# LECTURE #9 – SUMMARY

## Section II.5 Uniform and Non-Uniform Circular Motion

Definition: uniform circular motion (UCM) is the motion of an object that is following a circular path at constant speed.

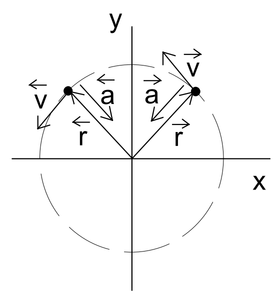
$\rightarrow$ it is accelerated motion in 2-D (because direction of velocity changes)

### (1) General Considerations

arc length s = R$\theta$ (by definition, with $\theta$ in radians)

$$ds = R\,d\theta \quad \text{and} \quad \frac{ds}{dt} = R\frac{d\theta}{dt} \quad \text{where} \quad \frac{d\theta}{dt} \equiv \omega = \text{angular speed (units of radians/s)}$$

*Note: There are 2$\pi$ radians in a circle, but a radian is not really a dimensional unit. Be careful - never use degrees/s !*




Now, speed v is just $v = \frac{ds}{dt} = R\frac{d\theta}{dt} \quad \therefore \quad \boxed{v = R\omega}$

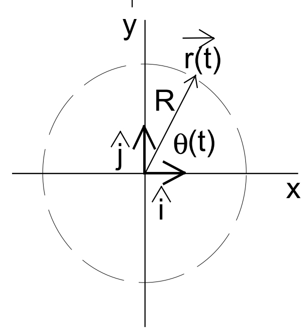
This equation is general, not just for UCM ($\omega$ can change).

### (2) Uniform Circular Motion

period (T) = the time it takes for a particle to go one full circle

distance $s = Tv = 2\pi R$ 2$\pi$ radians = 1 full circle

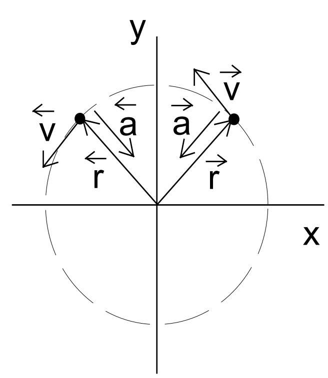
$$\therefore \quad T = \frac{2\pi R}{v} = \frac{2\pi R}{\omega R} = \frac{2\pi}{\omega} \quad \Rightarrow \quad \text{units of time (s)}$$




frequency = the number of full revolutions or cycles per unit time

$$f = \frac{1}{T} = \frac{\omega}{2\pi} \quad \Rightarrow \quad \text{units of cycles/s (Hertz = Hz)}$$

**Equations of motion for UCM:**




Position vector: $|\vec{r}(t)| = R$

$$\vec{r}(t) = x(t)\hat{i} + y(t)\hat{j} = R\cos\theta(t)\hat{i} + R\sin\theta(t)\hat{j} = (R\cos\omega t)\hat{i} + (R\sin\omega t)\hat{j}$$

using $\theta(t) = \omega t$ with $\theta$ = 0 at t = 0.

Velocity: $$\vec{v}(t) = \frac{dx(t)}{dt}\hat{i} + \frac{dy(t)}{dt}\hat{j} = (-\omega R\sin\omega t)\hat{i} + (\omega R\cos\omega t)\hat{j}$$

$$v = |\vec{v}(t)| = \sqrt{{v_x}^2 + {v_y}^2} = \sqrt{\omega^2R^2\sin^2\omega t + \omega^2R^2\cos^2\omega t} = \omega R$$

Scalar product $\vec{r}(t) \bullet \vec{v}(t) = r_x v_x + r_y v_y = ... = 0$, so $\vec{v} \perp \vec{r}$ and $\vec{v}$ || trajectory

Acceleration: $$\vec{a}(t) = \frac{dv_x(t)}{dt}\hat{i} + \frac{dv_y(t)}{dt}\hat{j} = (-\omega^2R\cos\omega t)\hat{i} + (-\omega^2R\sin\omega t)\hat{j}$$

$$\boxed{a = |\vec{a}(t)| = \omega^2 R = \frac{v^2}{R}}$$

Note: $\vec{a}(t) = -\omega^2\vec{r}(t)$, so $\vec{a} \parallel \vec{r}$ (anti-parallel).