PHY 140Y – FOUNDATIONS OF PHYSICS
2001-2002
Term Test #1 – Solutions
Thursday, October 25, 2001
6:30 PM - 8:30 PM

QUESTIONS:

1. Give BRIEF answers to each of the following. *[5 marks each for 20 total]*

(a) Define and briefly explain the difference between inertial and noninertial frames of reference. What is a fictitious force and why does it arise?

Solution:
Inertial frames of reference move at constant velocity. Non-inertial reference frames are accelerating.

Observers in two inertial reference frames may measure different velocities for an object, but they will measure the same acceleration and force on an object, i.e., velocities can be different in different inertial frames of reference, but accelerations and forces cannot be different in different inertial frames of reference.

An observer in a non-inertial frame of reference will measure velocities, accelerations, and forces on an object that are different from those seen by an observer in an inertial frame of reference.

e.g., $\vec{F}_B = \vec{F}_A + \vec{F}_f$

Observer B (non-inertial) sees a different force applied to object P than the force that observer A (inertial) sees. Observer B is mistaking his/her own acceleration for that of the object.

This error is the fictitious force $\vec{F}_f = m\vec{a}_B^A$. Newton's Laws of motion do not hold in non-inertial frames of reference. If Newton's Second Law is applied in accelerating frames of reference, then the fictitious force must be introduced to ensure $\vec{F}_{net} = m\vec{a}$.

(b) A passenger in a car travelling at 60 km/hour pours a cup of coffee for the tired driver, with it taking the coffee 0.10 seconds to reach the cup. Describe the path of the coffee as it moves from a Thermos bottle into a cup as seen by (i) the passenger, and (ii) someone standing beside the road and looking in the window of the car as it drives past. (iii) What happens if the car accelerates while the coffee is being poured?

Solution:

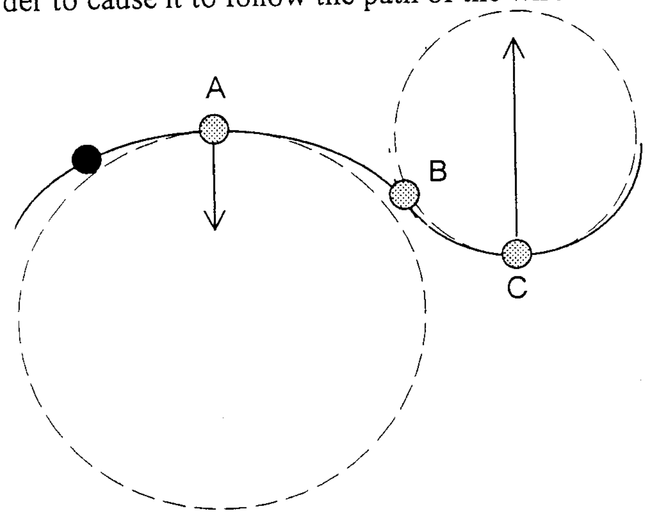
(i) The passenger sees the coffee pouring nearly vertically into the cup, just as if s/he were standing on the ground pouring it.

(ii) The stationary observer sees the coffee moving in a parabolic path with a constant horizontal velocity of 60 km/hour (17 m/s) and a downward acceleration of g. If it takes the coffee 0.10 seconds to reach the cup, then the stationary observer sees the coffee moving 1.7 m horizontally before it hits the cup.

(iii) If the cars slows suddenly, the coffee reaches the place where the cup would have been had there been no change in velocity and continues falling because the cup has not yet reached that location.

If the car rapidly speeds up, then the coffee falls behind the cup. If the car accelerates sideways, then the coffee ends up somewhere other than in the cup.

(c) A bead slides freely along a curved wire at constant speed, as shown in the following overhead view. At each of the points A, B, and C, describe the magnitude and direction of the force that the wire exerts on the bead in order to cause it to follow the path of the wire at that point.



