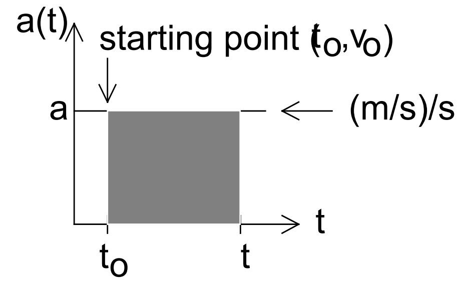
LECTURE #5 – SUMMARY



Alternative approach: use areas rather than integrals.

$v(t) = v_o +$ area under curve from t_o to t

$\therefore v(t) = v_o + a(t - t_o)$

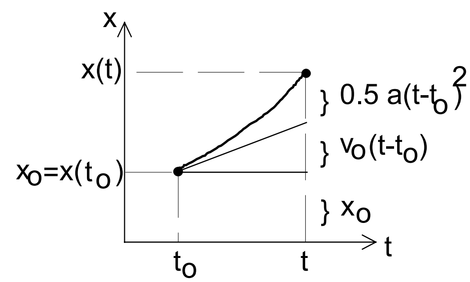
Velocity is increasing at a constant rate.

<u>Displacement</u>

$$\frac{dx(t)}{dt} = v(t) = v_o + a(t - t_o) \therefore x(t) = \int v(t)dt = \int [v_o - at_o + at]dt = v_o t - at_o t + \frac{at^2}{2} + D$$

Need initial conditions again to solve for D, the constant of integration.

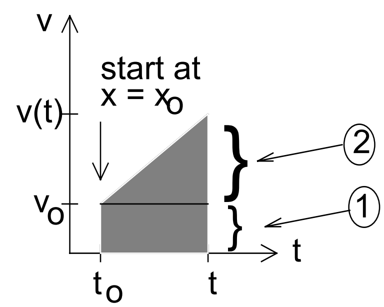
At $t=t_o$, let $x(t_o)=x_o$. Substitute in and rearrange terms to get $D = x_o - v_o t_o + \frac{1}{2} a t_o^2$.



$$\begin{aligned} \text{So} \quad x(t) &= \left(v_o t - at_o t + \tfrac{1}{2} a t^2\right) + \left(x_o - v_o t_o + \tfrac{1}{2} a t_o^2\right) \\ &= x_o + v_o t - v_o t_o - a t_o t + \tfrac{1}{2} a t^2 + \tfrac{1}{2} a t_o^2 \\ &= x_o + v_o(t - t_o) + \tfrac{1}{2} a (t - t_o)^2 \end{aligned}$$

Again, can do this using areas rather than integrals.

$x(t) = x_o +$ area under curve from t_o to t



area (1) = $v_o(t - t_o)$ $\rightarrow$ *change in position due to initial velocity*

area (2) $= \frac{1}{2}(t - t_o)[v(t) - v_o] = \frac{1}{2}(t - t_o)a(t - t_o) = \frac{1}{2}a(t - t_o)^2$

$\rightarrow$ *change in position due to increase in velocity from v_o to v(t) caused by the acceleration*

Two additional useful equations.

(1) We can get rid of "a" in the equation for x(t).

$$x(t) = x_o + v_o(t - t_o) + \tfrac{1}{2}(t - t_o)^2 \left[\frac{v(t) - v_o}{t - t_o}\right] = ... = x_o + \tfrac{1}{2}[v_o + v(t)](t - t_o)$$

(2) We can get rid of "t" and relate the velocity to the displacement.

$$x(t) - x_o = v_o(t - t_o) + \tfrac{1}{2}a(t - t_o)^2 = v_o\left[\frac{v(t) - v_o}{a}\right] + \tfrac{1}{2}a\left[\frac{v(t) - v_o}{a}\right]^2$$

$$2a[x(t) - x_o] = 2v_o v(t) - 2v_o^2 + v^2(t) - 2v_o v(t) + v_o^2 \Rightarrow v(t)^2 = v_o^2 + 2a[x(t) - x_o]$$

Summary of the equations for the case of <u>constant acceleration in 1D</u>:

$$\vec{a}(t) = a\,\hat{i}$$

$$\vec{v}(t) = v(t)\,\hat{i} = [v_o + a(t - t_o)]\,\hat{i}$$

$$\vec{x}(t) = x(t)\,\hat{i} = \left[x_o + v_o(t - t_o) + \tfrac{1}{2}a(t - t_o)^2\right]\hat{i}$$

$$\vec{x}(t) = \left\{x_o + \tfrac{1}{2}[v_o + v(t)](t - t_o)\right\}\hat{i}$$

$$v(t)^2 = v_o^2 + 2a[x(t) - x_o]$$