and away from the mass; and
3. the given horizontal force $\vec{F}$.

Choose a coordinate system with $\hat{i}$ parallel to $\vec{F}$ and with $\hat{j}$ pointing straight up (parallel to $-\vec{w}$). Let θ be the angle of the cord to the vertical.

b) Since the acceleration of the mass is zero, so is the total force on the mass.

x direction

$$F_{x\ \text{total}} = 0\ \text{N} \implies F - T\sin\theta = 0\ \text{N} \implies F = T\sin\theta.$$

y direction

$$F_{y\ \text{total}} = 0\ \text{N} \implies mg - T\cos\theta = 0\ \text{N} \implies T = \frac{mg}{\cos\theta}.$$

Substitute the expression for T from the y equation into the x equation.

$$F = \frac{mg}{\cos\theta}\sin\theta = mg\tan\theta.$$

Hence, F increases as θ increases, so $\vec{F}$ is not constant.

c) The work done by the applied force is not accounted for by a potential energy term in the CWE theorem. The work W done by $\vec{F}$ is included in the term $W_{\text{nonconservative}}$ in the CWE theorem. The work done by the gravitational force is accounted for in the CWE theorem by the change in the gravitational potential energy. The appropriate potential energy function here is mgy, since the entire path is close to the surface of the Earth.

Take the origin to be at the bottom of the path, so initially $y = 0$ m. The force of the cord on the system does zero work since $\vec{T}$ is at all points perpendicular to the circular path followed by the system. Therefore, from the CWE theorem,

$$W_{\text{nonconservative}} = \Delta(\text{KE} + \text{PE}) = (\text{KE}_f + \text{PE}_f) - (\text{KE}_i + \text{PE}_i) = (\frac{1}{2}mv^2 + mgy_f) - (\frac{1}{2}mv^2 + 0\ \text{m}) = mgy_f.$$

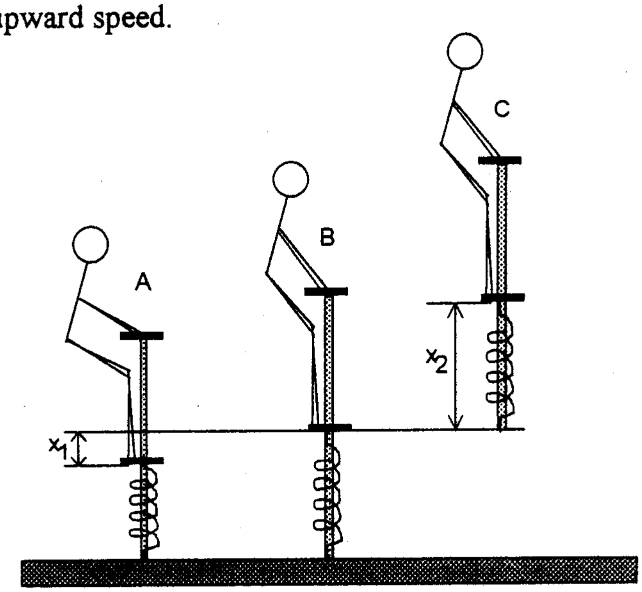
From the geometry,

$$y_f = \ell - \ell\cos\theta = \ell(1-\cos\theta)$$

where ℓ is the length of the cord. Hence

$$W_{\text{nonconservative}} = mg\ell(1-\cos\theta) = (80.0\text{ kg})(9.81\text{ m/s}^2)(25.0\text{ m})(1-\cos 35°) = 3.5\times10^3\text{ J}.$$

3. A child's pogo stick stores energy in a spring having spring constant $k = 2.5\times10^4$ N/m. At position A ($x = x_1 = -0.10$ m) in the figure below), the spring compression is a maximum and the child is momentarily at rest. At position B ($x = 0$), the spring is relaxed and the child is moving upwards. At position C, the child is again momentarily at rest at the top of the jump. Assuming that the combined mass of the child and pogo stick is 25 kg, determine the following:
(a) the total energy of the system if both potential energies (child and pogo stick) are zero at $x = 0$,
(b) x_2, the position at C,
(c) the speed of the child at $x = 0$,
(d) the value of x for which the kinetic energy of the system is a maximum, and
(e) the child's maximum upward speed.



