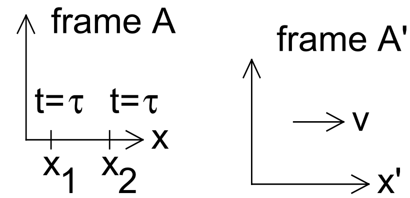
LECTURE #29 – SUMMARY

Applications: (1) Simultaneity

Say two events happen at the same time and different places as measured in A. Do the two events occur at the same time as measured in reference frame A'?



Event 1: $x = x_1 \quad t = t_1 = \tau$

Event 2: $x = x_2 \quad t = t_2 = \tau$

Apply the Lorentz transformations:

$$t'_1 = \gamma\left(t_1 - \frac{v}{c^2}x_1\right) = \gamma\left(\tau - \frac{v}{c^2}x_1\right) \text{ and } t'_2 = \gamma\left(\tau - \frac{v}{c^2}x_2\right)$$

$$\boxed{\therefore t'_2 - t'_1 = -\frac{\gamma v}{c^2}(x_2 - x_1) \neq 0}$$

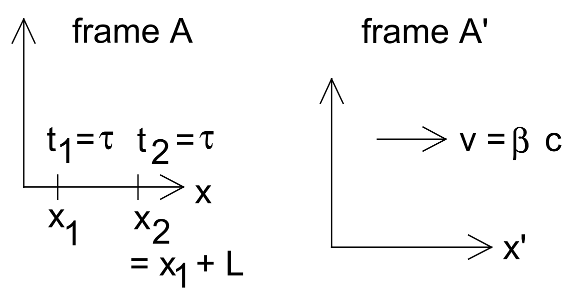
So the two events do NOT occur simultaneously in frame A'.

Mathematically: $t' = \gamma\left(t - \frac{v}{c^2}x\right)$ and $dt' = \gamma\left(dt - \frac{v}{c^2}dx\right)$

And so, if the events are simultaneous in frame A, then dt=0, giving $\boxed{dt' = -\frac{\gamma v}{c^2}dx}$

Example:

Say I am standing in the middle of a room of width L. I throw two pieces of chalk in opposite directions and they hit the walls simultaneously. If an observer is moving with velocity v=βc relative to the room, what will the observer measure as the time difference between impacts?



$$t'_2 - t'_1 = -\frac{\gamma v}{c^2}(x_1 + L - x_1) = -\frac{1}{\sqrt{1-\beta^2}}\frac{\beta L}{c}$$

If β = 0.6, L = 10 m, then: $t'_2 - t'_1 = -2.50 \times 10^{-8}$ sec

If β = 0.999999, L = 1000 m = 1 km, then:

$t'_2 - t'_1 = -2.36 \times 10^{-3}$ sec

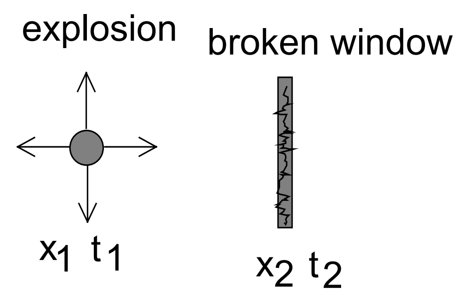
- Answer does not depend on the velocity of the objects.
- If $\beta > 0$, then $t'_2 < t'_1$. If $\beta < 0$, then $t'_2 > t'_1$.

Applications: (2) Causality

Let's say that an event at (x_1, t_1) as measured in reference frame A "causes" a second event at (x_2, t_2) as also measured in reference frame A, where $t_2 > t_1$.

Example:

Is it possible that, seen from another reference frame, say A', the second event (broken window) occurs BEFORE the first event (explosion)?



i.e., Is it possible that $t'_1 > t'_2$?
If so, then we have a problem:
(1) causality is violated (idea of cause and effect)
(2) unpalatable because the principle of relativity requires that "the laws of physics are the same in all inertial reference frames"

Apply the Lorentz transformations:

$$t'_2 = \gamma\left(t_2 - \frac{v}{c^2}x_2\right) \quad \text{and} \quad t'_1 = \gamma\left(t_1 - \frac{v}{c^2}x_1\right) \qquad \therefore t'_2 - t'_1 = \gamma(t_2 - t_1)\left[1 - \frac{v}{c^2}\frac{(x_2 - x_1)}{(t_2 - t_1)}\right]$$

$\frac{(x_2 - x_1)}{(t_2 - t_1)}$ is the velocity of the causal agent, v_c, i.e., the velocity of the explosion.

Thus: $\quad t'_2 - t'_1 = \gamma(t_2 - t_1)\left[1 - \frac{v}{c^2}v_c\right]$ (where v is velocity of A' with respect to A)

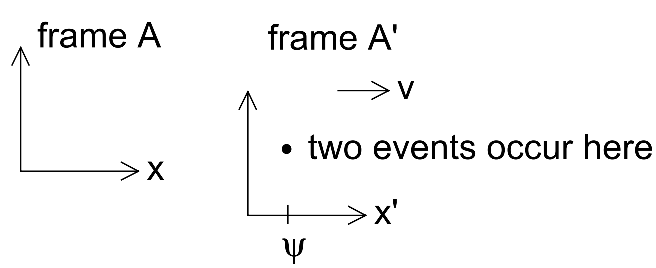
Now: (1) $t_2 > t_1$, (2) $\gamma > 0$, and (3) $1 - \frac{v}{c^2}v_c > 0$ because $|v_c| < c$ and $|v| < c$.

Therefore, $t'_2 - t'_1 > 0$ and $t'_2 > t'_1$ and so causality is not violated. Phew!

V.4 Time Dilation, the Twin Paradox, and the Lorentz Contraction

Time Dilation

Say we have two events that occur at the same place in reference frame A'.
In reference frame A':
Event (1): $t'_1 = 0 \quad x'_1 = \psi$
Event (2): $t'_2 = \tau \quad x'_2 = \psi$



In reference frame A:

Event (1) $\quad t_1 = \gamma\left(t'_1 + \frac{v}{c^2}x_1\right) = \gamma\left(0 + \frac{v}{c^2}\psi\right)$

Event (2) $\quad t_2 = \gamma\left(t'_2 + \frac{v}{c^2}x_2\right) = \gamma\left(\tau + \frac{v}{c^2}\psi\right)$

Thus: $\quad \boxed{t'_2 - t'_1 = \tau}$ and $\boxed{t_2 - t_1 = \gamma\tau > \tau}$

The elapsed time appears longer in A than in A' (recall τ is in A') $\rightarrow$ time dilation

In general: $\boxed{\Delta t = \gamma \Delta t' = \frac{\Delta t'}{\sqrt{1 - v^2/c^2}}}$ or $\boxed{\Delta t' = \frac{\Delta t}{\gamma} = \Delta t\sqrt{1 - v^2/c^2}}$

where A' is the frame in which the two events occurred at the same place.