LECTURE #27 – SUMMARY

SECTION V. SPECIAL RELATIVITY

Section V.1 The Special Theory of Relativity

Principle of Galilean Relativity:
"The laws of mechanics are the same in all inertial reference frames."

Special Theory of Relativity:
"The laws of physics are the same in all inertial reference frames."
(Einstein, 1905, age 26)

Why so important?
- led to the application of the notions of relativity to all physical phenomena, all the laws of physics, not just to a restricted range of phenomena.
- if a theory does not hold in all inertial reference frames, then it is wrong!
- if the applications of this theory are strange or counter-intuitive, so be it!
- brought consistency and harmony to a number of troubling problems in physics

Section V.2 Implications of Special Relativity

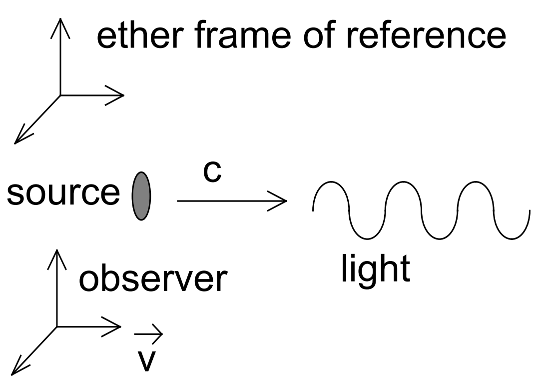
Ether and the Speed of Light

Maxwell's Equations (19th century) assumed that EM waves propagate at speed "c" in free space regardless of the motion of the source. This created a problem: What is "c" relative to? Intuition suggested that light might behave like sound. Sound requires a medium to propagate and the speed of sound is the speed relative to this medium (e.g., air, water, etc.). By extension, light was thought to have a medium and "c" is the speed of light relative to this medium. $\Rightarrow$ ether

Scientists widely believed in ether:
- permeated the universe
- had low viscosity, allowing it to creep everywhere and provide no resistance to motion
- was very stiff since "c" is very large (the harder the medium, the faster sound travels, e.g., waves travel faster along a stiff spring)

Consider light emitted by a source (1D):

$$v_{\text{observer}}^{\text{light}} = v_{\text{ether}}^{\text{light}} + v_{\text{observer}}^{\text{ether}} = c - v$$



This implies that Maxwell's Eqns only hold in the ether frame of reference because only here do EM waves have speed "c". An observer moving wrt the ether would measure a different speed of light.

This violates the principle of relativity since:

- Maxwell's Equations only hold in the ether frame
- an absolute (ether) frame could presumably be detected

There are two options:
(1) The principle of relativity does not hold for all laws. (try to detect the ether)
(2) Einstein's Special Theory of Relativity which implies that
"The speed of light is the same in all inertial reference frames."
$\rightarrow$ only in this way that Maxwell's Eqns could hold in all inertial reference frames
$\rightarrow$ all observers in inertial reference frames must see the same speed of light "c"!
$\rightarrow$ this is often written as a restatement of STR (or synomous with STR), but this is not true: it is an implication of STR
$\rightarrow$ ether does not exist

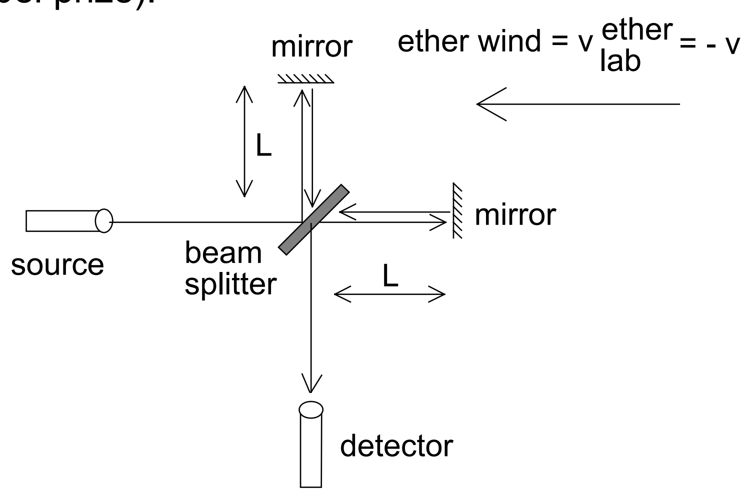
Tests to detect the ether in order to support option (1) can be seen as an important test of STR. Let's consider approaches to detecting the ether.

(Q1) Is the ether dragged by the Earth? $\rightarrow$ Aberration of Starlight
Statement: If Earth does not drag the ether with it, then the direction from which starlight appears to come will depend on the motion of Earth wrt the ether. If ether drag does occur, then starlight will always come from the same direction.
Observation: As Earth travels around its orbit, the position of the stars seems to change.
Conclusion: Earth does NOT drag the ether with it and must move through it.