Solution:

(a) Given that the potential energies of the child and and of the pogo stick are zero at x = 0, the total energy of the system for case A is:

$$\begin{aligned}E_A &= K_A + U_A = \tfrac{1}{2}mv_A^2 + \tfrac{1}{2}kx_A^2 + mgx_A\\ &= 0 + \tfrac{1}{2}kx_1^2 + mgx_1\\ &= \tfrac{1}{2}(2.5\times10^4\text{ N/m})(-0.10\text{ m})^2 + (25\text{ kg})(9.81\text{ m/s}^2)(-0.10\text{ m})\\ &= 100.5\text{ J}\\ &= 1.0\times10^2\text{ J (to two significant figures)}\end{aligned}$$

(b) At position B, the spring is relaxed, so this represents the equilibrium position of the spring. The spring remains in this position as the pogo stick leaves the ground, so the potential energy of the spring is zero for both position B and position C.

For case C: $E_C = K_C + U_C = \frac{1}{2}mv_C^2 + \frac{1}{2}kx_C^2 + mgx_C = 0 + 0 + mgx_2$

Since no nonconservative forces are acting, we can apply Conservation of Mechanical Energy: $E_C = E_A$

$$mgx_2 = E_A$$

Thus:

$$x_2 = \frac{E_A}{mg} = \frac{100.5 \text{ J}}{(25 \text{ kg})(9.81 \text{ m/s}^2)} = 0.41 \text{ m}$$

(c) At position B, x = 0, define the speed of the child (and pogo stick) to be v_B. Again apply Conservation of Mechanical Energy to get:

$$E_B = K_B + U_B = \frac{1}{2}mv_B^2 + \frac{1}{2}kx_B^2 + mgx_B = \frac{1}{2}mv_B^2 + 0 + 0$$

$$\frac{1}{2}mv_B^2 = E_A$$

Thus:

$$v_B = \sqrt{\frac{2E_A}{m}} = \sqrt{\frac{2(100.5 \text{ J})}{25 \text{ kg}}} = 2.8(4) \text{ m/s}$$

(d) The speed and kinetic energy of the child are both at a maximum when the acceleration, a, is 0, and hence when: $\sum F = 0$

i.e., when the spring force upwards balances the gravity force downwards. Call this position D.

$$F_s + F_g = 0$$

$$-kx_D - mg = 0$$

$$x_D = -\frac{mg}{k} = -\frac{(25 \text{ kg})(9.81 \text{ m/s}^2)}{2.5 \times 10^4 \text{ N/m}} = -0.0098 \text{ m} = -9.8 \text{ mm}$$

This is the value of x for which the kinetic energy of the system is a maximum.

(e) The child's maximum upward speed is obtained by applying Conservation of Mechanical Energy again.

$$E_D = K_D + U_D = \frac{1}{2}mv_D^2 + \frac{1}{2}kx_D^2 + mgx_D = E_A$$

$$\frac{1}{2}mv_D^2 + \frac{1}{2}kx_D^2 + mgx_D = E_A$$

$$\frac{1}{2}mv_D^2 = E_A - \frac{1}{2}kx_D^2 - mgx_D$$

$$v_D = \sqrt{\frac{2}{m}\left(E_A - \frac{1}{2}kx_D^2 - mgx_D\right)}$$

$$= \sqrt{\frac{2}{25 \text{ kg}}\left[100.5 \text{ J} - \frac{1}{2}(2.5\times10^4 \text{ N/m})(-9.8\times10^{-3} \text{ m})^2 - (25 \text{ kg})(9.81 \text{ m/s}^2)(-9.8\times10^{-3} \text{ m})\right]}$$

$$= \sqrt{\frac{2}{25 \text{ kg}}\left[100.5 \text{ J} - 1.2005 \text{ J} + 2.4035 \text{ J}\right]} \Rightarrow 2.8(5) \text{ m/s}$$

4. A 0.250 kg sugar plum fairy is attached to a vertically suspended spring whose spring constant is 2.00 N/m. The sugar plum fairy is pulled down 10.0 cm and released from rest.