Solution:
At point A, the path is along the circumference of the larger circle. Therefore, the wire must be exerting a force on the bead directed towards the centre of the circle, i.e., downwards in the figure. Because the speed is constant, there is no tangential force component. At point B, the path is not curved and so the wire exerts no force on the bead. At point C, the path is again curved and so the wire again exerts a force on the bead. This time, the force is directed towards the centre of the smaller circle, i.e., upwards in the figure. Because the radius of this circle is smaller, the magnitude of the force exerted on the bead at C must be larger than it is at A (using $F = ma_r = mv^2/R$).

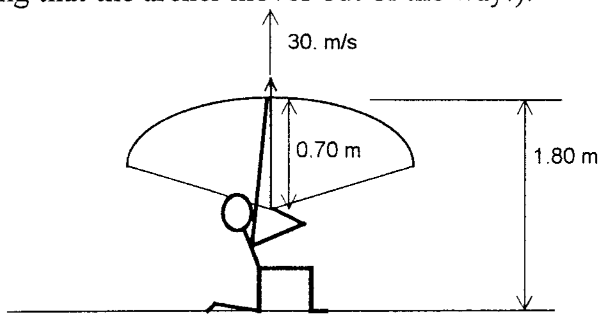
(d) A person steps from a boat towards a dock. Unfortunately s/he forgot to tie the boat to the dock, and the boot scoots away as s/he steps from it. Analyze this situation in terms of Newton's Third Law. Would the outcome be the same for a small dog jumping from the boat? Why or why not?

Solution:
As the person steps out of the boat, s/he pushes against it with her/his foot, expecting the boat to push back so that s/he accelerates towards the dock. However, because the boat is untied, the force exerted by the foot causes the boat to scoot away from the dock. As a result, the person is not able to exert a very large force on the boat before it move out of reach. Therefore, the boat does not exert a very large force on the person, who thus ends up not being accelerated sufficiently to make it to the dock. Consequently, the person falls into the water instead.

If a small dog were to jump from the untied boat towards the dock, the force exerted by the boat on the dog would probably be enough to ensure the dog's successful landing because of the dog's small mass.

[Each of the following five questions is worth 16 marks.]

2. The draw on Robin Hood's archery bow is 0.70 m, as shown in the figure below.
   (a) Calculate the magnitude of the acceleration of an arrow that leaves the bow with a speed of 30. m/s. Assume that the arrow experiences a constant acceleration over the 0.70 m distance before it leaves the bow.
   (b) If the arrow is shot vertically upwards and leaves the bow at distance 1.80 m from the ground, find the maximum height to which the arrow rises and the total time of flight until the arrow hits the ground (assuming that the archer moves out of the way!).



Solution:

a) Choose $\hat{\mathbf{i}}$ to be along the direction of the arrow with the origin at the rear of the arrow, and let $t = 0$ s at the instant of release. Then $v_{x0} = 0$ m/s and $x_0 = 0$ m, so for constant acceleration, the equations for $v_x(t)$ and $x(t)$ are

$$v_x(t) = a_x t, \qquad \text{and} \qquad x(t) = a_x \frac{t^2}{2}.$$

At the time t_f, when the arrow leaves the bow, these equations become

$$30 \text{ m/s} = a_x t_f, \qquad \text{and} \qquad 0.70 \text{ m} = a_x \frac{t_f^2}{2}.$$

These equations may be solved simultaneously for a_x and t_f: Solve the first one for t_f, $t_f = \dfrac{30 \text{ m/s}}{a_x}$, then substitute this value for t_f into the second:

$$0.70 \text{ m} = a_x \frac{\left(\dfrac{30 \text{ m/s}}{a_x}\right)^2}{2} = 4.5 \times 10^2 \left(\frac{\text{m}}{\text{s}}\right)^2 \frac{1}{a_x}.$$

Therefore, $a_x = 6.4 \times 10^2 \text{ m/s}^2$.

