# LECTURE #4 – SUMMARY

(E) Acceleration

average acceleration: $\vec{a}_{avg} \equiv \dfrac{\vec{v}(t_2)-\vec{v}(t_1)}{t_2-t_1} = \dfrac{v(t_2)-v(t_1)}{t_2-t_1}\hat{i}$

instantaneous acceleration $\vec{a}(t) \equiv \lim\limits_{\Delta t\to 0}\left(\vec{a}_{avg}\right) = \lim\limits_{\Delta t\to 0}\left[\dfrac{v(t+\Delta t)-v(t)}{\Delta t}\right]\hat{i}$

$$\vec{a}(t) = \frac{d\vec{v}(t)}{dt} = \frac{dv(t)}{dt}\hat{i} = \frac{d}{dt}\left[\frac{d\vec{x}(t)}{dt}\right] = \frac{d^2\vec{x}(t)}{dt^2} = \frac{d^2x(t)}{dt^2}\hat{i}$$

Summary of the general vector relationships for motion in a straight line:

$$\vec{x}(t) = x(t)\hat{i} \qquad \vec{v}(t) = \frac{d\vec{x}(t)}{dt} = \frac{dx(t)}{dt}\hat{i} \qquad \vec{a}(t) = \frac{d\vec{v}(t)}{dt} = \frac{dv(t)}{dt}\hat{i} = \frac{d^2\vec{x}(t)}{dt^2} = \frac{d^2x(t)}{dt^2}\hat{i}$$

## Motion in a Straight Line - A Special Case with Constant Acceleration

We will now consider a special case: $a(t) = a$ $\rightarrow$ acceleration is constant

Analysis - Our Approach

- we will drop vector notation because everything is in the same direction
- previously we took derivatives to proceed from $x(t) \rightarrow v(t) \rightarrow a(t)$
- now we want to go in the opposite direction because we know a(t), i.e., we will proceed from $a(t) \rightarrow v(t) \rightarrow x(t)$
- this is somewhat harder - we will do this by integrating

Starting Point: $\boxed{a(t) = a}$

We know that $a(t) = \dfrac{dv(t)}{dt}$, so $\dfrac{dv(t)}{dt} = a(t) = a$

$$\therefore v(t) = \int a(t)dt = \int a\,dt = at + C$$

Need initial conditions to solve for C, the constant of integration.
Let's say that at $t=t_o$, $v(t_o)=v_o$.

$$v(t_o) = at_o + C = v_o$$
$$C = v_o - at_o$$
$$v(t) = at + (v_o - at_o)$$

Thus $\boxed{v(t) = v_o + a(t - t_o)}$