(a) Find x(t).

(b) At what position is the fairy when t = 3.5 s?

(c) What is the velocity of the fairy when t = 3.5 s?

(d) What is the acceleration of the fairy when t = 3.5 s?

Solution:

a) Choose a coordinate system with $\hat{\imath}$ pointing down and with origin at the sugar plum fairy's equilibrium position.

The position at any time t is given by

$$x(t) = A\cos(\omega t + \phi).$$

We need to find the constants $A \geq 0$ m, ω, and ϕ. The constant ω is straight-forward:

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{2.00 \text{ N/m}}{0.250 \text{ kg}}} = 2.83 \text{ rad/s}.$$

When $t = 0$ s, the sugar plum fairy is at rest, so her velocity component is zero,

$$v_x(t) = \frac{dx(t)}{dt} = -A\omega\sin(\omega t + \phi) \implies 0 \text{ m/s} = v_x(0 \text{ s}) = A\sin[\omega(0 \text{ s}) + \phi] = A\sin\phi.$$

So either $A = 0$ m, or $\sin\phi = 0$ in which case $\phi = 0$ rad or $\phi = \pi$ rad. But when $t = 0$ s we have

$$0 \text{ m} < 0.10 \text{ m} = x(0 \text{ s}) = A\cos[\omega(0 \text{ s}) + \phi] = A\cos\phi,$$

so we cannot have $A = 0$ m, nor can we have $\phi = \pi$ rad. Therefore

$$\phi = 0 \text{ rad}.$$

Finally, when $t = 0$ s,

$$x(t) = A\cos\omega t \implies 0.10 \text{ m} = A\cos[\omega(0 \text{ s})] = A.$$

Putting it all together,

$$x(t) = (0.10 \text{ m})\cos[(2.83 \text{ rad/s})t].$$

b) When $t = 3.5$ s, the sugar plum fairy's position is

$$x(3.5 \text{ s}) = (0.10 \text{ m})\cos[(2.83 \text{ rad/s})(3.5 \text{ s})] = -0.089 \text{ m}.$$

She is 0.089 m *above* her equilibrium point.

c) The velocity component of the sugar plum fairy is

$$v_x(t) = \frac{dx(t)}{dt} = -A\omega\sin(\omega t + \phi) = -(0.10 \text{ m})(2.83 \text{ rad/s})\sin[(2.83 \text{ rad/s})t]$$
$$\implies v_x(3.5 \text{ s}) = -(0.10 \text{ m})(2.83 \text{ rad/s})\sin[(2.83 \text{ rad/s})(3.5 \text{ s})] = 0.13 \text{ m/s}.$$

d) The acceleration component of the sugar plum fairy is

$$a_x(t) = \frac{d^2x}{dt^2} = -A\omega^2\cos(\omega t + \phi) = -(0.10)(2.83 \text{ rad/s})^2\cos[(2.83 \text{ rad/s})t]$$
$$\implies a_x(3.5 \text{ s}) = -(0.10)(2.83 \text{ rad/s})^2\cos[(2.83 \text{ rad/s})(3.5 \text{ s})] = 0.71 \text{ m/s}^2.$$

5. In reference frame S', a flashbulb goes off at the origin and the light propagates towards positive x' and negative x' to two detectors, each located 24.0×10^8 m from the S' origin as measured in S' (see the figure below).

