LECTURE #8 – SUMMARY

A particle is accelerating if its velocity changes (i) magnitude, or (ii) direction.

- The component of acceleration that is parallel to velocity changes only the magnitude of the velocity, NOT its direction.
- The component of acceleration that is always perpendicular to velocity changes only the direction of the velocity, NOT its magnitude.

Section II.4 Projectile Motion

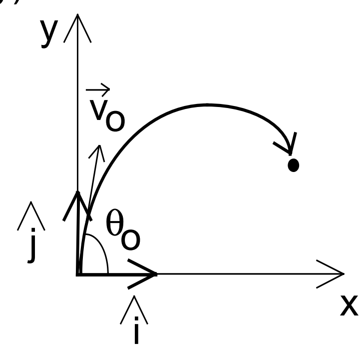
projectile - an object that is launched into the air and then moves predominantly under the influence of gravity.

Projectile motion can usually be treated in 2-D:

→ acceleration is constant in the x direction = zero (usually)

→ acceleration is constant in the y direction = gravity

→ assume negligible air resistance



Initial conditions? Set $t = t_o = 0$.

Position: at $t = 0$, $x_o = 0$, $y_o = 0$

Velocity: at $t = 0$, $v_{xo} = v_o \cos\theta_o$, $v_{yo} = v_o \sin\theta_o$

Equations governing the motion of the projectile:

Acceleration: $a_x = 0$ $a_y = -g$

Velocity:

$$v_x(t) = v_{xo} + a_x(t - t_o) = v_{xo} = v_o \cos\theta_o$$

$$v_y(t) = v_{yo} + a_y(t - t_o) = v_{yo} - gt = v_o \sin\theta_o - gt$$

Position:

$$x(t) = x_o + v_{xo}(t - t_o) + \tfrac{1}{2}a_x(t - t_o)^2 = v_{xo}t = v_o t\cos\theta_o$$

$$y(t) = y_o + v_{yo}(t - t_o) + \tfrac{1}{2}a_y(t - t_o)^2 = v_{yo}t - \tfrac{1}{2}gt^2 = v_o t\sin\theta_o - \tfrac{1}{2}gt^2$$

(1) Trajectory (y as a function of x)

$$y(t) = (\tan\theta_o)x(t) - \left(\frac{g}{2v_o^2\cos^2\theta_o}\right)x(t)^2$$

so the trajectory is a parabola.

(2) Range (how far the projectile travels if it traverses a level ground)

$$x_R = \frac{v_o^2}{g}\sin 2\theta_o$$

→ increases as v_o increases, and as g decreases

(3) Flight Time (how long the projectile is in flight if it traverses a level ground)

$$t_f = \frac{2v_o}{g}\sin\theta_o$$

→ increases as v_o increases, g decreases, θ_o goes from 0 to 90°

(4) Maximum Height (how high the projectile goes)

$$y_{max} = \frac{v_o^2}{2g}\sin^2\theta_o$$

→ increases as v_o increases, g decreases, θ_o goes from 0 to 90°