PHY 140Y – FOUNDATIONS OF PHYSICS
2001-2002
Problem Set #1 – Solutions

HANDED OUT: Friday, September 21, 2001 (in class).

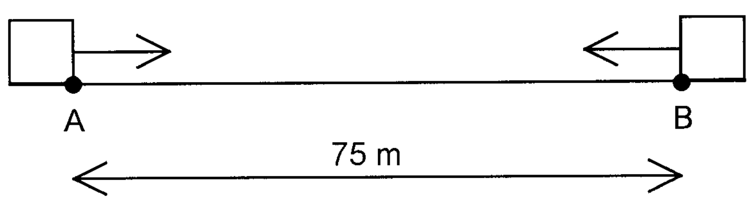
DUE: **5:00 PM, Thursday, October 4, 2001** in the appropriate box, labeled by tutorial group, in the basement at the bottom of the stairs leading down from MP202.

LATE PENALTY: 5 marks/day (which also applies to weekend days!) until 2:00 PM, Tuesday, October 9 (revised from October 8), after which it will not be accepted as solutions will then be available in tutorials and on the WWW.

NOTES: Answer all questions. A selected subset (3-4) will be marked out of 100%. Marks will be given for workings and units, as well as for final answers.

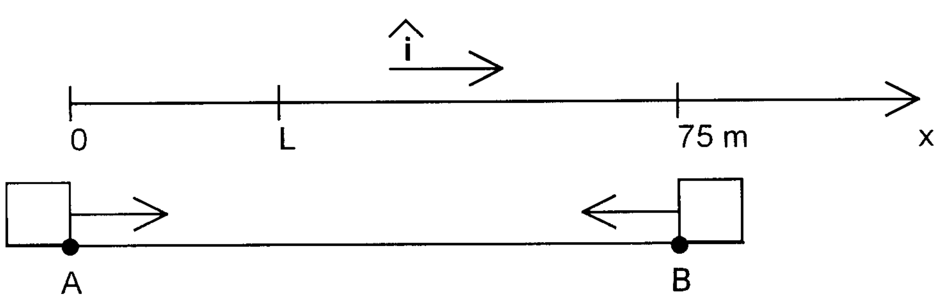
QUESTIONS:

1. One car starts out from point A with initial velocity 10.0 m/s to the right but is decelerating at the rate of 2.0 m/s^2. A second car starts out from rest at point B (75 m from A) and accelerates towards A at 4.0 m/s^2.
(a) When do the two cars collide?
(b) Where are the two cars when they collide?
(c) How fast is each car going when they collide?



Solution:

First, sketch the situation and set up the coordinate system. Define the origin to be at A.



Then, set up the initial conditions for both cars.

At $t = t_o$:

$$\vec{x}_{Ao} = 0 \qquad \vec{x}_{Bo} = 75\,\hat{i}\ \text{m}$$
$$\vec{v}_{Ao} = 10.0\,\hat{i}\ \text{m/s} \qquad \vec{v}_{Bo} = 0$$
$$\vec{a}_{Ao} = -2.0\,\hat{i}\ \text{m/s}^2 \qquad \vec{a}_{Bo} = -4.0\,\hat{i}\ \text{m/s}^2$$

Next, apply the equations of motion for the case of 1-D motion with constant acceleration.

For car A:

$$a_A = -2.0$$
$$v_A = v_{Ao} + a_A(t - t_o) = 10.0 - 2.0t$$
$$x_A = x_{Ao} + v_{Ao}(t - t_o) + \tfrac{1}{2}a_A(t - t_o)^2 = 10.0t - t^2$$

For car B:

$$a_B = -4.0$$
$$v_B = v_{Bo} + a_B(t - t_o) = -4.0t$$
$$x_B = x_{Bo} + v_{Bo}(t - t_o) + \tfrac{1}{2}a_B(t - t_o)^2 = 75 - 2.0t^2$$

Now apply these equations to answer the given questions.

(a) When the two cars collide, $x_A(t) = x_B(t) = L$

Thus:

$$10.0t - t^2 = L \qquad \text{(Eqn.1)}$$
$$75 - 2.0t^2 = L \qquad \text{(Eqn.2)}$$

Subtract: Eqn. 1 – Eqn. 2 gives:

$$10.0t - t^2 - (75 - 2.0t^2) = 0$$
$$t^2 + 10.0t - 75 = 0$$
$$t = -15 \ \text{or}\ t = 5.0$$

The positive root makes physical sense, so the cars collide at t = 5.0 seconds after they start.

(b) To find where are the two cars are when they collide, use Eqn. 1 or Eqn. 2 to solve for L.

$$L = 75 - 2.0t^2 = 75 - 2.0(5.0)^2 = 25\ \text{m}$$

When the two cars collide, they are 25 m to the right of point A, or 50 m to the left of point B.

(c) Apply the equations for velocity to determince how fast each car going when they collide.

