PHY 140Y - FOUNDATIONS OF PHYSICS
2000-2001
Term Test #2
Thursday, December 7, 2000
6:30 PM - 8:30 PM

INSTRUCTIONS:

Please give your name, student number, and TA's name on ALL examination booklets used.
Answer ALL questions. Total marks = 100.
Marks, shown in brackets, will be given for workings and units as well as for final answers.
[Non-]programmable calculators may be used. No aid/crib sheets are allowed.

Constants: $g = 9.81\ m/s^2$
$c = 3.00 \times 10^8\ m/s$

Lorentz transformations:
$$x' = \gamma(x - vt) \qquad t' = \gamma\left(t - \frac{vx}{c^2}\right)$$
$$x = \gamma(x' + vt') \quad \textit{and} \quad t = \gamma\left(t' + \frac{vx'}{c^2}\right) \quad \textit{with} \quad \gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

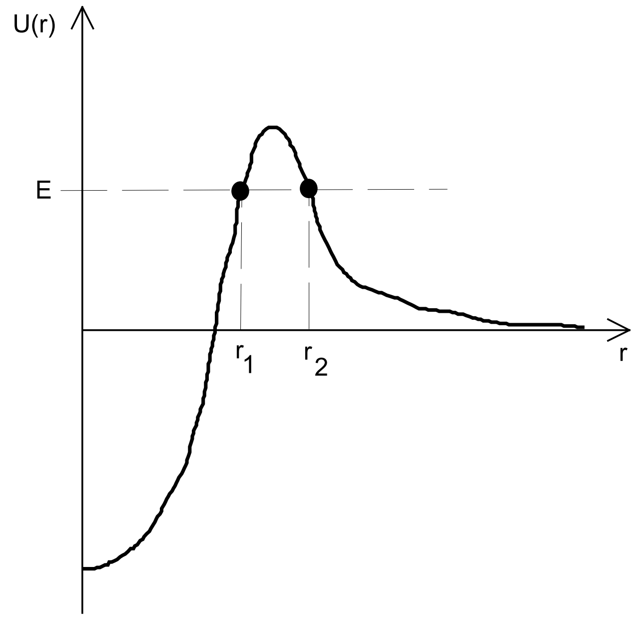
Lorentz velocity addition:
$$u_x = \frac{u_x' + v}{1 + \frac{v}{c^2}u_x'} \qquad u_x' = \frac{u_x - v}{1 - \frac{v}{c^2}u_x}, \quad u_y = \frac{u_y'}{\gamma\left(1 + \frac{v}{c^2}u_x'\right)}, \quad u_z = \frac{u_z'}{\gamma\left(1 + \frac{v}{c^2}u_x'\right)}$$

QUESTIONS:

1. Give BRIEF answers to each of the following. ***[5 marks each for 20 total]***

(a) A block of mass M is first allowed to hang from a spring in static equilibrium. It stretches the spring a distance L beyond the spring's unstressed length. The block and spring are then set into oscillation. Is the period of this system less than, equal to, or greater than the period of a simple pendulum consisting of a mass M suspended a length L? Explain your reasoning.

(b) A graph of the potential energy of an α-particle (the nucleus of a helium atom) in the immediate vicinity of a massive nucleus has the general form shown in the figure below. If the total energy, E, of the α-particle is that indicated in the sketch, qualitatively describe the motion of the α-particle
(i) if it is located in the region to the left of the hump on the diagram, and
(ii) if it is located to the right of the hump.
Explain why it is (classically) impossible for the α-particle to be in the region between r_1 and r_2. One of the strange features of quantum mechanics is that it is possible for the α-particle to "tunnel" through the region where it is (classically) forbidden to be; among other things, this phenomenon describes radioactivity.



(c) Sketch a resonance curve, indicating A_{max}, ω_d^{max}, and the full width at half maximum (FWHM). Briefly explain the significance of each of these terms. Under what conditions does resonance occur? What happens to A_{max}, ω_d^{max}, and the FWHM at resonance?

(d) Explain how a space-time diagram can be used to show that travel into the past is impossible, although travel into the future is not. Include a sketch in your answer. Also, draw a space-time diagram for the two twins of "twin paradox" fame: indicate the world lines of both twins on this diagram. How is acceleration indicated on a space-time diagram?

[Each of the following five questions is worth 16 marks.]