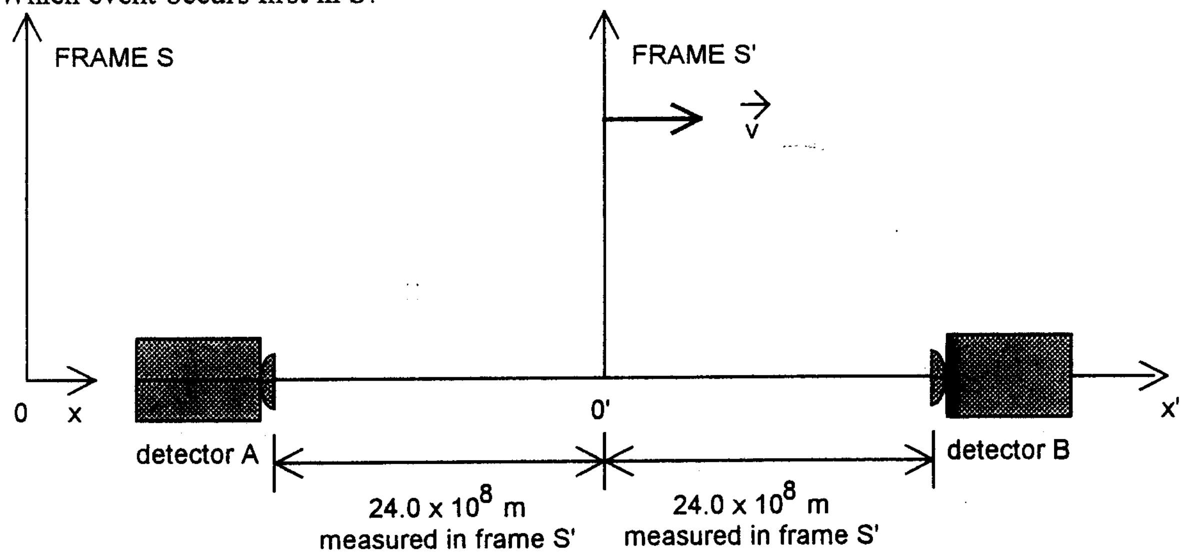
(a) Define two events in S' corresponding to the detectors' reception of the light.

(b) Reference frame S' is moving at speed v = 0.995c relative to reference frame S in the standard geometry. Find the relativistic factor γ.

(c) Find the spatial separation Δx between the two events in S.

(d) Fin the time interval Δt between the two events in S.

(e) Which event occurs first in S?



Solution:

a) Since the two detectors are equidistant, in S', from the flash bulb, they will detect the flash at the same time in S'. Let t_0' be the time that they detect the flash. Let Event A be the time and position of light detection by detector A, and Event B be the time and position of detection by detector B. Then in S' the two events have the following coordinates.

(Event A) $x' = x_A' = -24.0 \times 10^8$ m and $t' = t_A' = t_0'$,

(Event B) $x' = x_B' = 24.0 \times 10^8$ m and $t' = t_B' = t_0'$,

b)

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{1}{\sqrt{1-(0.995)^2}} = 10.0.$$

c) and

d) and

e) Use the inverse Lorentz transformation (Equations 25.37 and 25.40 on page 1164 of the text) to transform the coordinates from S' to S. To do this we will also need

$$v = 0.995c = 0.995(3.00 \times 10^8 \text{ m/s}) = 2.99 \times 10^8 \text{ m/s}.$$

The transformed coordinates are

(Event A)

$$x = x_A = \gamma(x'_A + vt'_A) = 10.0[-24.0 \times 10^8 \text{ m} + (2.99 \times 10^8 \text{ m/s})t'_0] = -24.0 \times 10^9 \text{ m} + (2.99 \times 10^9)t'_0$$

$$t = t_A = \gamma\left(t'_A + \frac{v}{c^2}x'_A\right) = 10.0\left(t'_0 + \frac{2.99 \times 10^8 \text{ m/s}}{(3.00 \times 10^8 \text{ m/s})^2}(-24.0 \times 10^8 \text{ m})\right) = 10.0(t'_0 - 7.97 \text{ s}),$$

and

(Event B)

$$x = x_B = \gamma(x'_B + vt'_B) = 10.0[24.0 \times 10^8 \text{ m} + (2.99 \times 10^8 \text{ m/s})t'_0] = 24.0 \times 10^9 \text{ m} + (2.99 \times 10^9)t'_0$$

$$t = t_B = \gamma\left(t'_B + \frac{v}{c^2}x'_B\right) = 10.0\left(t'_0 + \frac{2.99 \times 10^8 \text{ m/s}}{(3.00 \times 10^8 \text{ m/s})^2}(24.0 \times 10^8 \text{ m})\right) = 10.0(t'_0 + 7.97 \text{ s}),$$