For car A: $v_A = 10.0 - 2.0t = 10.0 - 2.0(5.0) = 0\ \text{m/s}$

For car B:

$$v_B = -4.0t = -4.0(5.0) = -20.\ \text{m/s}$$
$$\vec{v}_B = -20.\,\hat{i}\ \text{m/s} \quad \text{(i.e., 20 m/s to the left)}$$

2. A water balloon is thrown straight up and reaches its maximum height after 2.00 s. Sketch the situation and introduce an appropriate coordinate system to answer the following questions.
(a) What is the acceleration of the balloon after it leaves the hand and is rising?
(b) What is the acceleration of the balloon as it is falling?
(c) What is the acceleration of the balloon at the instant it is motionless at it maximum height?
(d) What was the initial velocity component of the balloon?
(e) How high did the balloon rise above the point of its release?

Solution:

Introduce a coordinate system whose origin is at the point of release of the balloon, and with $\hat{\imath}$ pointing straight up.

a) The acceleration of the balloon — due to gravity — is $\vec{a} = (-9.81\ \text{m/s}^2)\hat{\imath}$, which is constant during its entire flight.

b) Likewise $\vec{a} = (-9.81\ \text{m/s}^2)\hat{\imath}$.

c) Although the velocity is zero at the peak height, the acceleration is still $(-9.81\ \text{m/s}^2)\hat{\imath}$ since acceleration measures *change* in velocity (which is definitely not zero at this point — the balloon has even changed its direction!).

d) We know that when $t = 2.00$ s the velocity of the balloon is zero, and that its acceleration is $(-9.81\ \text{m/s}^2)\hat{\imath}$. Therefore, solving

$$v_x(t) = v_{x0} + a_x t$$

for v_{x0}, we have

$$v_{x0} = 0\ \text{m/s} - (-9.81\ \text{m/s}^2)(2.00\ \text{s}) = 19.6\ \text{m/s}.$$

e) The position of the balloon at any instant is

$$x(t) = x_0 + v_{x0}t + a_x\frac{t^2}{2}$$

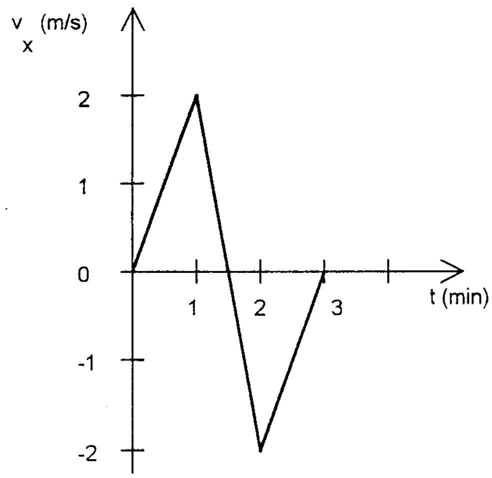
which, in our case, is

$$x(t) = 0\ \text{m} + (19.6\ \text{m/s})t - (9.81\ \text{m/s}^2)\frac{t^2}{2}.$$

The ball is at the maximum height when t = 2.00 (s), so

$$x(2.00\ \text{s}) = 0\ \text{m} + (19.6\ \text{m/s})(2.00\ \text{s}) - (9.81\ \text{m/s}^2)\frac{(2.00\ \text{s})^2}{2} = 19.6\ \text{m}.$$

3. The x-component of the velocity of a garbage barge is indicated in the graph shown below.
(a) What is the average acceleration of the barge during the time interval between 0 min and 1.0 min on the clock? What is the instantaneous acceleration of the barge when t = 0.5 min?
(b) What is the average acceleration of the barge during the time interval between 1.0 min and 2.0 min on the clock? What is the instantaneous acceleration of the barge when t = 1.5 min?
(c) What is the average acceleration of the barge during the time interval between 2.0 min and 3.0 min on the clock? What is the instantaneous acceleration of the barge when t = 2.5 min?
(d) Explain what is happening to the barge in descriptive English.
(e) Sketch a graph of the position x of barge as a function of time consistent with and as complete as the information provided allows. Assume that the barge initially is at the Cartesian origin.



Solution:

a) The average acceleration is the change in the velocity divided by the time interval during which the change took place:

$$\vec{\mathbf{a}}_{\text{ave}} = \frac{\Delta \vec{\mathbf{v}}}{\Delta t} = \frac{(2.0\text{ m/s})\hat{\mathbf{i}} - (0\text{ m/s})\hat{\mathbf{i}}}{60\text{ s}} = (3.3 \times 10^{-2}\text{ m/s}^2)\hat{\mathbf{i}}.$$

The instantaneous acceleration component when $t = 0.5$ min is the slope of the v_x versus t graph at that instant. Since the graph is a straight line between 0 min and 1 min, the instantaneous acceleration at each instant between these times is the same as the average acceleration during this interval. Hence, the instantaneous acceleration when $t = 0.5$ min is $(3.3 \times 10^{-2}\text{ m/s}^2)\hat{\mathbf{i}}$.

b) The average acceleration is the change in the velocity divided by the time interval during which the change took place:

$$\vec{\mathbf{a}}_{\text{ave}} = \frac{\Delta \vec{\mathbf{v}}}{\Delta t} = \frac{(-2.0\text{ m/s})\hat{\mathbf{i}} - (2.0\text{ m/s})\hat{\mathbf{i}}}{60\text{ s}} = (-6.7 \times 10^{-2}\text{ m/s}^2)\hat{\mathbf{i}}.$$

