PHY 140Y – FOUNDATIONS OF PHYSICS
2001-2002
Problem Set #4

HANDED OUT: Friday, November 16, 2001 (in class).

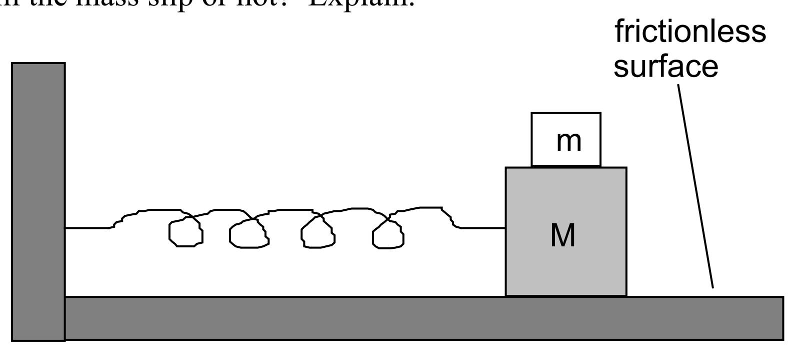
DUE: <u>**5:00 PM, Thursday, November 29, 2001**</u> in the appropriate box, labeled by tutorial group, in the basement at the bottom of the stairs leading down from MP202.

LATE PENALTY: 5 marks/day (which also applies to weekend days!) until 1:00 PM, Monday, December 3, after which it will not be accepted as solutions will then be available in tutorials and on the WWW.

NOTES: Answer all questions. A selected subset (3-4) will be marked out of 100%. Marks will be given for workings and units, as well as for final answers.

QUESTIONS:

1. A mass m rests on a block of mass M that is attached to a horizontal spring as indicated in the figure below. The block M rests on a frictionless surface. The coefficient of static friction for the contact surfaces of the two masses is μ_s and the corresponding coefficient of kinetic friction is μ_k. The system is set into simple harmonic oscillation with amplitude A. Identify the forces acting on mass m. What spring constant ensures that m is on the verge of slipping? If m is increased, will the mass slip or not? Explain.

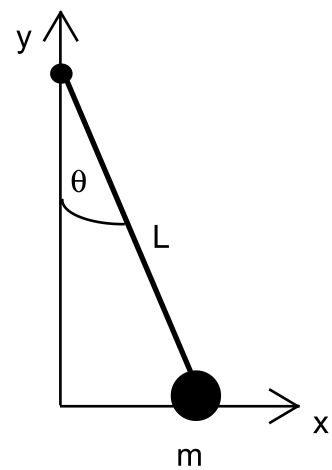


2. A 1.50-kg mass on a horizontal frictionless surface is attached to a horizontal spring with spring constant k = 200. N/m. The mass is in equilibrium at x = 0 m. The mass is released when t = 0 s at coordinate x = 0.100 m with a velocity (2.00 m/s) $\hat{i}$.
   (a) Find the constants A, ω, and φ (or δ as in class) in the equation: $x(t) = A\cos(\omega t + \phi)$.
   (b) Find the period of the oscillation.
   (c) Determine the maximum speed of the oscillation and the magnitude of the maximum acceleration.
   (d) Plot x(t) during the time interval from t = 0 s to t = 2T.

3. The pendulum inside a grandfather clock has a half period of 1.0000 s at a location where the magnitude of the local acceleration of gravity is 9.800 m/s^2. The clock is carefully moved to another location at the same temperature and is found to run slow by 89.0 s per day. What is the magnitude of the local acceleration due to gravity at the new location?

4. A plumb bob of length L is swung so that the mass on the end of the string moves in a horizontal circle at constant angular speed with the string making a constant small angle θ with the vertical direction, as shown in the adjacent figure. Show that if the motion is viewed sideways in the plane of the horizontal circle, then the motion is simple harmonic and of period

$$T = 2\pi\sqrt{\frac{L\cos\theta}{g}}.$$



5. (a) A 0.950-kg mass hangs vertically from a spring that has a spring constant of 8.50 N/m. The mass is set into vertical oscillation and after 600. s, you find that the amplitude of the oscillation is 1/10 that of the initial amplitude. What is the damping constant associated with this motion?

   (b) Show that if the position of a damped oscillator is given by $x(t) = Ae^{-\alpha t}\cos(\omega t)$, where $\alpha = \beta/(2m)$, the time interval for the amplitude to decrease to half its initial value is $(2m/\beta)\ln 2$.

