LECTURE #26 – SUMMARY

(1) What value of ω_d maximizes A ? Set $\dfrac{dA}{d\omega_d} = 0$: $\boxed{\omega_d^{max} = \sqrt{\omega_o^2 - \dfrac{b^2}{2m^2}}}$

(2) What is the value of A_{max} ? Evaluate A for ω_d^{max} : $\boxed{A_{max} = F_o \Big/ b\sqrt{\omega_o^2 - \dfrac{b^2}{4m^2}}}$

(3) What is the full width at half maximum?

Want values of ω_d for which $A = \frac{1}{2} A_{max}$, so that $\text{FWHM} = \Delta\omega = \left|\omega_d^+ - \omega_d^-\right|$.

$\boxed{\omega_d^{\pm} \cong \omega_o \pm \dfrac{b}{m}}$ and $\boxed{\text{FWHM} = \dfrac{2b}{m}}$ assuming that damping is relatively small.

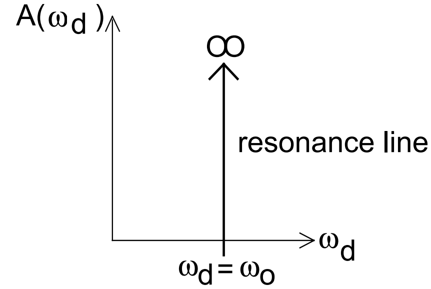
Resonance

Special case of driven harmonic motion with b = 0, no damping.

As $b \to 0$: $\omega_d^{max} \to \omega_o$, $A_{max} \to \infty$, $\text{FWHM} \to 0$

and as $\omega_d \to \omega_o$: $\qquad A \to \dfrac{F_o}{m(\omega_d^2 - \omega_o^2)} \to \infty$

$\Rightarrow$ this is resonance



As the driving frequency approaches the natural frequency, the amplitude goes into resonance. The output response can be huge. Damping "smears" out the resonance response.

If $\omega_d \neq \omega_o$, resonance still occurs, but A_{max} is finite.

Tacoma Narrows Bridge – collapsed in 1940

(1) Conceptual explanation: Oscillation of forces applied by the wind were at the natural frequency of the bridge, causing “sympathetic vibrations” and an increase in the amplitude until the bridge could no longer withstand the stress.

(2) Causal explanation: When the wind exceeded a minimum speed and blew around the bridge, vortices formed downwind, broke loose, and flowed away. As these broke away, they exerted a transverse driving force on the bridge.

(3) Quantitative explanation: Model the bridge as if suspended by two springs with equal k. Apply Newton’s Second Law for translation and for rotation:

$$F_{net}^{translation} = -k(y_1 + y_2) = ma \qquad \text{and} \qquad \tau_{net}^{rotation} = -\tfrac{1}{2}kL(y_1 - y_2) = I\alpha$$

Solutions (SHM): $y_1 = A_1 \sin(\omega_1 t)$ and $y_2 = A_2 \sin(\omega_2 t)$

Two normal modes of vibration:

vertical motion: $\omega_1 = 2k/m \Rightarrow f_1 = 8$ Hz

torsional motion: $\omega_2 = kL^2/2I \Rightarrow f_2 = 10$ Hz ($\approx f_1$, but usually 2-4 times f_1)

Torsional oscillations set in (due to mechanical failure) and were closely coupled to the vertical motion, leading to the collapse of the bridge.