The instantaneous acceleration component when $t = 1.5$ min is the slope of the v_x versus t graph at that instant. Since the graph is a straight line between 1 min and 2 min, the instantaneous acceleration at each instant between these times is the same as the average acceleration during this interval. Hence, the instantaneous acceleration when $t = 1.5$ min is $(-6.7 \times 10^{-2}\text{ m/s}^2)\hat{\mathbf{i}}$.

c) The average acceleration is the change in the velocity divided by the time interval during which the change took place:

$$\vec{\mathbf{a}}_{\text{ave}} = \frac{\Delta \vec{\mathbf{v}}}{\Delta t} = \frac{(0\text{ m/s})\hat{\mathbf{i}} - (-2.0\text{ m/s})\hat{\mathbf{i}}}{60\text{ s}} = (3.3 \times 10^{-2}\text{ m/s}^2)\hat{\mathbf{i}}.$$

The instantaneous acceleration component when $t = 2.5$ min is the slope of the v_x versus t graph at that instant. Since the graph is a straight line between 2 min and 3 min, the instantaneous acceleration at each instant between these times is the same as the average acceleration during this interval. Hence, the instantaneous acceleration when $t = 2.5$ min is $(3.3 \times 10^{-2}\text{ m/s}^2)\hat{\mathbf{i}}$.

d) The barge begins at rest when $t = 0$ s and accelerates to a speed of 2.0 m/s when $t = 60$ s. The barge then begins to slow down until it is instantaneously at rest when $t = 90$ s. It then reverses direction until it reaches a speed of 2.0 m/s when $t = 120$ s. The barge then slows down and stops when $t = 180$ s.

e) During the first 60 seconds the acceleration component is $a_x = 3.3 \times 10^{-2}\text{ m/s}^2$. When $t = 0$ s, $x_0 = 0$ m, $v_{x0} = 0$ m/s, hence

$$x(t) = x_0 + v_{x0}t + a_x\frac{t^2}{2} = (3.3 \times 10^{-2}\text{ m/s}^2)\frac{t^2}{2}.$$

This is the equation of a parabola with vertex at the origin, opening upwards. (A happy parabola!) Here are the values of x corresponding to various values of t:

t (s)	x (m)
0	0
10	1.7
20	6.6
30	15
40	26
50	41
60	59

The next time interval of constant acceleration begins when $t_1 = 60$ s and ends when $t_2 = 120$ s. To find $x(t)$ in this interval, we replace $(t - 0\text{ s})$ by $(t - t_1)$ and v_{x0} by $v_x(t_1)$ in the general equation for position, giving

$$x(t) = x(t_1) + v_x(t_1)(t - t_1) + a_x \frac{[t - t_1]^2}{2},$$

where now $a_x = -6.7 \times 10^{-2}$ m/s^2. The initial position is $x(60\text{ s}) = 59$ m, and the initial velocity component is the final velocity component from the previous time interval:

$$v_x(60\text{ s}) = 0\text{ m/s} + (3.3 \times 10^{-2}\text{ m/s}^2)(60\text{ s}) = 2.0\text{ m/s}.$$

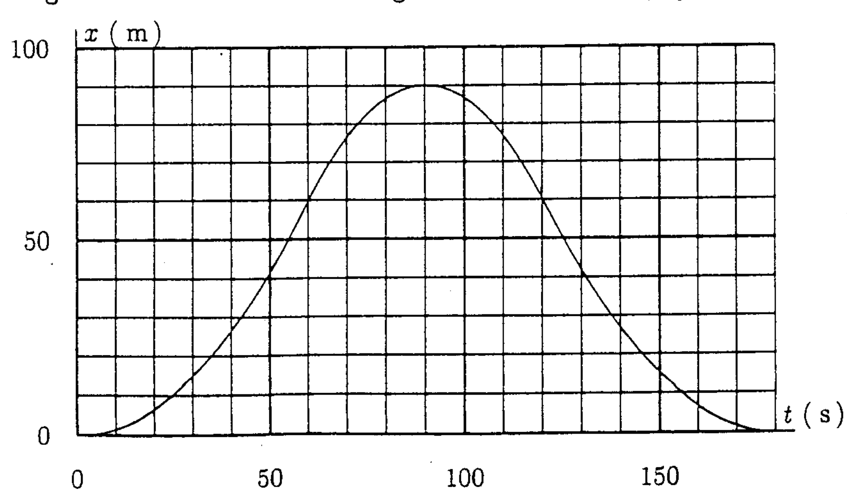
So, substituting these values into the new equation for $x(t)$:

$$x(t) = 59\text{ m} + (2.0\text{ m/s})(t - 60\text{ s}) + (-6.7 \times 10^{-2}\text{ m/s}^2)\frac{(t - 60\text{ s})^2}{2}.$$

From this equation we find:

t (s)	x (m)
60	59
70	76
80	86
90	89
100	86
110	76
120	59

During the third minute, the acceleration component returns to what it was during the first minute, and the barge has positions which are the reverse of those during the first minute. Hence the graph of the position of the barge as a function of time during the three minute voyage is:



4. A ball is tossed vertically upward with a speed of 25.0 m/s.
(a) Indicate an appropriate coordinate system for analyzing its motion.
(b) What is its maximum altitude above the point from which it was thrown?
(c) How long does it take to return to the point of release?
(d) Make a graph of the acceleration component a_x of the particle versus time, indicating appropriate numerical values along the axes.
(e) Make a graph of the velocity component v_x of the particle versus time, indicating appropriate numerical values along the axes.
(f) Make a graph of the position vector component x as a function of time, indicating appropriate numerical values along the axes.