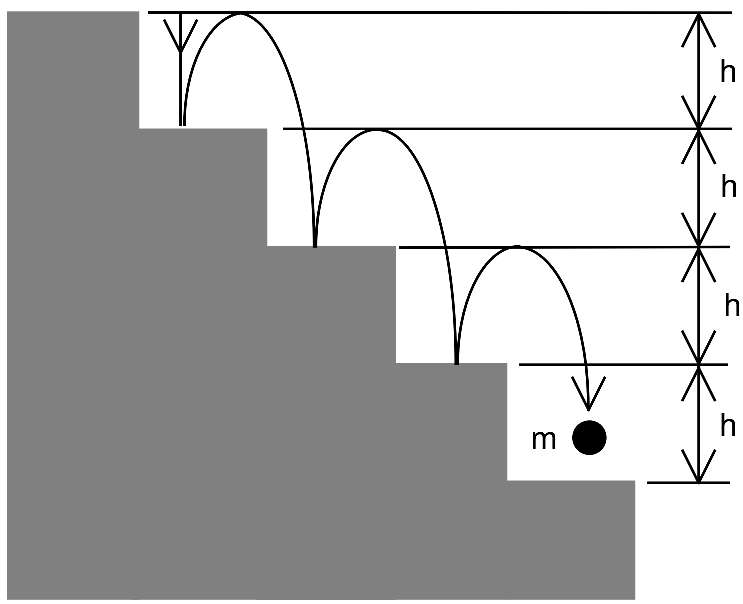
2. You may have enjoyed the childhood experience of dropping a marble down a set of hardwood steps and watching it bounce its way down (if not, it's never too late to try it!). Imagine a marble of mass m dropped down a set of steps of single-step-height h. The rebounding marble bounces to a height equal to the height of the previous step, as shown in the figure below (use this figure to solve part (b) only). The coefficient of restitution, ε, of the bouncing marble is defined to be

$$\varepsilon = \frac{\text{speed immediately after the bounce}}{\text{speed immediately before the bounce}}.$$

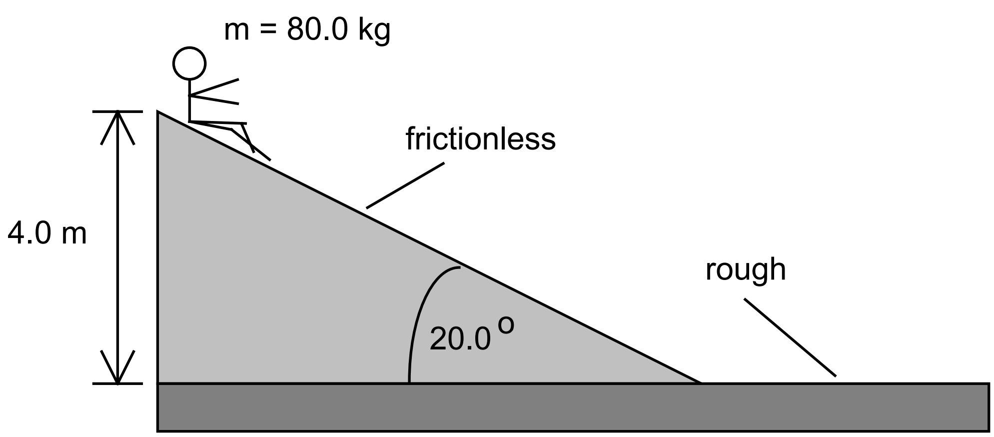
(a) If the coefficient of restitution is zero, what is happening?
(b) Neglect the horizontal speed of the marble. Show that the coefficient of restitution for the situation shown in the figure below is

$$\varepsilon = \sqrt{\frac{1}{2}}.$$

(c) Sketch what happens to the marble if its coefficient of restitution is equal to 1.
(d) The rules governing the construction and composition of tennis balls are quite stringent. When dropped from rest vertically from a height of 2.54 m, the ball must, after bouncing from a concrete floor, rebound to a height of at least 1.35 m, but no more than 1.47 m. What is the range of permissible coefficients of restitution of tennis balls?



3. An 80.0 kg student decides to liven things up by sliding (from a standing start) down a slippery (i.e., frictionless) 25° mud slope as indicated in the figure below. At the end of the slope, the student encounters a horizontal stretch of ground for which the coefficient of kinetic friction is $\mu_k = 0.20$.
(a) Set up an appropriate coordinate system clearly indicating the origin and the direction of the coordinate axes.
(b) Write down an expression for the total mechanical energy of the student at the top of the slide.
(c) What is the speed of the student at the bottom of the inclined slope?
(d) How far will the student travel along the rough horizontal ground before coming to a stop?
(e) What is the total work done by the normal force of the ground on the student during the descent? What is the total work done by the normal force of the ground on the student along the horizontal ground?
(f) What is the total work done by the kinetic frictional force on the student along the horizontal ground?
(g) What is the power of the kinetic frictional force as the student begins to move along the horizontal ground? Is this power constant over the entire horizontal distance travelled? Explain your reasoning.



