PHY 140Y - FOUNDATIONS OF PHYSICS
2000-2001
Problem Set #3

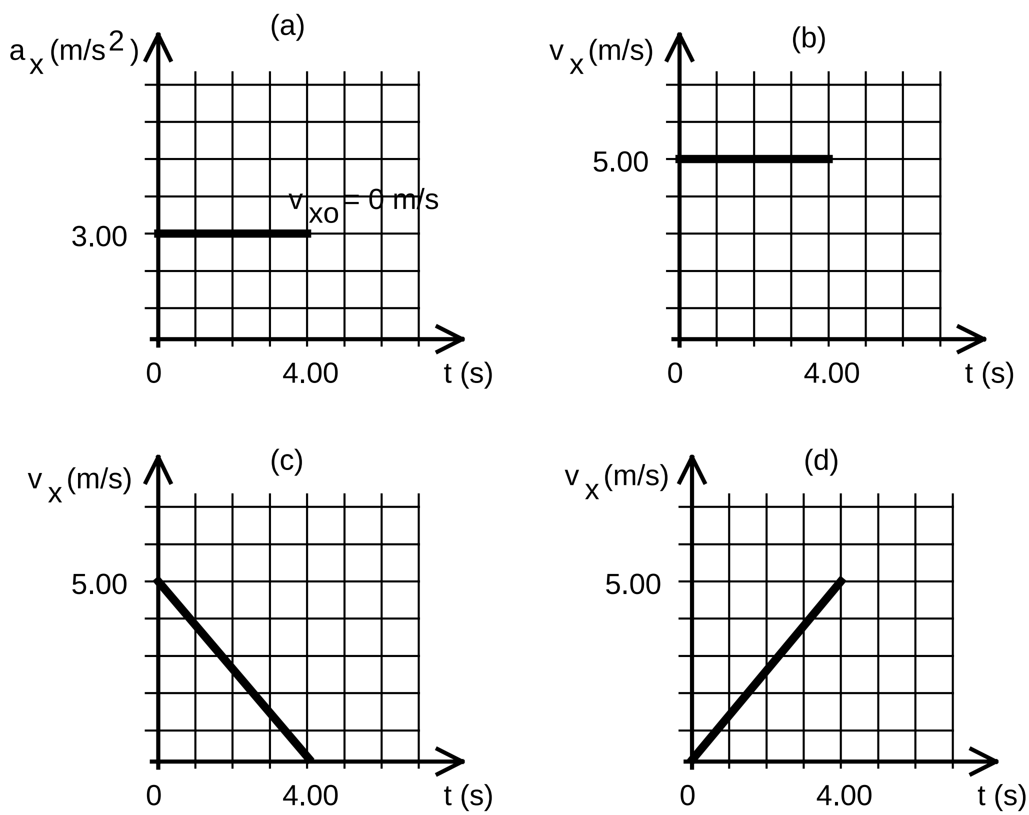
HANDED OUT: Friday, November 3, 2000 (in class).

DUE: **<u>5:00 PM, Thursday, November 16, 2000 (in tutorial group drop-off boxes).</u>**
Late penalty = 5 marks/day (which also applies to weekend days!) until 1:00 PM, Monday, November 20, after which it will not be accepted (as solutions will then be available in tutorials and on the WWW).

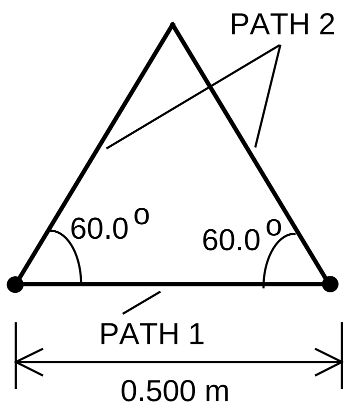
NOTES: Answer all questions.
50% will be awarded for making a reasonable attempt at all questions.
50% will be awarded for the answers to a selected subset of the questions.
Marks will be given for workings and units, as well as for final answers.