Solution:

a) Choose a coordinate system with $\hat{\mathbf{i}}$ pointing up and with origin at the point where the ball is released.

b) With this coordinate system, we have $x_0 = 0$ m, $v_{x0} = 25.0$ m/s, and $a_x = -9.81$ m/s^2. The equation for the x-component of the velocity is

$$v_x(t) = v_{x0} + a_x t = 25.0 \text{ m/s} - (9.81 \text{ m/s}^2)t.$$

When the ball reaches its maximum height, its velocity is zero; hence

$$0 \text{ m/s} = 25.0 \text{ m/s} - (9.81 \text{ m/s}^2)t.$$

Solve this for t:

$$t = 2.55 \text{ s}.$$

The equation for the position of the ball is

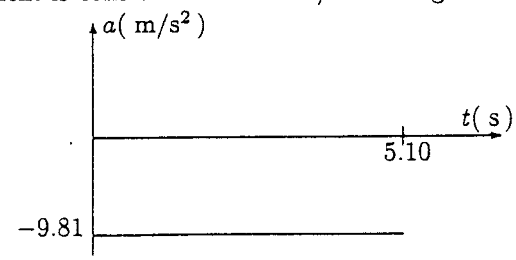
$$x(t) = x_0 + v_{x0}t + a_x\frac{t^2}{2} = 0 \text{ m} + (25.0 \text{ m/s})t - (9.81 \text{ m/s}^2)\frac{t^2}{2}.$$

To find its maximum altitude, let $t = 2.55$ s:

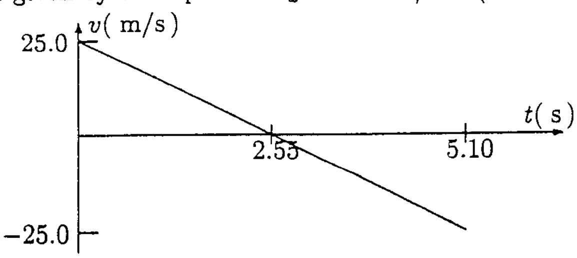
$$x(2.55 \text{ s}) = (25.0 \text{ m/s})(2.55 \text{ s}) - (9.81 \text{ m/s}^2)\frac{(2.55 \text{ s})^2}{2} = 31.9 \text{ m}.$$

c) Since the ball returns to its initial position, the time the ball rises is equal to the time it falls, so the total time is $2(2.55 \text{ s}) = 5.10$ s. (This also can be found by setting $x = 0$ m in the equation for the position and solving for t.)

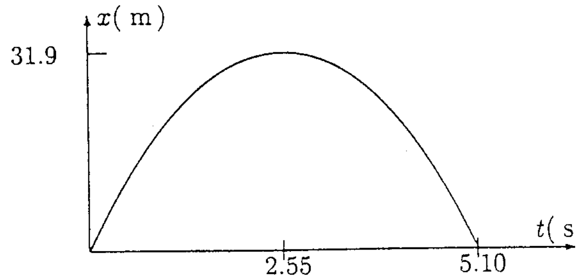
d) The acceleration component is constant at -9.81 m/s^2 throughout the flight. Its graph is



e) The velocity component is given by the equation $v_x = 25.0 \text{ m/s} + (-9.81 \text{ m/s}^2)t$. Its graph is



f) The x-component of the position is $x(t) = (25.0\text{ m/s})t - (9.81\text{ m/s}^2)\frac{t^2}{2}$. Its graph is



It has a maximum at the point(2.55 s, 31.9 m).

5. When a ball bounces from the floor, the ratio of the speed at the instant just after the bounce to the speed at the instant just before the bounce is called the coefficient of restitution, ε, of the ball.

(a) Show that if a ball, initially at rest, is dropped from a height h_o and rebounds to a height h_1, then the coefficient of restitution is

$$\varepsilon = \sqrt{h_1/h_o}.$$

(b) The height of rebound is difficult to measure precisely. An alternative way of determining the coefficient of restitution is to drop a ball from height h_o and measure the total time T from its release to the instant of its third impact with the floor. Show that the coefficient of restitution is related to T by

$$T = \sqrt{2h_o/g}\left(1 + 2\varepsilon + 2\varepsilon^2\right).$$

Solution:

a) Take $\hat{\mathbf{i}}$ to point up and let the origin be at floor level. For a ball falling from rest from an initial height h_0, we have $x_0 = h_0$, $v_{x0} = 0$ m/s, and $a_x = -g$. The equation for the position is thus

$$x(t) = h_0 - g\frac{t^2}{2}.$$

When the ball impacts the floor its position is 0 m, so

$$0\text{ m} = h_0 - g\frac{t^2}{2}.$$

Solve for t to find

$$t = \sqrt{\frac{2h_0}{g}}.$$

The equation for the velocity component is $v_x(t) = -gt$, so the velocity component when the ball impacts the floor is