b) Change coordinate systems. Let $\hat{\mathbf{i}}$ point straight up and let the origin be at ground level. Let $t = 0$ s be the time that the arrow leaves the bow. Then $x_0 = 1.80$ m, $v_{x0} = 30$ m/s, and $a_x = -9.81 \text{ m/s}^2$, so the equations for $v_x(t)$ and $x(t)$ are

$$v_x(t) = 30 \text{ m/s} + (-9.81 \text{ m/s}^2)t, \qquad \text{and} \qquad x(t) = 1.80 \text{ m} + (30 \text{ m/s})t - (9.81 \text{ m/s}^2)\frac{t^2}{2}.$$

At the time t_f that the arrow is at maximum height, $v_x(t_f) = 0$ m/s. So from the first equation, $0 \text{ m/s} = 30 \text{ m/s} - (9.81 \text{ m/s}^2)t_f$. Solve this to get $t_f = 3.1$ s. Substitute this value for t_f into the equation for $x(t)$ to get $x(t_f) = 1.80 \text{ m} + (30 \text{ m/s})(3.1 \text{ s}) - (9.81 \text{ m/s}^2)\dfrac{(3.1 \text{ s})^2}{2} = 48$ m. So the arrow reaches a maximum height of 48 m.

Now let t_f denote the total flight time (the time when the arrow hits the ground). Then $x(t_f) = 0$ m, so the equation for $x(t)$ becomes

$$0 \text{ m} = 1.80 \text{ m} + (30 \text{ m/s})t_f - (9.81 \text{ m/s}^2)\frac{t_f^2}{2}.$$

Use the quadratic equation to solve this for the positive root, and find that $t_f = 6.2$ s.

3. A naturalist observes a frog leap vertically to a height h.
(a) With what speed did the frog leave the ground? Express your answer in terms of h and g.
(b) If the frog used the same speed to leap horizontally for a maximum range, what distance could it cover?
(c) To what height does the frog ascend in making the leap for maximum horizontal range?

Solution:

a) Choose a coordinate system with origin at the frog's launch point, $\hat{\imath}$ pointed to the right, and $\hat{\jmath}$ pointed up. In this coordinate system

$$\begin{aligned} y_0 &= 0 \text{ m} \\ v_{y0} &= v_0, \\ a_y &= -g. \end{aligned}$$

So, the kinematic equations are

$$\begin{aligned} v_y(t) &= v_0 - gt \\ y(t) &= v_0 t - g\frac{t^2}{2}. \end{aligned}$$

When the frog reaches its maximum height, $v_y = 0$ m/s. Hence $0 \text{ m/s} = v_0 - gt$, so $t = \dfrac{v_0}{g}$.
When the frog is a maximum height, $y = h$. Hence

$$h = v_0\frac{v_0}{g} - g\frac{\left(\frac{v_0}{g}\right)^2}{2} = \frac{v_0^2}{2g}.$$

Therefore $v_0 = \sqrt{2gh}$.

b) When the frog leaps for maximum horizontal range using the same speed v_0, use the same coordinate system but now

$$\begin{aligned} y_0 &= 0 \text{ m} \\ v_{y0} &= v_0 \sin\theta \\ a_y &= -g \end{aligned} \qquad\qquad \begin{aligned} x_0 &= 0 \text{ m} \\ v_{x0} &= v_0 \cos\theta \\ a_x &= 0 \text{ m/s}^2. \end{aligned}$$

So, the kinematic equations are

$$\begin{aligned} v_y(t) &= v_0 \sin\theta - gt \\ y(t) &= (v_0 \sin\theta)t - g\frac{t^2}{2} \end{aligned} \qquad\qquad \begin{aligned} v_x(t) &= v_0 \cos\theta \\ x(t) &= (v_0 \cos\theta)t. \end{aligned}$$