(Q2) Can we detect Earth's motion through the ether? $\rightarrow$ Michelson-Morley Expt

Michelson-Morley Experiment (1881-1887 - before STR)
This experiment had sufficient sensitivity to detect the ether wind if it existed. It used a Michelson Interferometer (Nobel prize).

Since the Earth revolves around the Sun, the Earth must be moving relative to the ether. Let's say that the ether wind is from right to left (i.e., Earth moves through the ether from left to right). What is the difference in travel time between the two arms of the interferometer?



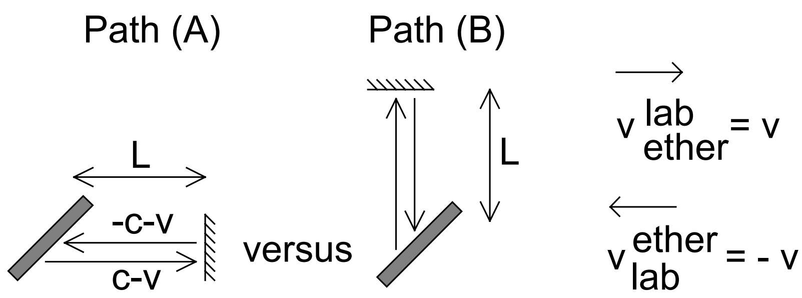
(A) First consider the round trip parallel to the motion (ether wind) and back.

Define: $v_{ether}^{lab} = v \quad (= v_{ether}^{Earth})$

So: $u = v_{lab}^{light} = v_{ether}^{light} + v_{lab}^{ether} = \begin{cases} c - v & (\text{left} \rightarrow \text{right}) \\ -c - v & (\text{right} \rightarrow \text{left}) \end{cases}$

Therefore, the round-trip travel time along path (A) is:

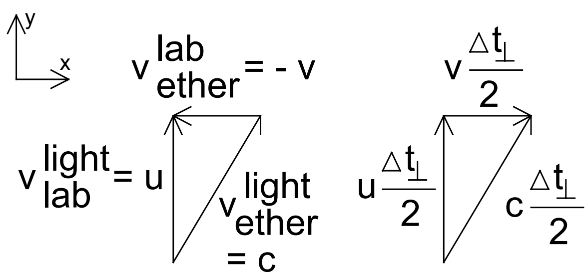
$$\Delta t_{||} = \frac{L}{v_{ether}^{light} + v_{lab}^{ether}}_{(\text{left} \rightarrow \text{right})} + \frac{L}{v_{ether}^{light} + v_{lab}^{ether}}_{(\text{right} \rightarrow \text{left})} = \frac{L}{|c-v|} + \frac{L}{|-c-v|} = \frac{2L/c}{1 - v^2/c^2}$$



(B) Now consider the round trip perpendicular to the motion (ether wind) and back.

$\vec{v}_{lab}^{light} = \vec{v}_{ether}^{light} + \vec{v}_{lab}^{ether}$ so $c^2 = u^2 + |-v|^2$

$\vec{u} = \vec{c} + \vec{v}$ $\quad \therefore u = \sqrt{c^2 - v^2}$



The round-trip travel time along path (B) is:

$$\Delta t_{\perp} = \frac{2L}{u} = \frac{2L}{\sqrt{c^2 - v^2}} = \frac{2L/c}{\sqrt{1 - v^2/c^2}} \neq \Delta t_{||}$$

The difference in the round-trip travel times is thus:

$$\Delta t_{||} - \Delta t_{\perp} = \frac{2L/c}{1 - v^2/c^2} - \frac{2L/c}{\sqrt{1 - v^2/c^2}} = \frac{2L}{c}\left[\frac{1}{1 - v^2/c^2} - \frac{1}{\sqrt{1 - v^2/c^2}}\right]$$

Now, we need to use the approximations:

$$\frac{1}{1 - \Delta x} \cong 1 + \Delta x \quad (\Delta x << 1) \qquad \frac{1}{\sqrt{1 - \Delta x}} \cong 1 + \frac{\Delta x}{2} \quad (\Delta x << 1)$$

So $$\Delta t_{||} - \Delta t_{\perp} = \frac{2L}{c}\left[\left(1 + \frac{v^2}{c^2}\right) - \left(1 + \frac{v^2}{2c^2}\right)\right] = \frac{2L}{c}\frac{v^2}{2c^2} = \frac{L}{c}\frac{v^2}{c^2}$$

Path difference between (A) and (B): $c(\Delta t_{||} - \Delta t_{\perp}) = \frac{Lv^2}{c^2}$