LECTURE #3 – SUMMARY

Integration

definite integral: $\int_{t_i}^{t_f} v(t)dt = \lim_{\substack{\Delta t \to 0 \\ (N\to\infty)}} \sum_{k=1}^{N} v(t_k)\Delta t$

= continuum equivalent of a discrete summation (= area under curve)

If $v(t) = \frac{dx(t)}{dt}$, then $\int_{t_i}^{t_f} v(t)dt = \int_{t_i}^{t_f} \frac{dx(t)}{dt} dt = \int_{t_i}^{t_f} dx(t) = x(t_f) - x(t_i) = x(t)\Big|_{t_i}^{t_f}$

indefinite integral: $x(t) = \int v(t)dt + C$

e.g. for $v(t) = at^m$, the integral is $x(t) = \frac{a}{m+1} t^{m+1} + C$

SECTION II. CLASSICAL KINEMATICS

Section II.1 Motion in a Straight Line

(A) Position Vectors - define distance and direction to a point w.r.t. some origin
At t=t_1, $\vec{x}(t_1) = x(t_1)\hat{i}$. At t=$t_2$, $\vec{x}(t_2) = x(t_2)\hat{i}$

(B) Displacement Vector
displacement = the change in position over some finite time interval
$\Delta\vec{x} \equiv \vec{x}(t_2) - \vec{x}(t_1) = [x(t_2) - x(t_1)]\hat{i} = \Delta x\hat{i}$

(C) Average Velocity
= change in position over some finite time interval, over the elapsed time
i.e. average velocity = displacement / time interval

$\vec{v}_{avg} \equiv \frac{\vec{x}(t_2) - \vec{x}(t_1)}{t_2 - t_1} = \frac{x(t_2) - x(t_1)}{t_2 - t_1}\hat{i} = \frac{\Delta\vec{x}}{\Delta t} = \frac{\Delta x}{\Delta t}\hat{i}$ = slope of line

(D) Instantaneous Velocity
= change in the position vector, NOT over some finite time interval, but rather over an infinitesimal time interval

Substitute t_1 = t and t_2 = t + Δt to get: $\vec{v}_{avg} \equiv \frac{x(t+\Delta t) - x(t)}{\Delta t}\hat{i}$

To get the instantaneous velocity at time t, shrink Δt→0.

Then $\vec{v}(t) \equiv \lim_{\Delta t\to 0}(\vec{v}_{avg}) = \lim_{\Delta t\to 0}\left[\frac{x(t+\Delta t) - x(t)}{\Delta t}\right]\hat{i} = \frac{d\vec{x}(t)}{dt} = \frac{dx(t)}{dt}\hat{i}$

= tangent to the curve at t = rate of change of displacement at time t
Note: this is a vector quantity $\vec{v} \parallel \vec{x}$, with the instantaneous speed = $|\vec{v}|$