$$v_x = -g\sqrt{\frac{2h_0}{g}} = -\sqrt{2gh_0}.$$

The speed of the ball on impact is the magnitude of the velocity vector, so $v_{\text{impact}} = \sqrt{2gh_0}$. The time for the ball to rise to a given height is the same as the time it takes to fall from that height. Therefore, if the rebounding ball attains a height h_1, its initial speed must have been $v_{\text{rebound}} = \sqrt{2gh_1}$. The coefficient of restitution ϵ is therefore

$$\epsilon = \frac{v_{\text{rebound}}}{v_{\text{impact}}} = \frac{\sqrt{2gh_1}}{\sqrt{2gh_0}} = \sqrt{\frac{h_1}{h_0}}.$$

b) Let t_0 be the time for the initial fall to the first bounce. Let $2t_1$ be the time between the first and second bounce, and $2t_2$ be the time between the second and the third bounce. The total time T between the initial release of the ball and the third bounce is $T = t_0 + 2t_1 + 2t_2$. The time to fall from height h_0 to the floor was calculated in part (a) to be $t_0 = \sqrt{\frac{2h_0}{g}}$. The time for the ball to rise to a given height is the same as the time it takes to fall from that height. Hence $t_1 = \sqrt{\frac{2h_1}{g}}$. Let h_2 be the height to which the ball rebounds after the second bounce. Then $t_2 = \sqrt{\frac{2h_2}{g}}$ Substitute these times into the expression for T:

$$T = \sqrt{\frac{2h_0}{g}} + 2\sqrt{\frac{2h_1}{g}} + 2\sqrt{\frac{2h_2}{g}}.$$

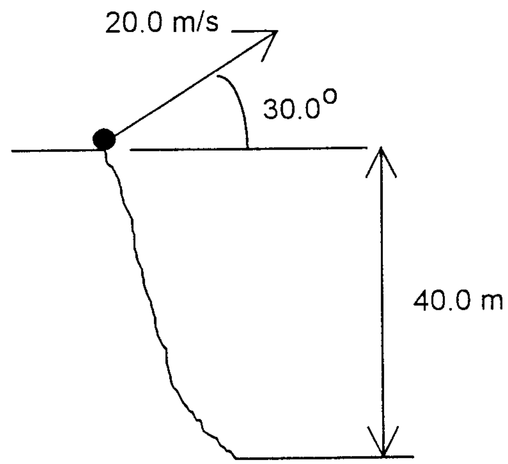
Factor out the expression $\sqrt{\frac{2h_0}{g}}$.

$$T = \sqrt{\frac{2h_0}{g}}\left(1 + 2\sqrt{\frac{h_1}{h_0}} + 2\sqrt{\frac{h_2}{h_0}}\right).$$

In part (a) we showed that $\sqrt{\frac{h_1}{h_0}} = \epsilon$. Also note that $\sqrt{\frac{h_2}{h_0}} = \sqrt{\frac{h_2 h_1}{h_1 h_0}} = \sqrt{\frac{h_2}{h_1}}\epsilon$ Finally, since h_2 is the height to which the ball rebounds after falling through the height h_1, we also have $\sqrt{\frac{h_2}{h_1}} = \epsilon$. Thus

$$T = \sqrt{\frac{2h_0}{g}}\left(1 + 2\epsilon + 2\epsilon\epsilon\right) = \sqrt{\frac{2h_0}{g}}\left(1 + 2\epsilon + 2\epsilon^2\right).$$

6. A soccer ball is kicked off a cliff at a speed of 20.0 m/s as indicated in the figure below.
(a) In a sketch, indicate a choice for a coordinate system to analyze the problem.
(b) Give equations for the initial x and y components of position, velocity, and acceleration. Also give equations for the x and y components of position and velocity as functions of time.
(c) Determine the time of flight of the ball.
(d) Determine where the ball hits the ground the first time.



Solution:

a) Choose a coordinate system with $\hat{\imath}$ pointing right, $\hat{\jmath}$ pointing up, and origin at the launch point of the soccer ball.

b) In this coordinate system,

$$\begin{aligned} y_0 &= 0 \text{ m} \\ v_{y0} &= (20.0 \text{ m/s}) \sin 30.0^\circ \\ &= 10.0 \text{ m/s} \\ a_y &= -g \end{aligned} \qquad \begin{aligned} x_0 &= 0 \text{ m} \\ v_{x0} &= (20.0 \text{ m/s}) \cos 30.0^\circ \\ &= 17.3 \text{ m/s} \\ a_x &= 0 \text{ m/s}^2 . \end{aligned}$$