QUESTIONS:

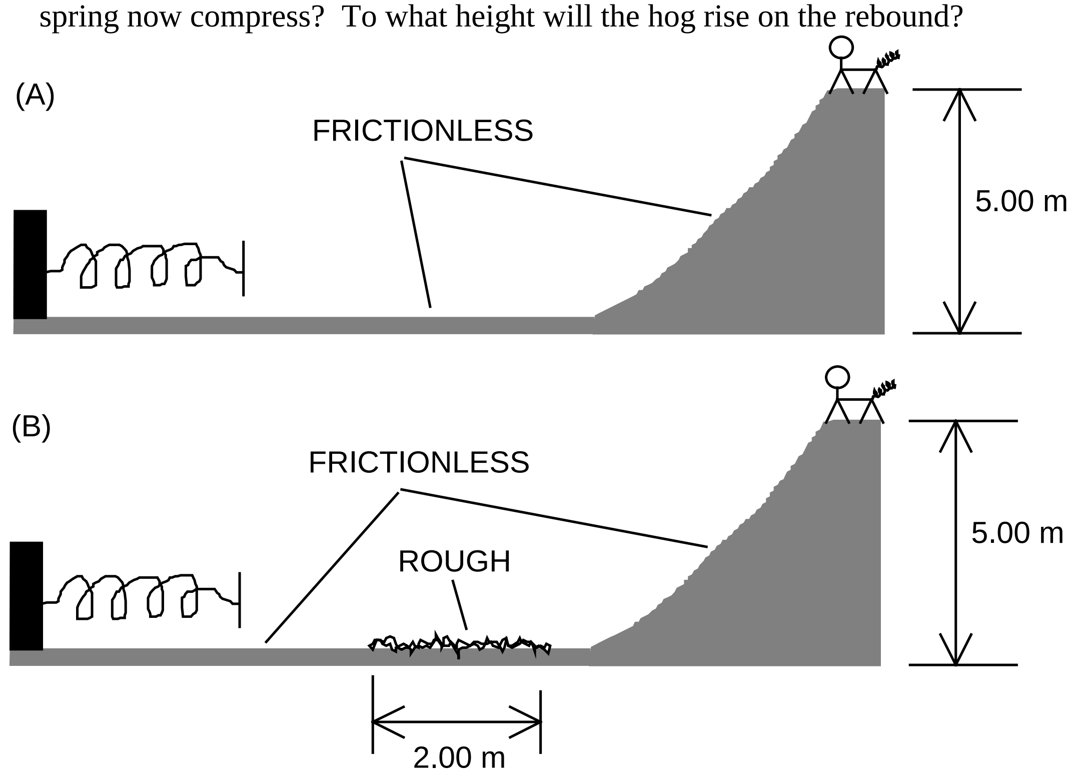
1. Each graph in the figure below describes the one-dimensional motion of a 5.00 kg system during a 4.00 s interval. For each case, what is the work done on the system by the total force on the system during the interval?



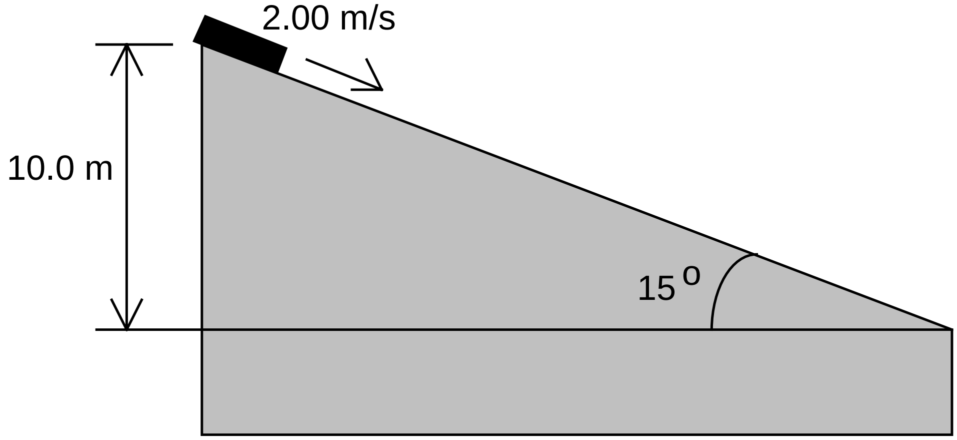
2. A 3.00 kg mass is on a rough horizontal table. The coefficient of kinetic friction for the mass on the table surface is $\mu_k = 0.25$. The mass is pushed slowly between points A and B via the two different routes shown in the figure below. Find the work done by the kinetic frictional force along each path. Is this force conservative or nonconservative? Explain your answer.



