PHY 140Y - FOUNDATIONS OF PHYSICS
2000-2001
Problem Set #4

HANDED OUT: Friday, November 17, 2000 (in class).

DUE: **5:00 PM, Thursday, November 30, 2000 (in tutorial group drop-off boxes).**
Late penalty = 5 marks/day (which also applies to weekend days!) until 1:00 PM, Monday, December 4, after which it will not be accepted (as solutions will then be available in tutorials and on the WWW).

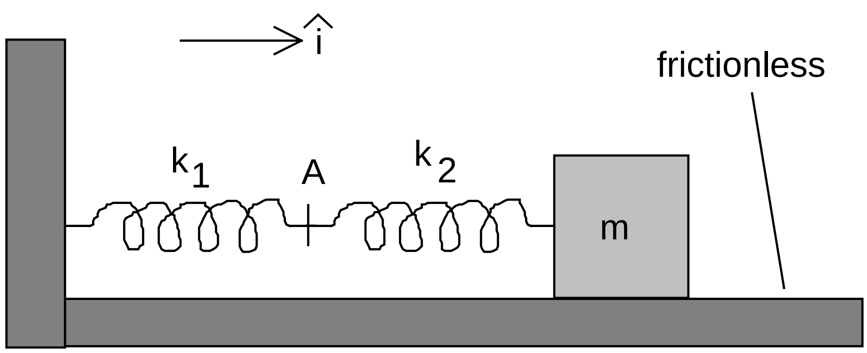
NOTES: Answer all questions.
50% will be awarded for making a reasonable attempt at all questions.
50% will be awarded for the answers to a selected subset of the questions.
Marks will be given for workings and units, as well as for final answers.

QUESTIONS:

1. Two springs with spring constants k_1 and k_2 are connected as shown in the figure below, with a mass m attached on a frictionless surface. We pull on m with a force $\vec{F}$ and hold m at rest. The mass m moves a distance x and point A moves a distance x' from their equilibrium positions.

   (a) By examining the forces on point A, show that $x' = \dfrac{k_2}{k_1 + k_2} x$.

   (b) By examining the forces on mass m, show that if the system is modelled with a single spring stretching and moving mass m a distance x, then the effective spring constant of the single spring is $k_{eff} = \dfrac{k_1 k_2}{k_1 + k_2}$.



2. A vertical spring is attached to the ceiling. An 8.00 kg frozen turkey is attached to the bottom of the spring and causes the spring to stretch an additional 10. cm. The turkey is then pulled down 4.0 cm beyond the equilibrium position and released at rest. Take the positive x-direction to be downward.

(a) What is the spring constant?
(b) If the oscillation is described by $x(t) = A\cos(\omega t + \phi)$, find A, ω, and ϕ.
(c) What is the maximum speed of the turkey?
(d) What is the magnitude of its maximum acceleration?

3. A phonograph record with a diameter of 30.0 cm has a cricket sitting on its rim. The record and cricket are rotating at 33.3 rev/min. The setting Sun casts a shadow of the cricket on a wall directly facing the Sun.
(a) Does the shadow move in simple harmonic motion? Explain.
(b) What is the amplitude of the motion of the shadow?
(c) What is its period?
(d) What is the maximum speed of the shadow?
(e) What is the magnitude of the maximum acceleration of the shadow?
(f) When the acceleration of the shadow is zero, is the velocity of the shadow also zero?
(g) What are the forces acting on the cricket? Sketch an appropriate force diagram.

4. (a) Show that if the position of a damped oscillator is given by $x(t) = Ae^{-\alpha t}\cos(\omega t)$, where $\alpha = \beta/(2m)$, the time interval for the amplitude to decrease to half its initial value is $(2m/\beta)\ln 2$.
(b) The suspension system of your 1.20×10^3 kg car has a natural oscillation frequency of 2.0 Hz without functioning shock absorbers. The shock absorbers usually provide critical damping to the system. Along a worn section of concrete highway, you notice that the concrete joints, spaced 10.0 m apart, set up a violent oscillation of the car, making it almost uncontrollable. At what speed are you driving?
(c) Country dirt roads frequently become rippled, and when you drive over them at a certain speed, the bumps set up resonance in the car's suspension system, making control of the car problematic. If the ripples are 0.50 m apart and you find that resonance occurs when you are driving at 25 km/hour over them, what is the natural oscillation frequency of the suspension system in your car?

5. A clock at rest in an inertial reference frame S' is moving directly away from you (in another inertial reference frame S) at speed v = 0.949c.
(a) What is the relativistic factor γ?
(b) If the moving clock indicates an interval of 24.0 hr has elapsed, what interval has elapsed on your watch?
(c) Are your answers the same if the moving clock is coming directly toward you?

6. You are rushing off to an 8 AM class at a constant speed. Your roommate, at rest, nonchalantly observes that you travel the 100.0 m length of a long corridor in 5.00×10^{-7} s.
(a) What is your speed?
(b) According to rulers at rest with respect to you, what is the length of the corridor?
(c) How long does it take you to cover the distance according to your watch?