The equations for the velocity and position components are

$$\begin{aligned} v_y(t) &= 10.0 \text{ m/s} - gt \\ y(t) &= (10.0 \text{ m/s})t - g\frac{t^2}{2} \end{aligned} \qquad \begin{aligned} v_x(t) &= 17.3 \text{ m/s} \\ x(t) &= (17.3 \text{ m/s})t. \end{aligned}$$

c) The soccer ball impacts when $y(t) = -40.0$ m. Substitute this into the equation for $y(t)$,

$$-40.0 \text{ m} = (10.0 \text{ m/s})t - g\frac{t^2}{2},$$

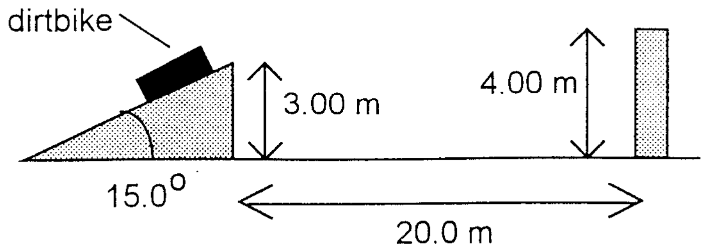
and then use the quadratic formula to solve for t. The two roots are $t = 4.05$ s and $t = -2.01$ s. Since impact occurs after $t = 0$ s (the time when the ball is kicked), choose the positive root. Thus, the flight time is 4.05 s.

d) The x-coordinate of the impact point is determined from the equation for $x(t)$ with $t = 4.05$ s.

$$x(t) = (17.3 \text{ m/s})t = (17.3 \text{ m/s})(4.05) = 70.1 \text{ m}.$$

The coordinates of the impact point are therefore $x = 70.1$ m and $y = -40.0$ m.

7. A dirt biker races up a 15.0° incline at a constant speed of 120. km/hr. The end of the ramp is 3.00 m off the ground as indicated in the figure below. A 4.00 m high obstacle is located 20.0 m from the base of the ramp.
(a) Indicate an appropriate coordinate system to attack the problem.
(b) Will the daredevil clear the obstacle? If so, where will she land?



Solution:

First convert the speed from km/h to m/s.

$$120 \text{ km/h} = (120 \text{ km/h})\left(\frac{10^3 \text{ m}}{1 \text{ km}}\right)\left(\frac{1 \text{ h}}{3600 \text{ s}}\right) = 33.3 \text{ m/s}.$$

a) Choose a coordinate system with $\hat{\imath}$ pointing to the right, $\hat{\jmath}$ pointing up, and origin at the ground level directly below the daredevil's launch point.

b) In this coordinate system

$$\begin{aligned} y_0 &= 3.00 \text{ m} \\ v_{y0} &= (33.3 \text{ m/s}) \sin 15.0° \\ &= 8.62 \text{ m/s} \\ a_y &= -g \end{aligned} \qquad \begin{aligned} x_0 &= 0 \text{ m} \\ v_{x0} &= (33.3 \text{ m/s}) \cos 15.0° \\ &= 32.2 \text{ m/s} \\ a_x &= 0 \text{ m/s}^2. \end{aligned}$$

So, the equations for the velocity and position components are

$$\begin{aligned} v_y(t) &= 8.62 \text{ m/s} - gt \\ y(t) &= 3.00 \text{ m} + (8.62 \text{ m/s})t - g\frac{t^2}{2} \end{aligned} \qquad \begin{aligned} v_x(t) &= 32.2 \text{ m/s} \\ x(t) &= (32.2 \text{ m/s})t. \end{aligned}$$

To see if the daredevil clears the obstacle, find the y-coordinate when the x-coordinate is 20.0 m. To do this, first substitute 20.0 m for $x(t)$ in the x position equation and solve it for the time t at which the daredevil reaches the obstacle.

$$20.0 \text{ m} = (32.2 \text{ m/s})t \qquad \text{so} \qquad t = 0.621 \text{ s}.$$

Now substitute $t = 0.621$ s into the equation for $y(t)$.

$$y(0.621 \text{ s}) = 3.00 \text{ m} + (8.62 \text{ m/s})(0.621 \text{ s}) - g\frac{(0.621 \text{ s})^2}{2} = 6.46 \text{ m}.$$

Since the barrier is only 4.00 m high, the daredevil easily clears it.

The daredevil lands where $y(t) = 0$ m. So, substitute 0 m into the equation for $y(t)$ to find the landing time.

$$0 \text{ m} = 3.00 \text{ m} + (8.62 \text{ m/s})t - g\frac{t^2}{2}.$$

Use the quadratic formula to solve this for t. Since landing occurs after the time $t = 0$ s when the daredevil leaves the ramp, choose the positive root, $t = 2.06$ s.

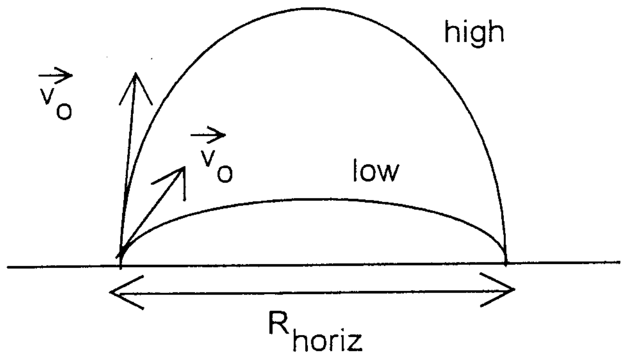
The x-coordinate of the landing point is $x(2.06 \text{ s}) = (32.2 \text{ m/s})(2.06 \text{ s}) = 66.3$ m.