Impact occurs where $y = 0$ m, so

$$0 \text{ m} = (v_0 \sin\theta)t - g\frac{t^2}{2}.$$

Use the quadratic equation to solve for t. The two roots are

$$t = 0 \text{ s} \quad \text{and} \quad t = \frac{2v_0 \sin\theta}{g}.$$

The zero root is the launch time, so we want the nonzero root. The horizontal range R is found by substituting this time into the equation for x:

$$R = v_0 \cos\theta \frac{2v_0 \sin\theta}{g} = \frac{2v_0^2 \sin\theta\cos\theta}{g} = \frac{v_0^2 \sin 2\theta}{g}.$$

This a maximum when $2\theta = 90° \implies \theta = 45°$, so

$$R_{\text{max}} = \frac{v_0^2}{g}.$$

Substitute the expression for v_0^2 from part a):

$$R_{\text{max}} = \frac{2gh}{g} = 2h.$$

c) The time to the maximum height is half that for the horizontal range, [which was found in part b)], or

$$t = \frac{v_0 \sin\theta}{g}.$$

The maximum height then is

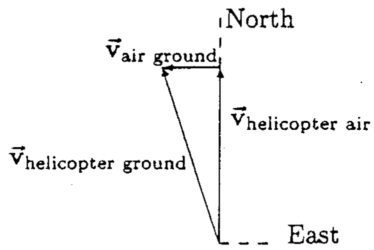
$$y_{\text{max}} = v_0 \sin\theta \frac{v_0 \sin\theta}{g} - g\frac{\left(\frac{v_0 \sin\theta}{g}\right)^2}{2} = \frac{v_0^2 \sin^2\theta}{2g}.$$

Substitute $\theta = 45°$ and $v_0^2 = 2gh$ to find $y_{\text{max}} = \dfrac{2gh\sin^2 45°}{2g} = \dfrac{h}{2}$.

4. Sergeant Pepper is on a water bombing run in a helicopter aimed horizontally northward above a road. The speed of the helicopter is 40.0 m/s relative to the air. The wind is blowing to the west at a constant speed of 8.6 m/s relative to the ground.
   (a) What is the speed of the helicopter relative to the ground? Provide a sketch of the geometry.
   (b) A water bomb is dropped from an altitude of 150. m in the direction of a fire, and just barely misses its target. How long will it take the bomb to hit the ground?
   (c) Where, relative to the helicopter, will the water bomb hit?
   (d) What is the speed of the water bomb the instant before impact?

Solution:

a) The geometry of the situation and a coordinate system are shown below.



The three velocities are related by the relative velocity addition equation.

$$\vec{v}_{\text{helicopter ground}} = \vec{v}_{\text{helicopter air}} + \vec{v}_{\text{air ground}}.$$

Since the three vectors form a right triangle, we may use the Pythagorean theorem to find the speed of the helicopter with respect to the ground.

$$v_{\text{helicopter ground}} = \sqrt{(40.0\text{ m/s})^2 + (8.6\text{ m/s})^2} = 41\text{ m/s}.$$

b) Take the z-axis to be vertical with the origin on the ground and $\hat{k}$ upward. The z-components of the bomb's velocity and position at a time t after it is dropped, are

$$v_z(t) = v_{z0} + a_z t \qquad \text{and} \qquad z(t) = z_0 + v_{z0}t + a_z\frac{t^2}{2}$$

where z_0, v_{z0}, and a_z are the initial height, initial z component of velocity, and the z component of acceleration for the water bomb. From the statement of the problem,

$$z_0 = 150\text{ m}, \qquad v_{z0} = 0\text{ m/s}, \qquad \text{and} \qquad a_z = -g.$$

Hence

$$v_z(t) = -gt \qquad \text{and} \qquad z(t) = 150\text{ m} - g\frac{t^2}{2}.$$

Impact is at $z = 0$ m, thus, from the position equation, at the time of impact

$$0\text{ m} = 150\text{ m} - g\frac{t^2}{2}.$$

Solve for the impact time t.

$$t = \sqrt{\frac{2(150\text{ m})}{9.81\text{ m/s}^2}} = 5.53\text{ s}.$$

(As usual, we use the positive root, since impact is after the time of release.)