4. The force of a spring on a mass m when it is a distance x from the equilibrium position at x = 0 m is

$$\vec{F}_{spring} = -kx\,\hat{i}.$$

Suppose, instead, that the only force acting on mass m is a force given by

$$\vec{F} = +kx\,\hat{i}.$$

(a) Qualitatively describe the force $\vec{F}$ (i.e., its magnitude and direction) as the position of the mass moves toward increasing positive values of x.

(b) Qualitatively describe the force $\vec{F}$ (i.e., its magnitude and direction) as the position of the mass moves toward increasing negative values of x.

(c) Use Newton's Second Law to show that the differential equation that describes the motion of the mass when $\vec{F}$ is the only force acting on it, is

$$\frac{d^2x}{dt^2} - \frac{k}{m}x = 0 \ \text{m/s}^2.$$

(d) Does $x(t) = A\cos(\omega t + \phi)$ satisfy this differential equation? If so, what is the value of ω that emerges from this?

(e) Does $x(t) = A\exp(\omega t)$ satisfy this differential equation? If so, what is ω? Qualitatively describe what happens to the particle for times t > 0 s if $x(t) = A\exp(\omega t)$.

(f) Do you think that such a force exists in nature? Explain.

5. In reference frame S', a flashbulb goes off at the origin and the light propagates toward positive x' and negative x' to two detectors, each located 24.0×10^8 m from the S' origin as measured in S' (see figure below).

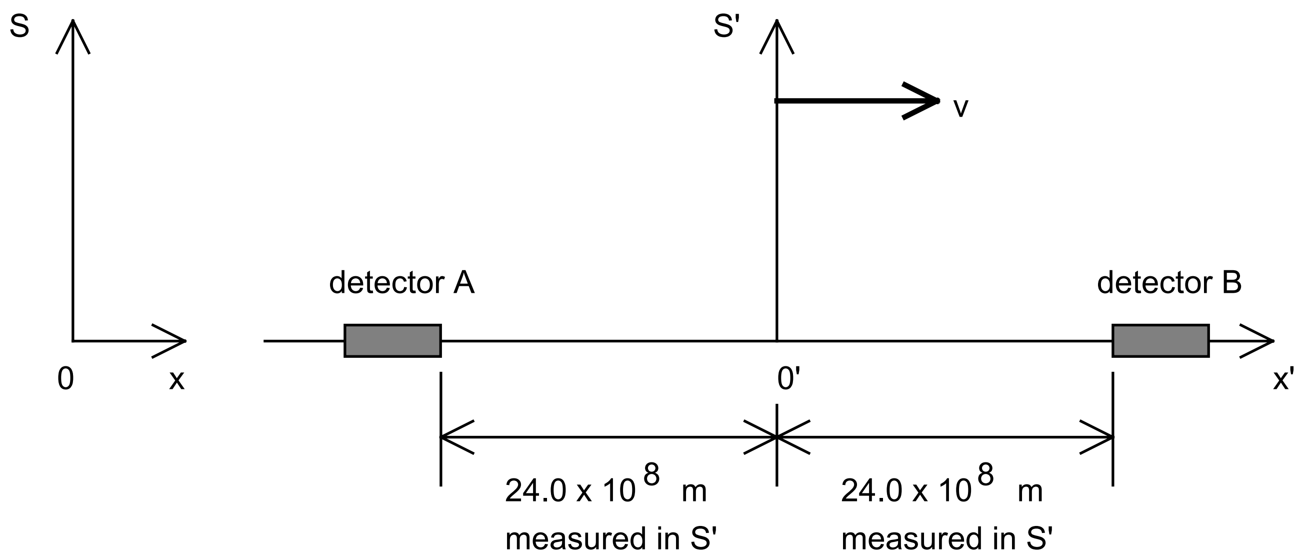
(a) Define two events in reference frame S' corresponding to the detectors' reception of the light.

(b) Reference frame S' is moving at speed v = 0.995c relative to reference frame S in the standard geometry. Find the relativistic factor γ.

(c) Find the spatial separation Δx between the two events in reference frame S.

(d) Find the interval Δt between the two events in reference frame S.

(e) Which event occurs first in S?



6. In inertial reference frame S', a particle is fired along the y'-axis at speed 0.995c parallel to $\hat{j}'$, measured using rulers and clocks in the S' frame. This S' reference frame is moving at speed 0.990c with respect to inertial reference frame S in the standard geometry.
(a) Sketch the geometry. Determine the velocity components u_x and u_y of the particle in reference frame S.
(b) What angle does velocity vector $\vec{u}$ makes with the x-axis in reference frame S?
(c) Assume that the particle is an electron of mass 9.11×10^{-31} kg. What is the electron's total relativistic energy as measured in reference frame S'?
(d) What is the electron's kinetic energy as measured in reference frame S'?
(e) What is the electron's rest energy as measured in reference frame S'?
(f) What is the magnitude of the electron's relativistic momentum as measured in reference frame S'?

END