8. A projectile is launched at speed v_o towards a target located a distance R_{horiz} away over level terrain as indicated in the figure below.

(a) Show that there are *two* possible launch angles, hence that there is a high trajectory and a low trajectory, *except* when the launch angle is 45° for maximum horizontal range, in which case the two launch angles are identical to each other. The two launch angles are symmetrical about the 45° angle.

(b) If $R_{horiz} = 100.$ m and $v_o = 40.0$ m/s, find the two possible angles for launch.

(c) Which launch angle is a quarterback on a football team likely to use?



Solution:

Choose a coordinate system with origin at the launch point, $\hat{\imath}$ in the horizontal direction, and $\hat{\jmath}$ pointing up. In this coordinate system

$$\begin{aligned} y_0 &= 0\text{ m} \\ v_{y0} &= v_0 \sin\theta \\ a_y &= -g \end{aligned} \qquad\qquad \begin{aligned} x_0 &= 0\text{ m} \\ v_{x0} &= v_0 \cos\theta \\ a_x &= 0\text{ m/s}^2\,. \end{aligned}$$

So, the equations for the velocity and position components are

$$\begin{aligned} v_y(t) &= v_0 \sin\theta - gt \\ y(t) &= (v_0 \sin\theta)t - g\frac{t^2}{2} \end{aligned} \qquad\qquad \begin{aligned} v_x(t) &= v_0 \cos\theta \\ x(t) &= (v_0 \cos\theta)t \end{aligned}$$

a) Impact occurs where the y-coordinate is zero, so at impact time

$$0\text{ m} = (v_0 \sin\theta)t - g\frac{t^2}{2}.$$

Solve for t. The two roots are

$$t = 0\text{ s} \qquad \text{and} \qquad t = \frac{2v_0 \sin\theta}{g}.$$

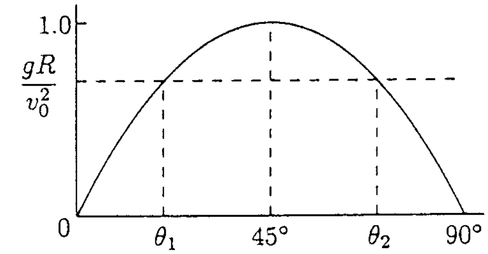
The zero root is the time of the launch, so we want the nonzero root. The horizontal range R is found by substituting the impact time into the equation for x.

$$R = (v_0 \cos\theta)\frac{2v_0 \sin\theta}{g} = \frac{2v_0^2 \sin\theta\cos\theta}{g} = \frac{v_0^2 \sin 2\theta}{g}$$

This implies that

(1) $$\sin 2\theta = \frac{gR}{v_0^2}.$$

Here's a graph of $\sin 2\theta$ as a function of θ:



Notice that for $\frac{gR}{v_0^2} < 1$, there are two solutions, θ_1 and θ_2, to equation (1), symmetrically located with respect to 45°. When $\frac{gR}{v_0^2} = 1$ there is only one, $\theta = 45°$. If $\frac{gR}{v_0^2} > 1$ then there is no solution, the initial velocity is too small for the ball to get there.

b) Solve equation (1) for $\sin 2\theta$.

$$\sin 2\theta = \frac{gR}{v_0^2} = \frac{(9.81\ \text{m/s}^2)(100\ \text{m})}{(40.0\ \text{m/s})^2} = 0.613$$

The arcsine of 0.613 is 37.8°. Hence one solution is

$$\theta_1 = \frac{38.8°}{2} = 18.9°.$$

The other solution is obtained by subtracting this solution from 90.0°.

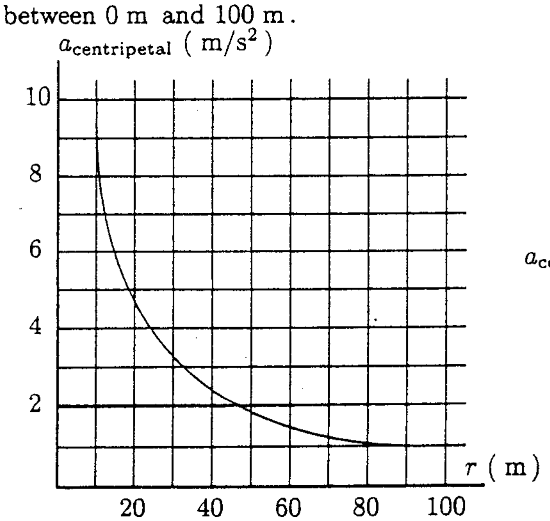
$$\theta_2 = 90° - 18.9° = 71.1°.$$

c) A quarterback is more likely to use the smaller angle, since a football thrown at this angle spends less time in the air and is therefore less likely to be intercepted.

9. A particle is moving at constant speed 10.0 m/s in circles of various radii.
(a) Make a graph of the magnitude of the centripetal acceleration of the particle versus the radius of the circle for radii between r = 0 m and 100 m.
(b) At what radius is the magnitude of the centripetal acceleration of the particle equal to the magnitude of the local acceleration due to gravity ($g = 9.81\ \text{m/s}^2$)? What is the angular speed ω of the particle with speed 10.0 m/s at this radius?