We may use the equation for $v_z(t)$ to find the z component of velocity at impact.

$$v_z(t) = (-9.81\text{ m/s}^2)(5.53\text{ s}) = -54.2\text{ m/s}.$$

c) Since the motion along z is independent of motion along x and y, the bomb will hit directly below the helicopter.

d) The speed of the bomb the instant before impact is

$$v = \sqrt{v_x^2 + v_y^2 + v_z^2} = \sqrt{(-8.6\text{ m/s})^2 + (40\text{ m/s})^2 + (-54.2\text{ m/s})^2} = 68\text{ m/s}.$$

5. A small sphere of mass m is attached to the end of a cord of length R that rotates counter-clockwise in a vertical circle about a fixed point O as shown. Define θ as the angle that the cord makes with the vertical.

(a) Draw the free-body diagram for mass m when it is at points: (i) A - the bottom of the circle, (ii) B - the top of the circle, and (iii) C – at angle θ as shown.

(b) What are the radial and tangential components of the acceleration?

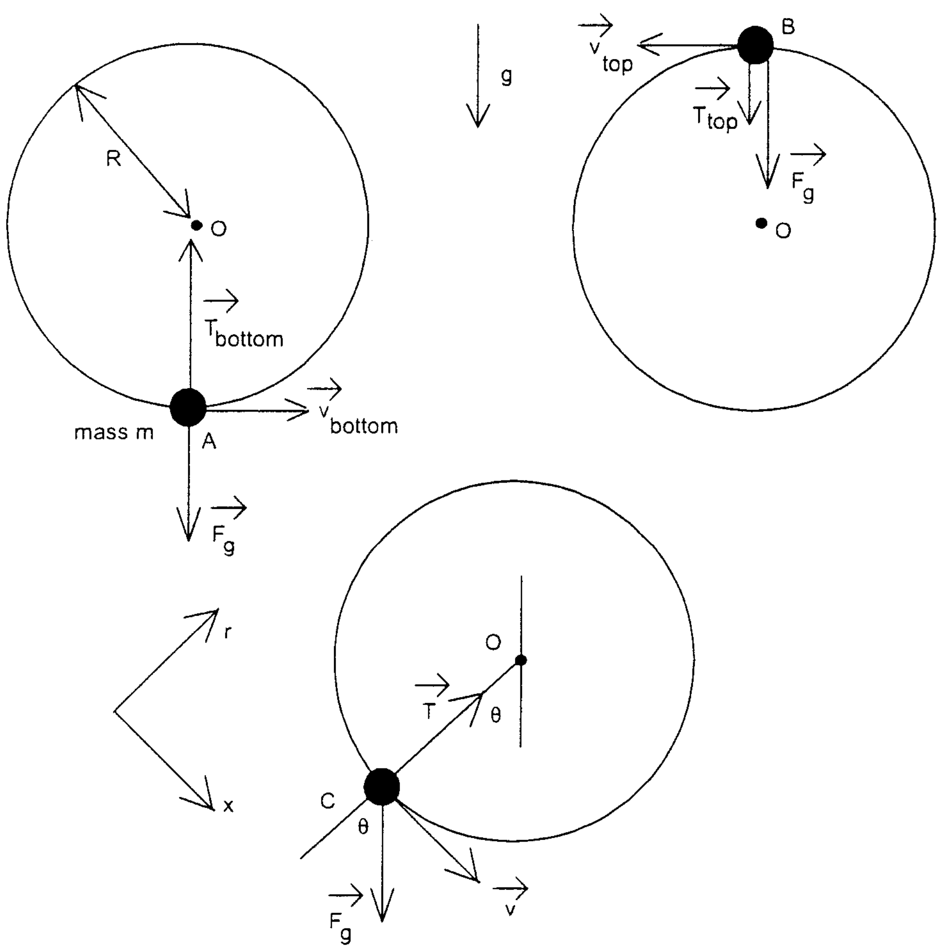
(c) Find an expression for the tension in the cord at time t in terms of the speed of the sphere, v(t), θ(t), m, and R.

