PHY 140Y - FOUNDATIONS OF PHYSICS
2000-2001
Problem Set #2

HANDED OUT: Friday, October 6, 2000 (in class).

DUE: **5:00 PM, Thursday, October 19, 2000 (in tutorial group drop-off boxes).**
Late penalty = 5 marks/day (which also applies to weekend days!) until 1:00 PM, Monday, October 23, after which it will not be accepted (as solutions will then be available in tutorials and on the WWW).

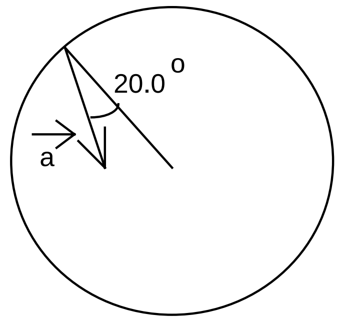
NOTES: Answer all questions.
50% will be awarded for making a reasonable attempt at all questions.
50% will be awarded for the answers to a selected subset of the questions.
Marks will be given for workings and units, as well as for final answers.

QUESTIONS:

1. A pilot wishes to fly due southeast. A wind is blowing to the west at a speed of 120 km/hr. The airspeed of the airplane is 700. km/hr. Determine
   (a) the speed of the plane with respect to the ground
   (b) the angle at which the plane should be aimed with respect to the direction south.

2. Two identical swimmers have stroke frequencies of one complete stroke per second (a right and a left stroke make up one complete stroke). They swim at a speed of 3.00 m/s in still water. They start swimming simultaneously from a fixed rock in a uniformly flowing river whose current speed is v. One swimmer goes 100. m downstream and then returns to the starting point. The other swimmer moves directly across the stream (along a path perpendicular to the current) to a point 100. m across the river, and returns via the same route to the starting point. Both swimmers execute perfect turns, taking zero time. The swimmer who went downstream and back returns to the starting point 5.00 s later than the cross-stream swimmer.
   (a) By how many stroke periods (the time for one right plus one left stroke) does the cross-stream swimmer win?
   (b) Find expressions for the times the swimmers take to complete their courses in terms of the current speed v. Draw a vector diagram showing the relevant velocities of the swimmers and the current.
   (c) Calculate the speed of the current v from the given data.

3. A particle moving along a circle of radius 4.00 m has an instantaneous total acceleration of magnitude 40.0 m/s^2 directed as indicated in the figure below.
(a) What is the magnitude of the centripetal acceleration?
(b) What is the magnitude of the tangential acceleration?
(c) What is the angular velocity at the instant indicated?
(d) Is the particle speeding up or slowing down in its circular motion?
(e) Does the centripetal acceleration have constant magnitude?



4. You may have noticed that as a cassette tape is played, the radius of the tape left on the supply spool decreases slowly with time as the tape is drawn through the machine at a constant speed v_o. Because the radius of the tape remaining changes slowly compared with the speed of the tape through the machine, to a good approximation we can write:

$$v_o = r\frac{d\theta}{dt} = r\omega > 0$$

where ω is the angular speed of the tape reel. Notice that the tape speed v_o is constant, but radius r changes slowly with time. Therefore the angular speed of the reel also changes slowly with time. Assume that r represents the average radius of each coiled layer (since the tape is actually wound in a spiral). Let r_o be the initial radius of the tape when t = 0 s and $\theta_o = 0$ rad, where θ is the total angle of tape unwound from the spool. Represent the thickness of the tape by b. When the tape has unwound through a total angle θ, the radius of tape remaining is

$$r = r_o - b\frac{\theta}{2\pi}.$$

(a) Show that $\frac{dr}{dt} = -\frac{b}{2\pi}\frac{d\theta}{dt} = -\frac{b}{2\pi}\frac{v_o}{r}$.

(b) Integrate the result of part (a) to show that $\pi(r_o^2 - r^2) = bv_o t$.

What is the geometrical interpretation of the left-hand side of this equation? What is the physical interpretation of the right-hand side of this equation? Illustrate each with a sketch.

(c) Solve the result of part (b) for r and substitute this for r in Equation (1). Integrate the resulting expression to show that the total angle θ through which the tape has turned at time t is $\theta = \frac{2\pi}{b}\left\{r_o - \sqrt{r_o^2 - \frac{bv_o t}{\pi}}\right\}$.