Solution:

a) For a fixed speed of 10.0 m/s, here is a graph of the magnitude of the centripetal acceleration versus the radius of the circle for radii between 0 m and 100 m.



$$a_{\text{centripetal}} = \frac{(10.0\ \text{m/s})^2}{r}$$

b) Solve $a_{\text{centripetal}} = \dfrac{v^2}{r}$ for r, and then substitute 10.0 m/s for v and g for $a_{\text{centripetal}}$.

$$r = \frac{v^2}{a_{\text{centripetal}}} = \frac{(10.0\ \text{m/s})^2}{9.81\ \text{m/s}^2} = 10.2\ \text{m}.$$

Now solve $v = r\omega$ for ω and then substitute 10 m/s for v and 10.2 m for r.

$$\omega = \frac{v}{r} = \frac{10.0\ \text{m/s}}{10.2\ \text{m}} = 0.980\ \text{rad/s}.$$

Alternatively, combine the solutions for ω and r, simplify, and then substitute 10.0 m/s for v and g for $a_{\text{centripetal}}$.

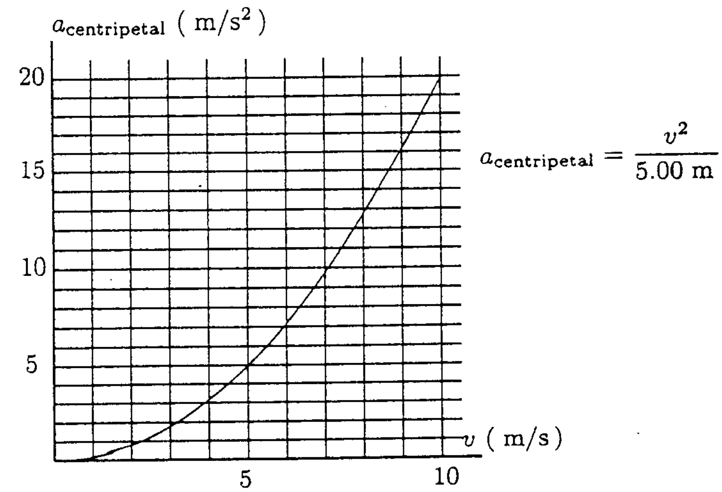
$$\omega = \frac{v}{r} = \frac{v}{\left(\dfrac{v^2}{a_{\text{centripetal}}}\right)} = \frac{a_{\text{centripetal}}}{v} = \frac{9.81\ \text{m/s}^2}{10\ \text{m/s}} = 0.981\ \text{rad/s}.$$

The difference in the answers is due to roundoff error.

10. A particle is moving in a circle of radius 5.00 m.
(a) Make a graph of the magnitude of the centripetal acceleration versus the speed of the particle for speeds ranging between 0 m/s and 10.0 m/s. What is the slope of the graph when v = 0 m/s?
(b) Make a corresponding graph of the magnitude of the centripetal acceleration versus the angular speed ω of the particle for the same speeds indicated in part (a). What is the slope of this graph when ω = 0 rad/s?

Solution:

a) For a circle of fixed radius 5.00 m, here is a graph of the magnitude of the centripetal acceleration versus the speed of the particle from 0 m/s to 10.0 m/s.



Since the magnitude of the centripetal acceleration is

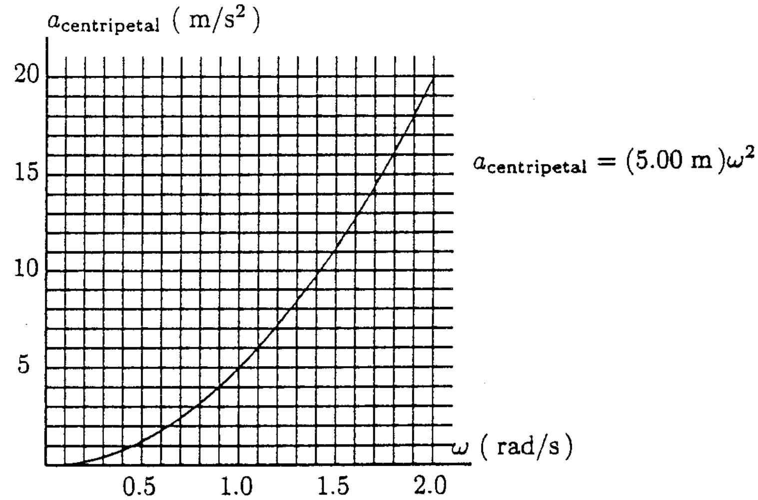
$$a_{\text{centripetal}} = \frac{v^2}{r}.$$

The slope of the graph is, for fixed r,

$$\frac{da}{dv} = \frac{2v}{r}.$$

For $v = 0$ m/s, the slope is zero.

b) Here is a graph of the magnitude of the centripetal acceleration versus angular speed ω. When v varies from 0 m/s to 10.0 m/s, then for a circle of radius 5.00 m, the angular speed ω varies from 0 rad/s to 2.00 rad/s.



Since

$$a_{\text{centripetal}} = r\omega^2$$

the slope of this graph is

$$\frac{da_{\text{centripetal}}}{d\omega} = 2r\omega.$$

For $\omega = 0$ rad/s, the slope is zero.