7. An RCMP officer is riding along a straight trail into town at a speed of 0.867c. Two gunslingers are glaring at each other with only 50.0 m of dirt between them (oriented parallel to the line along which the officer is riding), according to witnesses cowering behind hitching posts along the street. The two desperadoes draw and fire simultaneously according to them.

(a) Determine the value of the relativistic factor γ.

(b) How far apart are the desperadoes according to the RCMP officer?

(c) Carefully sketch and define two appropriate reference frames. Then define two events corresponding to the shots fired by the two desperadoes. Transform the events into the other reference frame.

(d) Which gunslinger fired first according to the reference frame of the RCMP officer?

(e) How does your answer to part (d) change if one gunslinger fires 0.5 μs after the other?

8. Two plane mirrors are located in the A' reference frame. One mirror, M_1, is at x' = 0 m, while the other mirror, M_2, is at x' = 5.0 m, as shown in the figure below. The A' reference frame, with its mirrors, is moving at high speed v = 0.90c relative to a reference frame A (in the standard geometry). The origins of the two reference frames coincide when t = t' = 0 s. A photon (a particle of light) bounces back and forth between the two mirrors and is reflected from mirror M_1 when t = t' = 0 s.

(a) In reference frame A', when does the photon arrive at mirror M_2?

(b) Specify the space-time coordinates (i.e., x' and t') of the two events in A' corresponding to the photon at mirror M_1 and the photon at mirror M_2.

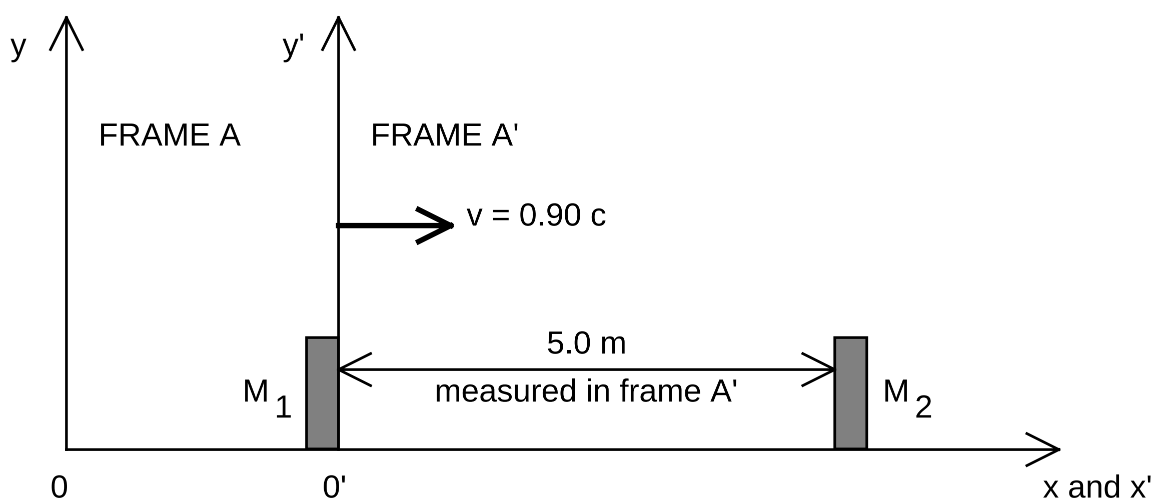
(c) In reference frame A, when does the photon arrive at mirror M_2?

(d) In reference frame A, where on the x-axis is mirror M_2 when the photon arrives at that mirror?

(e) What is the distance Δx in reference frame A that the photon travels while moving from mirror M_1 to mirror M_2?

(f) What is the time interval Δt in reference frame A that it takes the photon to travel from mirror M_1 to mirror M_2?

(g) Calculate the ratio $\Delta x/\Delta t$. Is this what you expect to obtain? Why or why not?



9. A particle in the S' reference frame has a velocity $\vec{u}' = 0.779c\hat{i}' + 0.450c\hat{j}'$. The S' frame is moving in the standard geometry with speed 0.950c with respect to reference frame S.
(a) Sketch the situation.
(b) What is the angle θ' that $\vec{u}'$ makes with the x'-axis in S'?
(c) What is the speed u' of the particle in the S' frame when measured with clocks and rulers at rest in S'?
(d) Find the velocity components u_x and u_y of the particle in the S reference frame.
(e) What is the speed u of the particle in S, measured using rulers and clocks in that frame?
(f) What angle θ does the velocity vector $\vec{u}$ make with the x-axis in reference frame S?

10. A politician of mass 70.0 kg is ejected from office and sent at a speed of 0.980c to a planet located 20.0 LY from Earth.
(a) What is the kinetic energy of the politician according to us on Earth?
(b) What is the kinetic energy of the politician according to the politician?
(c) How many years (measured with clocks at rest on the Earth) will the one-way trip take?
(d) According to the politician, how far away will the Earth be when the trip is completed (but the politician is still moving at the same speed)?
(e) How many years will the trip take according to clocks at rest with respect to the politician?