LECTURE #30 – SUMMARY

Time Dilation *… continued*

Several points to note:

(1) The time difference between two events is always shortest in the reference frame for which the events occur at the same place (here, this is frame A').

(2) The time measured in this reference frame is called the proper time (this is always the shortest time).

(3) Time dilation does not depend on the direction of motion of the reference frame (v), because $\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$.

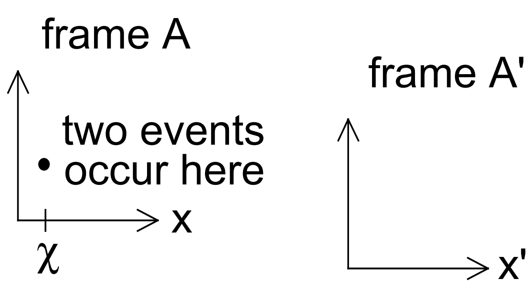
(4) Time dilation is symmetric. We don't really want to say that reference frame A' is in motion, as we don't know if A or A' is in motion.

Consider what happens if the two events occur at the same place in A.

In reference frame A:

Event (1): $t_1 = 0 \quad x_1 = \chi$

Event (2): $t_2 = \tau \quad x_2 = \chi$



In reference frame A':

Event (1): $t'_1 = \gamma\left(t_1 - \frac{v}{c^2}x_1\right) = \gamma\left(0 - \frac{v}{c^2}\chi\right)$

Event (2): $t'_2 = \gamma\left(t_2 - \frac{v}{c^2}x_2\right) = \gamma\left(\tau - \frac{v}{c^2}\chi\right)$

Thus $\boxed{t_2 - t_1 = \tau}$ and $\boxed{t'_2 - t'_1 = \gamma\tau}$ (the reverse of what we first derived!)

(5) The only way to apply time dilation is if you have two events that happen at the same PLACE in one reference frame.

e.g., clocks (this is why clocks are often used in STR examples - because we assume a clock remains in the same place in a given reference frame)

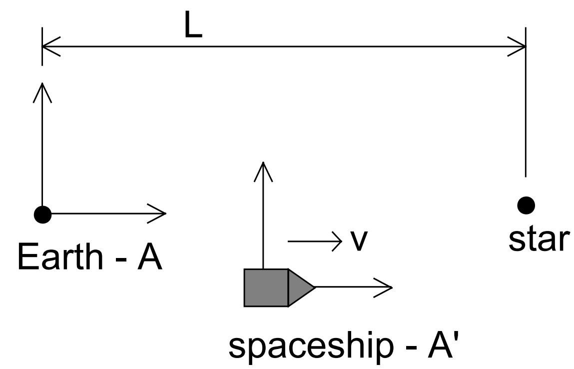
Note: We also derived the equations for time dilation by considering a "light box". See the textbook, Chapter 38, pages 1014-1016, for this approach.

The Twin Paradox

Time dilation allows travel into the future - the twin paradox is a famous example of this possibility.

One of two twins travels on a fast spaceship to a distant star, while the other stays on Earth. There are clocks on Earth and on the star (A), and on the moving spaceship (A'). After reaching the star, the first twin returns to Earth and is younger than the twin who remained behind because of time dilation. The difference in their ages can be arbitrarily large depending on how far and how fast the travelling twin goes.

Consider the view from the Earth-star reference frame (A):



First, let's define a lightyear.
One <u>lightyear</u> is the distance travelled by light, at speed c, in one year:

c = 1 lightyear / year

1 lightyear = c × 1 year

In relativity, often work in these units so c = 1.

One-way travel time for spaceship:

$$\Delta t_{one-way} = \frac{L}{v} = \frac{L\ \ (m)}{v\ \ (m/s)} = \frac{L\ \ (m)}{\beta c\ \ (m/s)} = \frac{L\ \ (lightyears)}{\beta\ \ (lightyears/yr)} = \frac{L}{\beta}\,(yr)$$

Say L = 10 lightyears, $\beta = 0.8$: $\Delta t_{one\text{-}way} = 12.5$ yr

Two-way travel time: $\Delta t_{two-way} = \frac{2L}{\beta} = 25$ yr

Can we apply time dilation here? Yes, because events occur at the same place relative to the spaceship (A').

In reference frame A':
Event (1): spaceship is at Earth $x'_1 = 0$
Event (2): spaceship back at Earth: $x'_2 = 0$
Spacecraft appears to have stayed at rest in A'.

Since events occur in reference frame A' at the same place, the time between events must be shortest in A': $\Delta t' = \frac{\Delta t}{\gamma} = \frac{2L}{\beta\gamma}$

(Could also use the "safe" approach – apply Lorentz transformations.)

Substituting in our numbers: $\Delta t' = \frac{\Delta t}{\gamma} = \frac{25}{1/\sqrt{1-0.8^2}} = 15\ \ yr$

Summary – two-way travel time (assuming c = 1 here):

Frame A (Earth)	**Frame A' (spaceship)**	**Difference**
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$\Delta t = \frac{2L}{\beta}$	$\Delta t' = \frac{2L}{\beta\gamma}$	$\Delta t - \Delta t' = \frac{2L}{\beta}\left(1-\frac{1}{\gamma}\right) = \frac{2L}{\beta}\left(1-\sqrt{1-\beta^2}\right)$

as $v \to 0$, then $\beta \to 0$ and $\gamma \to 1$, so:

$\Delta t = \frac{2L}{\beta}$	$\Delta t' = \frac{2L}{\beta\gamma}$	$\Delta t - \Delta t' \to 0$

as $v \to c$, then $\beta \to 1$ and $\gamma \to 0$, so:

$\Delta t \to 2L$	$\Delta t' \to 0$	$\Delta t - \Delta t'$ (yr) $\to 2L$ (ly)

The twin in the spaceship can go very very far and upon returning to Earth, will be almost the same age if v is close to c. The problem is that the twin on Earth will have aged an arbitrarily large amount (2L years, where L is arbitrary).

<u>The paradox is that from the spaceship, it is the Earth that moves away and back, so the Earth-bound twin should be younger.</u>

Does the twin paradox violate the Principle of Relativity?

After all, in our example, when the spaceship returns, 15 years have elapsed for the travelling twin but 25 years have elapsed for the stationary twin.
→ We will know which twin travelled and which one didn't, which should not be possible if they are both in inertial frames of reference.

However, STR is not violated!
The travelling twin turned around - this implies acceleration and therefore STR no longer applies to this case (remembering the "Special" in STR). While the Earth-bound twin remains in an inertial frame of reference, the travelling twin experiences two inertial reference frames and a period of acceleration. This makes it possible to determine which twin travelled and which did not.

What if the spacecraft did not turn around and return to Earth?

The Earth-bound twin would think that the spaceship's clock ran slower, while the travelling twin would think that Earth's clock ran slower (symmetry). However, the twins cannot compare their clocks unless they get together. Clocks synchronized in one reference frame will not be synchronized in another frame.