# PHY 140Y – FOUNDATIONS OF PHYSICS
# 2001-2002
# Problem Set #3

**HANDED OUT:** Friday, November 2, 2001 (in class).

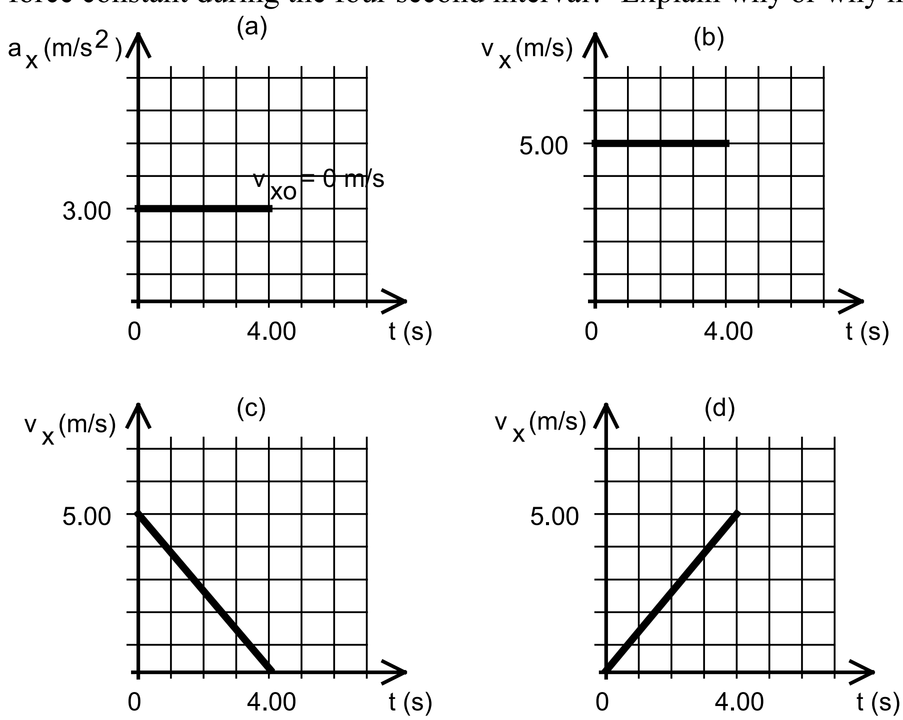
**DUE:** <u>5:00 PM, Thursday, November 15, 2001</u> in the appropriate box, labeled by tutorial group, in the basement at the bottom of the stairs leading down from MP202.

**LATE PENALTY:** 5 marks/day (which also applies to weekend days!) until 1:00 PM, Monday, November 19, after which it will not be accepted as solutions will then be available in tutorials and on the WWW.

**NOTES:** Answer all questions. A selected subset (3-4) will be marked out of 100%. Marks will be given for workings and units, as well as for final answers.

**QUESTIONS:**

1. Each graph in the figure below describes the one-dimensional motion of a 5.00 kg system during a 4.00 s interval. For each case, answer the following questions:
   (a) What is the work done on the system by the total force on the system during the interval?
   (b) Find an expression for the kinetic energy as a function of time that is valid during the four second interval.
   (c) What is the average power of the total force on the system?
   (d) Find P(t) at any instant during the four second interval. Is the instantaneous power of the total force constant during the four second interval? Explain why or why not.

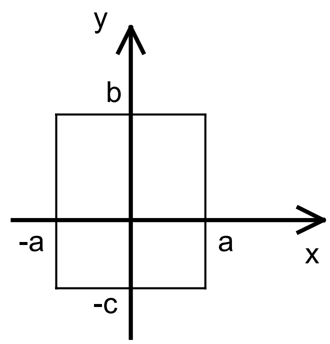


2. A force $\vec{F} = (2.00\text{N})\hat{i} + (1.00\text{N})\hat{j}$ is one force on a system as it executes the rectangular path shown in the figure below.

(a) Find the work done on the system by this force if the path is traversed in the clockwise sense.

(b) Find the work done on the system by this force if the path is traversed in the counterclockwise sense.

(c) Is this force conservative or nonconservative? Explain.




3. A system of two identical springs, each with spring constant k, is used to accelerate a mass m from rest to as great a speed as possible on a frictionless surface. Various arrangements of the springs are possible: (i) use just one spring; (ii) use two springs in series; or (iii) use two springs in parallel, as shown in the figure below. Regardless of which design is used, the spring system can be set the same distance $x_o$ from the equilibrium position of mass m.

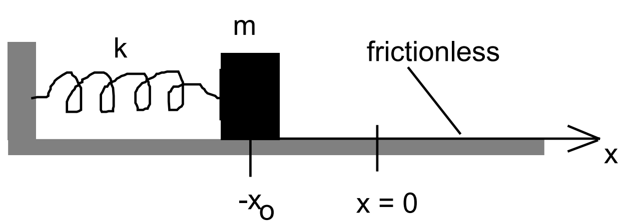
(a) Find the maximum potential energy of m in each spring system.