The spatial separation of the two events in S is

$$\Delta x = x_B - x_A = 24.0 \times 10^9 \text{ m} + (2.99 \times 10^9)t'_0 - (-24.0 \times 10^9 \text{ m} + (2.99 \times 10^9)t'_0) = 4.80 \times 10^{10} \text{ m}.$$

The separation in time of the two events in S is

$$\Delta t = t_B - t_A = 10.0(t'_0 + 7.97 \text{ s}) - 10.0(t'_0 - 7.97 \text{ s}) = 159 \text{ s}$$

Event A occurs before Event B.

6. A particle in the S' reference frame has a velocity $\vec{u}' = 0.779c\hat{i}' + 0.450c\hat{j}'$. The S' frame is moving in the standard geometry with speed 0.950c with respect to reference frame S.

(a) Sketch the situation.

(b) What is the angle θ' that $\vec{u}'$ makes with the x'-axis in S'?

(c) What is the speed u' of the particle in the S' frame when measured with clocks and rulers at rest in reference frame S'?

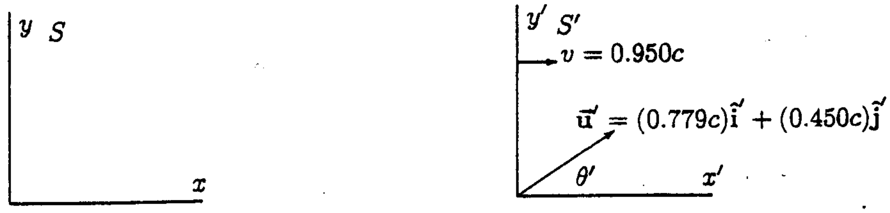
(d) Find the velocity components u_x and u_y of the particle in the S reference frame.

(e) What is the speed u of the particle in S, measured using rulers and clocks in that frame?

(f) What angle θ does the velocity vector $\vec{u}$ make with the x-axis in reference frame S?

<u>Solution:</u>

a) Here's the picture.



b) The angle θ' is found from the velocity components in S',

$$\tan\theta' = \frac{0.450c}{0.779c} \implies \theta' = 30.0°.$$

c)

$$u' = \sqrt{(0.779c)^2 + (0.450c)^2} = 0.900c.$$

d) Use the velocity component addition equations (Equations 25.45 and 25.47 on page 1172 of the text) to find u_x and u_y.

$$u_x = \frac{u'_x + v}{1 + \frac{vu'_x}{c^2}} = \frac{0.779c + 0.950c}{1 + \frac{(0.950c)(0.779c)}{c^2}} = \frac{0.779 + 0.950}{1 + (0.950)(0.779)}c = 0.994c = 2.98 \times 10^8 \text{ m/s}.$$

and

$$u_y = \frac{u'_y}{\gamma\left(1 + \frac{vu'_x}{c^2}\right)} = \frac{u'_y}{\left(\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}\right)\left(1 + \frac{vu'_x}{c^2}\right)} = \frac{u'_y\sqrt{1 - \frac{v^2}{c^2}}}{\left(1 + \frac{vu'_x}{c^2}\right)}$$

$$= \frac{0.450c\sqrt{1 - (0.950)^2}}{\left(1 + \frac{(0.950c)(0.779c)}{c^2}\right)} = \frac{0.450\sqrt{1 - (0.950)^2}}{1 + (0.950)(0.779)}c = 0.081c = 2.4 \times 10^7 \text{ m/s}.$$

Thus

$$\vec{\mathbf{u}} = (0.994)c\hat{\imath} + (0.081c)\hat{\jmath}.$$

e) The speed is

$$u = \sqrt{(0.994c)^2 + (0.081c)^2} = 0.997c.$$

f) The angle θ is found from the velocity components in S,

$$\tan\theta = \frac{0.081}{0.994} \implies \theta = 4.7°.$$

END