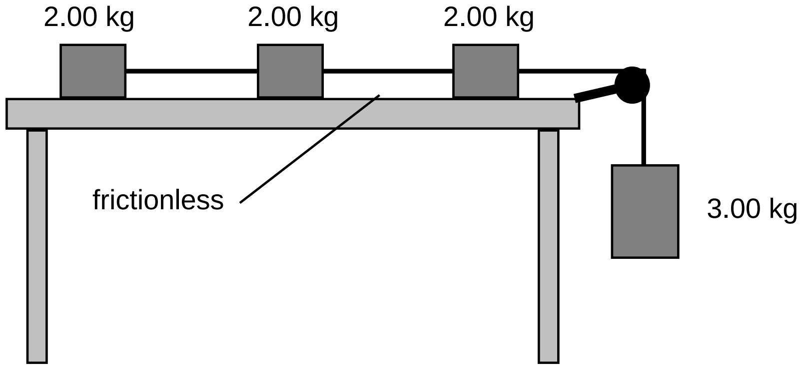
Note: The following integration formula may be useful: $\int \frac{-a}{\sqrt{b - at}}\,dt = 2\sqrt{b - at} + C$.

5. Two forces act simultaneously on a 4.0 kg mass. One force is $\vec{F}_1 = (-3.0\ \mathrm{N})\hat{i} + (6.5\ \mathrm{N})\hat{j}$ and the other has a magnitude 5.0 N directed at 240° counterclockwise from the +x-axis. Find the magnitude and direction of the acceleration.

6. A 1.50 kg rock is in the middle of the bottom of an open bucket of mass 2.00 kg. The bucket and rock rotate in a vertical circle of radius 1.20 m at a constant speed of 4.00 m/s.
(a) Draw a force diagram indicating the forces on the rock when it is at the lowest point in its motion.
(b) What is the magnitude of the total force on the rock when the bucket is at the lowest point of its circular path?
(c) What is the force of the bucket on the rock at this lowest point?
(d) At the highest point on the circular path, will the rock fall from the bottom of the bucket? Indicate your reasoning with an appropriate calculation and explanation.

7. You are sitting in the rear seat of an automobile that accelerates uniformly from rest to a speed of 120 km/hr during 6.00 s along a straight stretch of level highway. The seat has a horizontal surface and a vertical seat back. Your mass is 75.0 kg.
(a) Draw a force diagram indicating all forces on you in the horizontal and vertical directions, from the viewpoint of an observer at rest outside the car on the side of the highway.
(b) Determine the magnitude of your acceleration.
(c) What is the magnitude of the total force on you? In which direction does the total force act?
(d) Identify all of the forces which are the Newton's Third Law pairs to the forces identified in part (a), and state the objects on which they act and their direction.

8. Three 2.00 kg decoy ducks on a frictionless table are connected in series by strings as indicated in the figure below (pretend the blocks look like ducks). The final string passes over an ideal pulley at the edge of the table and suspends a larger 3.00 kg decoy. The ducks are initially at rest.
(a) Which string has the least tension and why?
(b) Which string has the greatest tension? Show that the greatest tension has a magnitude less than 29.4 N.
(c) Do you expect the magnitude of the acceleration of the ducky system to be less than, greater than, or equal to g?
(d) Calculate the magnitude of the acceleration of the system of ducks.



9. A candy bar of mass m rests on a tray of mass M that is on a horizontal frictionless surface. The coefficient of static friction between the candy bar and the upper surface of the tray is μ_s. A constant horizontal force F on the tray accelerates the tray and the candy bar to the right.

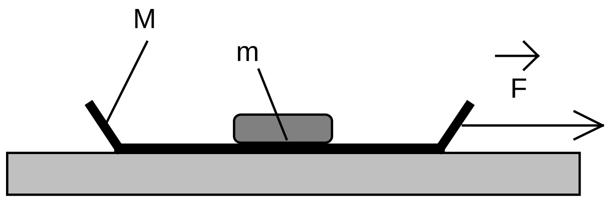
(a) Draw a free-body diagram indicating the forces acting on the tray and the candy bar, considering them as a single system.

(b) Determine the magnitude of the acceleration of this system in terms of the applied horizontal force and the appropriate masses.

(c) Draw another free-body diagram indicating the forces acting on the candy bar, considering it as a separate system from the tray. What force accelerates the candy bar?

(d) What is the maximum magnitude of the acceleration if the candy bar does not slip on the tray?

(e) Draw a third free-body diagram indicating the forces acting on the tray, considering it as a separate system from the candy bar. Using this diagram, determine the magnitude and direction of the tray's acceleration. How does this answer compare with that in part (b)?



10. A small sphere of mass m is attached to the end of a cord of length R which rotates counter-clockwise in a vertical circle about a fixed point O as shown. Define θ as the angle that the cord makes with the vertical.

(a) Draw the free-body diagram for mass m when it is at points: (i) A - the bottom of the circle, (ii) B - the top of the circle, and (iii) C – at angle θ as shown.

(b) What are the radial and tangential components of the acceleration?

(c) Find an expression for the tension in the cord at time t in terms of the speed of the sphere, v(t), θ(t), m, and R.

(d) What is the minimum magnitude of the tension and at what position does it occur? What is the maximum magnitude of the tension and at what position does it occur?

(e) At what position is the cord most likely to break if the speed increases? Why?