(d) What is the minimum magnitude of the tension and at what position does it occur? What is the maximum magnitude of the tension and at what position does it occur?

(e) At what position is the cord most likely to break if the speed increases? Why?

Solution:

(a) There are only two forces acting on the mass: gravity and tension. The magnitude and direction of the tension force varies around the circle.



(b) Note that the speed of the sphere is not uniform because there is a tangential component of acceleration due to the weight of the sphere. Use the free-body diagram for position C, with the x-r coordinate system shown.

Apply Newton's Second Law: $\vec{F}_{net} = \vec{F}_g + \vec{T} = m\vec{a}$

$$F_{net,x} = F_{g,x} + T_x = ma_x$$

acceleration in the x (tangential) direction:

$$mg \sin\theta = ma_t$$
$$a_t = g \sin\theta$$

$$F_{net,r} = F_{g,r} + T_r = ma_r$$

acceleration in the r (radial) direction:

$$-mg\cos\theta + T = ma_r$$
$$a_r = \frac{T}{m} - g\cos\theta$$

(c) The tension in the cord at time t can be found using the equation of motion in the r direction

$$T(t) = ma_r(t) + mg\cos\theta(t)$$
$$= m\left(\frac{v(t)^2}{R} + g\cos\theta(t)\right)$$

(d) The minimum magnitude of the tension occurs when $\theta = 180°$ and $\cos\theta = -1$, at the top of the circle, and is:

$$T(t) = m\left(\frac{v(t)^2}{R} + g\cos\theta(t)\right) = m\left(\frac{v_{top}^{\ 2}}{R} + g\cos 180°\right) = m\left(\frac{v_{top}^{\ 2}}{R} - g\right)$$

At this point, the tangential acceleration is zero and the total acceleration is radial and directed downwards.

The maximum magnitude of the tension occurs when $\theta = 0°$ and $\cos\theta = +1$, at the bottom of the circle, and is:

$$T(t) = m\left(\frac{v(t)^2}{R} + g\cos\theta(t)\right) = m\left(\frac{v_{bottom}^{\ 2}}{R} + g\cos 0°\right) = m\left(\frac{v_{bottom}^{\ 2}}{R} + g\right)$$

At this point, the tangential acceleration is again zero and the total acceleration is radial and directed upwards.

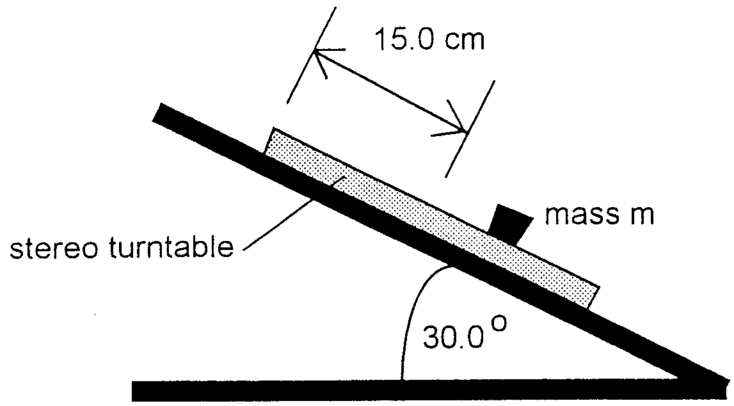
(e) If the average speed increases, then the cord is most likely to break at the bottom of the circle where the tension in the cord has its largest value.

6. An old stereo turntable of 15.0 cm radius turns at 33.0 rev/min while mounted on a plane inclined at 30.0° to the horizontal as shown in the figure below.