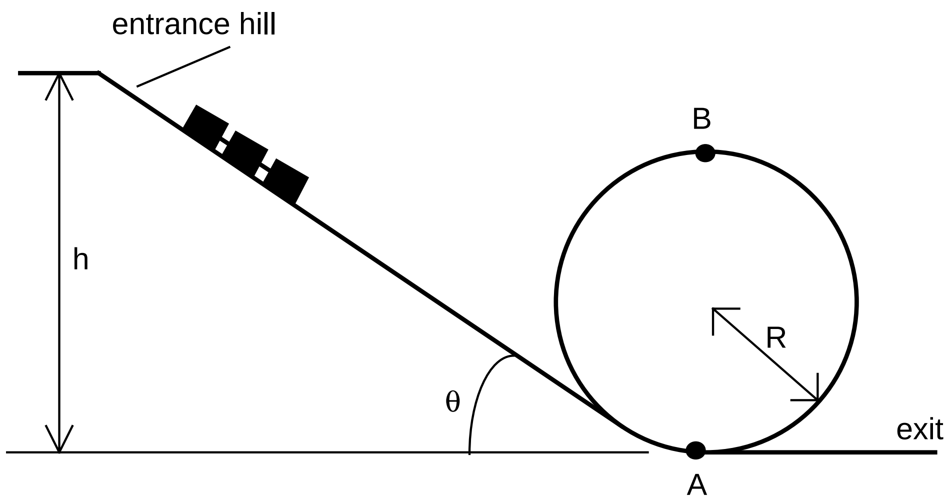
3. A 100. kg warthog squeals with delight as it slides from rest down the greased (frictionless) slope and plane in figure (A) below. The hog encounters a spring that compresses by 150. cm before bringing the hog to rest for an instant.
(a) What is the spring constant of this spring?
(b) The hog is sent back in the opposite direction by the spring. To what height will the hog rebound up the slope?
(c) Suppose the hog begins anew at the 5.00 m height, but now encounters a 2.00 m stretch of level rough ground on the way to the spring and back (see figure (B) below). The coefficient of kinetic friction for the hog on the rough ground is 0.30. How much will the spring now compress? To what height will the hog rise on the rebound?



4. A student flings a 2.00 kg physics textbook down a rough inclined plane with an initial speed of 2.00 m/s, as indicated in the figure below. As luck would have it, the book stops right at the bottom of the incline, just before the precipice.
   (a) What is the work done by the gravitational force on the book?
   (b) What is the work done by the normal force on the book?
   (c) What is the change in the kinetic energy of the book?
   (d) What is the change in the gravitational potential energy of the book?
   (e) What is the value of the coefficient of kinetic friction for the book on the incline?



5. You have been assigned the exciting task of designing new roller coasters. The track is to include a loop-the-loop of radius R as shown in the figure below. To a first approximation, ignore all frictional effects. The approach to the loop is made by starting the roller coaster vehicle (of mass m) at zero speed down an incline of height h, as indicated.
   (a) What is the speed of the vehicle at the beginning of the loop (point A)?
   (b) What is the speed of the vehicle at point B if the force of the track on the vehicle at this point is zero?
   (c) Find the ratio h/R such that the force of the track on the vehicle at point B becomes zero.
   (d) What is the speed of the vehicle as it exits the loop?
   (e) What is the total work done by the gravitational force on the vehicle from the top of the incline to the exit from the loop?
   (f) Which of your answers to parts (a)-(e) depend on the mass m of the vehicle?