6. (a) Show that as $v/c \to 1$, an approximate expression for the relativistic factor γ is

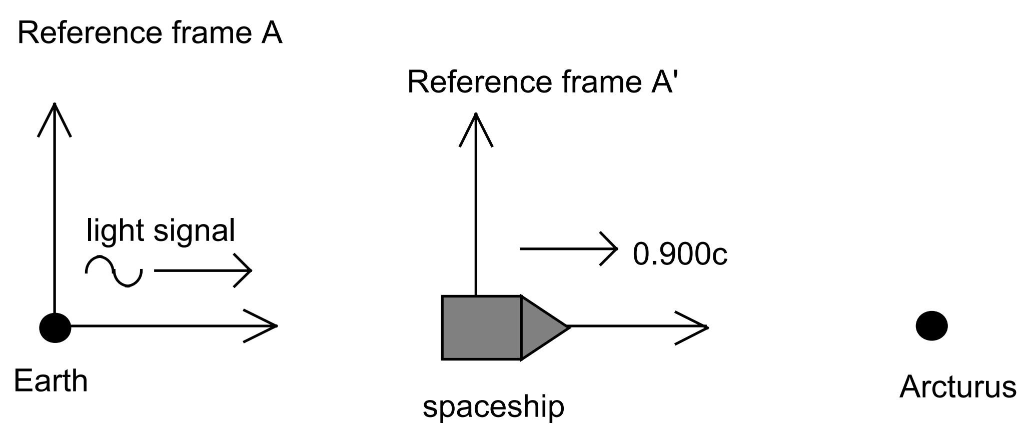
$$\gamma \cong \frac{1}{\sqrt{2\left(1-\frac{v}{c}\right)}}.$$

   (b) Close to the San Andreas Fault, south of San Francisco, lies the Stanford Linear Accelerator (SLAC) which is used to project subatomic particles to great speed. The accelerator is 3.0 km long and passes under a major highway. The accelerator is capable of accelerating electrons to speeds that have a relativistic factor $\gamma = 1.00 \times 10^4$. Calculate the ratio of the speed of the electrons to the speed of light. Use the result of part (a) to find v/c. Can you easily find v/c on your calculator without the result of part (a)? Comment.

   (c) In a reference frame in which the electrons are at rest, what is the length of the accelerator? Assume (incorrectly) that the speed of the electrons is constant along the length of the accelerator.

7. In an inertial reference frame S, the following observations are made: (i) a professor passes out a test at the origin and starts a stopwatch simultaneously, (ii) 10.0 s later, a student has a fit at $x = 9.0 \times 10^8$ m. The dean (located at the origin in another inertial reference frame S' in the standard geometry) is cruising by at a speed of 0.98c.
   (a) Calculate the relativistic factor γ.
   (b) Specify the space and time coordinates of the two events in S.
   (c) Use the Lorentz transformation equations to find the space and time coordinates of these events in the dean's reference frame.

8. You are in a spaceship of length 100. m moving at speed 0.900c to visit the star Arcturus. Back home, a friend notices that you forgot your toothbrush and so sends a light signal to the spaceship. Let A' be the reference frame of the spaceship and A be the reference frame of the friend back on Earth. The light signal arrives at the tail of the spaceship when t = t' = 0 sec.
   (a) When does the light signal reach the front of the spaceship according to the spaceship clock?
   (b) When does the light signal reach the front of the spaceship according to your friend's clock?
   (c) Are the answers to (a) and (b) related by the time dilation equation? Why or why not?
   (d) The light signal is reflected back to the tail of the spaceship by a mirror in the nose of the spaceship. When does the light signal reach tail according to the spaceship clock? When does the light signal reach tail according to your friend's clock?
   (e) Determine the total distance travelled by the light signal as it travels from the tail to the nose and then back to the tail according to (i) you, inside the spaceship, and (ii) your friend, back on Earth.



9. A politician of mass 70.0 kg is ejected from office and sent at a speed of 0.980c to a planet located 20.0 LY from Earth.
   (a) What is the kinetic energy of the politician according to us on Earth?
   (b) What is the kinetic energy of the politician according to the politician?
   (c) How many years (measured with clocks at rest on the Earth) will the one-way trip take?
   (d) According to the politician, how far away will the Earth be when the trip is completed (but the politician is still moving at the same speed)?
   (e) How many years will the trip take according to clocks at rest with respect to the politician?

10. A light source at point P, as shown in the figure below, is turned on at time $t = 0$ s for a very short interval and then extinguished.

(a) At what time does the light arrive at the origin O?

(b) At what time does the light arrive at a point A located a distance x from the origin along the x-axis?

(c) The spot of light moves from point O to point A in a time given by the difference between the answer to part (b) and the answer to part (a). Show that the average speed $\langle v \rangle$ of the light spot in moving along the x-axis from O to A is

$$\langle v \rangle = c\frac{a+\sqrt{a^2+x^2}}{x}.$$

(d) Show that when x is small, $\langle v \rangle \to \infty$, and when $x \to \infty$, $\langle v \rangle \to c$. The spot of light therefore, begins to move out along the x-axis at a speed *greater* than the speed of light and *slows down* to the speed of light as $x \to \infty$. Does this result contradict the special theory of relativity? Explain.