(a) If a mass m can be placed anywhere on the turntable without slipping, where is the most critical place on the disk where slipping might occur? Why?

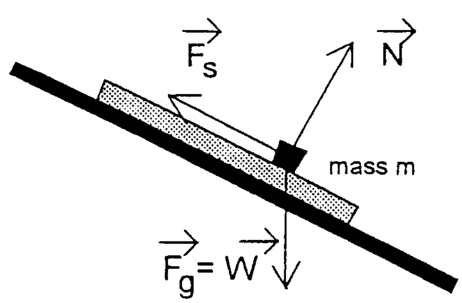
(b) Sketch a force diagram for mass m, indicating all of the forces acting on it.

(c) Calculate the minimum possible coefficient of friction that must exist if no slipping occurs. Is your answer independent of mass m? Is this a static or kinetic coefficient of friction?



Solution:

a) The location where the mass will first slip is at the lowest point in its circular path. At that location the static force of friction and the component of the weight along the incline are in opposite directions. Their difference must provide the total force toward the center of the circular motion that causes the centripetal acceleration.



b) The forces on the mass are:

1. the weight $\vec{w}$ of the particle, directed downward;
2. the normal force $\vec{N}$ of the surface on the particle, directed outward perpendicularly from the surface; and
3. the static force of friction $\vec{f}_s$ on the particle, acting in a direction to oppose slippage.

c) At the bottom of the circular path, $\vec{f}_s$ points up the incline and parallel to it towards the center of the circle.

Choose a coordinate system with $\hat{\imath}$ directed parallel to $\vec{f}_s$ when the particle is at the bottom of its circular path. Thus $\hat{\imath}$ points from the bottom of the circular path up the slope and parallel to it. Let $\hat{\jmath}$ point outward perpendicular to the surface and parallel to $\vec{N}$. Let $\theta = 30.0°$ be the angle of the incline, which is also the angle that both $\hat{\jmath}$ and $\vec{N}$ make with $-\vec{w}$. Write Newton's second law for each coordinate direction, and consider the mass about to slip so $f_{s\,\max} = \mu_s N$. Then the total force in the x direction creates the centripetal acceleration, while the total force in the y direction is zero.

x direction

$$F_{x\ \text{total}} = ma_x \implies f_{s\,\max} - mg\sin\theta = \frac{mv^2}{r}$$
$$\implies \mu_s N - mg\sin\theta = \frac{mv^2}{r}$$

y direction

$$F_{y\ \text{total}} = 0\ \text{N} \implies N - mg\cos\theta = 0\ \text{N}$$
$$\implies N = mg\cos\theta.$$

Substitute the expression for N from the y equation into the x equation.

$$\mu_s mg\cos\theta - mg\sin\theta = m\frac{v^2}{r} \implies \mu_s = \frac{mg\sin\theta + m\frac{v^2}{r}}{mg\cos\theta} = \tan\theta + \frac{v^2}{rg\cos\theta}.$$

(1) $$\mu_s = \tan\theta + \frac{v^2}{rg\cos\theta}.$$

Note that the result in independent of the mass. To evaluate this expression, you need to find the speed of the particle. First find its angular speed ω in rad/s.

$$\omega = 33\ \text{rev/min} = (33\ \text{rev/min})\left(\frac{2\pi\ \text{rad}}{\text{rev}}\right)\left(\frac{\text{min}}{60\ \text{s}}\right) = 3.5\ \text{rad/s}.$$

The speed of the particle in circular motion is

$$v = r\omega = (0.15\ \text{m})(3.5\ \text{rad/s}) = 0.53\ \text{m/s}.$$

Now substitute this value for v into equation (1).

$$\mu_s = \tan 30.0° + \frac{(0.53\ \text{m/s})^2}{(0.15\ \text{m})(9.81\ \text{m/s}^2)\cos 30.0°} = 0.80.$$

END