6. When $t = 0$ s, a mass m is projected at speed v_o vertically upward.

(a) Show that the instantaneous power of the gravitational force at any instant is

$$P(t) = mg^2 t - mgv_o .$$

(b) Sketch a graph of P(t) versus t, labelling appropriate points.

(c) At what point in the motion is the power zero?

(d) What prevents the power from increasing indefinitely?

(e) What is the physical significance of the area under the P(t) vs. t curve?

7. For each of the four cases shown in the figure for Problem 1, answer the following questions.

(a) Find an expression for the kinetic energy as a function of time that is valid during the four second interval.

(b) What is the average power of the total force on the system?

(c) Find P(t) at any instant during the four second interval. Is the instantaneous power of the total force constant during the four second interval? Explain why or why not.

8. A 3.00 kg brick is hurled vertically upward at an initial speed of 5.00 m/s from a height of 20.0 m. Choose $\hat{j}$ to be upward and place the y-coordinate origin at ground level.

(a) Find the total mechanical energy E of the brick.

(b) Make a graph of the potential energy of the brick as a function of y.

(c) Plot the total mechanical energy E on the graph.

(d) For a general y-coordinate, indicate on the graph what represents the kinetic and potential energies of the brick.

(e) Is there a turning point?

(f) What y-coordinates are prohibited for the brick?

9. The total force on a system of mass m varies with position according to $\vec{F} = F_o \cos\left(\frac{2\pi x}{L}\right)\hat{i}$.

(a) Calculate the work done on the system by this force as the system moves from $x = 0$ to $x = L/2$.

(b) Calculate the work done on the system by this force as the system moves from $x = L/2$ to $x = 0$.

(c) Is the force conservative? If so, proceed with the following questions. If not, take the rest of the day off.

(d) Show that the following potential energy function is appropriate for this force:

$$U(x) = PE(x) = -\frac{F_o L}{2\pi}\sin\left(\frac{2\pi x}{L}\right).$$

(e) Sketch the potential energy function over the domain $-L \le x \le L$.

(f) The system is released at x = 0 m with zero speed. What is the total mechanical energy E of the system? Plot this value of E on the energy diagram of part (c).

(g) Describe the motion of the system, giving particular attention to whether the system is bounded in its motion by turning points (and if so, at what x values), and whether the system is oscillatory (and if so, whether the oscillation is simple harmonic or not).

(h) If the total mechanical energy of the system is somehow decreased slightly to E' < E, what happens to the motion of the system?

(i) For what values of E is the motion unbounded?

10. A mass m is attached to a spring of spring constant k and is oscillating horizontally at angular frequency ω on a frictionless surface. The mass is released at rest when t = 0 s at the position x = A.

(a) What is the position x of the oscillator as a function of time?

(b) What is the velocity component $v = \frac{dx}{dt}$ as a function of time?

(c) What is the instantaneous power of the spring force when t = 0 s? Explain your answer.

(d) What is the instantaneous power of the spring force when the oscillator is at position x = 0 m? Explain your answer.

(The results of parts (c) and (d) imply that the oscillator reaches a peak power sometime during the first quarter of a complete oscillation.)

(e) Show that the instantaneous power P(t) of the oscillator force is $P(t) = \frac{k\omega A^2}{2}\sin(2\omega t)$.

(f) Show that the power is at a maximum for the first time when t is equal to one-eighth the period of oscillation.

(g) What is the average power of the oscillator force during one period of the oscillation?

(h) Make graphs of $F_{x,\ spring}(t)$, $v_x(t)$, and P(t) [the result of part (e)] versus time during one period of the oscillation if the mass is 1.00 kg, the spring constant is 49.0 N/m, and the amplitude of the oscillation is 0.100 m. What conclusions can be drawn from these graphs?