(b) Find an expression for the maximum speed of m as it leaves each system.

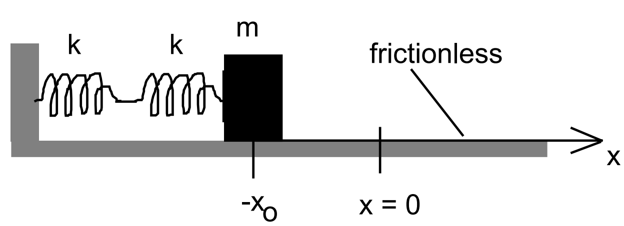
(c) To give m the greatest possible speed, which arrangement should be used (or does it not make any difference)?

*Hint: The effective spring constant of two identical springs, each having spring constant k, is k/2 when they are attached in series, and is 2k when they are connected in parallel.*

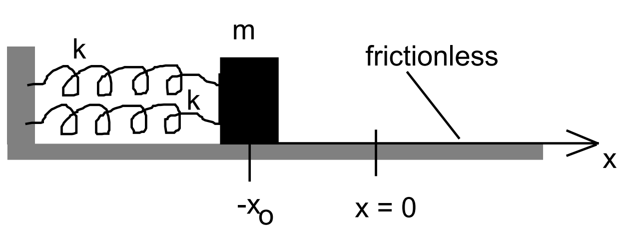
(i) single spring



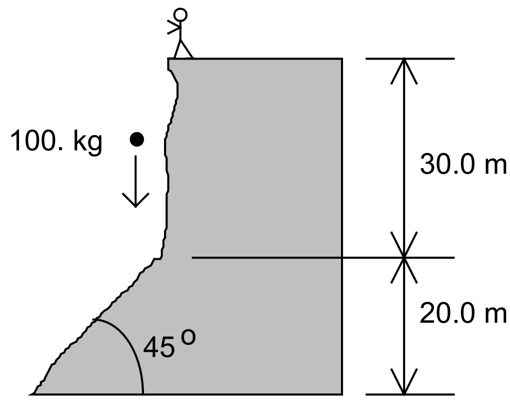

(ii) springs in series




(iii) springs in parallel



4. A hiker pushes a 100. kg boulder off a cliff as pictured in the figure below (not a smart idea!). The rock falls vertically for 30.0 m and then slides down a talus (gravel) slope. At the bottom of the slope and at a speed of 2.00 m/s, the rock passes some astonished park rangers who subsequently set off in hot pursuit of the hiker.
(a) How much work was done by the kinetic frictional force as the rock slid down the slope to the point where it passed the rangers?
(b) Assuming the kinetic frictional force is of constant magnitude, find the coefficient of kinetic friction for the boulder on the slope. In your answer, include a force diagram for the rock on the slope.




5. A popular recreation for the fearless is jumping from tall structures while attached to an elastic bungee cord to ensure that less than catastrophic consequences result from the fall. Model the bungee cord as an ideal spring. A student of mass m drops off a tower of height h, tethered to this bungee cord. The elasticity of the bungee cord is chosen so that its effective spring constant is $k = 2mg/h$. The jump is begun at zero speed at altitude h with the bungee cord fastened vertically above the jumper, barely taut (with no slack) but under no initial tension.
(a) Choose a suitable coordinate system to analyze this problem.
(b) What is the initial total mechanical energy of the bungee jumper?
(c) What is the kinetic energy of the jumper at the ground? What is the jumper's speed at this point?
(d) At what height above the ground is the speed of the jumper a maximum?
(e) Will the same bungee cord be safe to use for a jump from the same tower (of height h) for a person with mass $m'<m$? With mass $m'>m$? Explain your reasoning.

6. (a) A system of mass m is traveling at a large initial speed $v_o$. A small amount of work $\Delta W_{total}$ is done by all the forces acting on the system. Begin with the Work-Energy Theorum in the form $W_{total} = \Delta\ KE$ and show that the change in the speed $\Delta v$ of the system is approximately given by

$$\Delta v \approx \frac{\Delta W_{total}}{m v_o}.$$

Explain any assumptions made. Use this equation to estimate the change in the speed of a 10.0 kg mass initially travelling at 30.0 m/s when 1.00 J of work is done on it by the total force on the system.

(b) A system of mass m is undergoing vertical motion near the surface of the Earth with the gravitational force of the Earth the only force acting on the system. For such vertical motion near the surface of the Earth, the total mechanical energy of a system of mass m is

$$E = K + U = KE + PE = \frac{1}{2} m \left( \frac{dy}{dt} \right)^2 + mgy$$

where y is postive upwards. The system has constant vertical (downward) acceleration and the position of the system at any time is

$$y(t) = y_o + v_{oy} t + \tfrac{1}{2} g t^2 .$$

Use this in the expression for E to show that E is independent of time (that is, the total mechanical energy is conserved), and to find E in terms of $y_o$, $v_{yo}$, m, and g. The result should not be unexpected.

7. A particle of mass m is dropped from rest from a height h in the uniform gravitational field near the surface of the Earth as shown in the figure below. For convenience, the indicated coordinate system has been chosen to analyze this problem.

(a) For review, starting with constant acceleration, find expressions for the y-component of velocity, $v_y(t)$, and the y coordinate of the particle, y(t). Also show that the time-of-flight of the particle is $T = \sqrt{\frac{2h}{g}}$ .

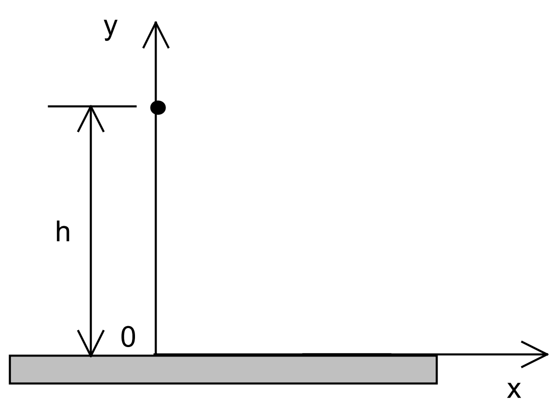
(b) The potential energy of the particle is time dependent. The following equation gives the average value of a time-dependent function f(t):

$$f_{ave} = \frac{1}{T} \int_0^T f(t)\, dt .$$

Use this equation to derive an equation for the time-average value of the potential energy during the time-of-flight T in terms of m, g, and h.

(c) The kinetic energy of the particle is also time dependent. Derive an equation for the time-average value of the kinetic energy of the particle during the time-of-flight T in terms of m, g, and h.

(d) How are $PE_{ave}$ and $K_{ave}$ related? *(This relationship is a consequence of a theorum in advanced mechanics known as the virial theorum.)*



8. A mass m is attached to a spring of spring constant k and is oscillating horizontally at angular frequency $\omega$ on a frictionless surface. The mass is released at rest when t = 0 s at the position x = A.

(a) What is the position x of the oscillator as a function of time?

(b) What is the velocity component $v = \frac{dx}{dt}$ as a function of time?

(c) What is the instantaneous power of the spring force when t = 0 s? Explain your answer.

(d) What is the instantaneous power of the spring force when the oscillator is at position x = 0 m? Explain your answer.

(The results of parts (c) and (d) imply that the oscillator reaches a peak power sometime during the first quarter of a complete oscillation.)

(e) Show that the instantaneous power P(t) of the oscillator force is $P(t) = \frac{k\omega A^2}{2}\sin(2\omega t)$.

(f) Show that the power is at a maximum for the first time when t is equal to one-eighth the period of oscillation.

(g) What is the average power of the oscillator force during one period of the oscillation?

(h) Make graphs of $F_{x,\,spring}(t)$, $v_x(t)$, and P(t) [the result of part (e)] versus time during one period of the oscillation if the mass is 1.00 kg, the spring constant is 49.0 N/m, and the amplitude of the oscillation is 0.100 m. What conclusions can be drawn from these graphs?

9. The potential energy function of a conservative force acting on a system moving in one dimension along the x-axis is described by U = PE = C |x|, where C is a positive constant. This conservative force is the only force on the system.

(a) What are the SI units of C?

(b) What is the x-component of the force on the system?

(c) Make a schematic graph of the potential energy as a function of x.

(d) Let the total mechanical energy E of the system be greater than zero. Sketch a representative value for E > 0 J on your graph of the potential energy function. Do turning points exist? If so, how many? Indicate prohibited values of x on the graph.

(e) Describe the motion of the system. Is the motion simple harmonic motion? In what ways is the motion similar to simple harmonic oscillation? In what ways is it different?

10. A pendulum of mass m and length l makes an angle $\theta$ with the vertical direction.

(a) Find an expression for the gravitational potential energy of m as a function of $\theta$ so that the potential energy is zero when $\theta$ = 0 rad.

(b) Make a sketch of this potential energy function in units of mgl as a function of $\theta$ over the range $-\pi/2$ rad $\le \theta \le \pi/2$ rad.

(c) Let $PE_{max}$ be the maximum potential energy of m over this range of $\theta$. Let the total mechanical energy E of m be constant and have a value of $E = PE_{max}/2$. What are the turning points of a pendulum with such a value for the total mechanical energy?