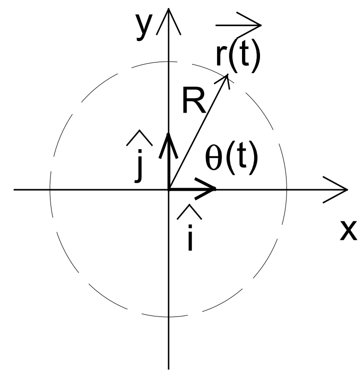
LECTURE #23 – SUMMARY

Uniform Circular Motion and Simple Harmonic Motion



For UCM, position vector: $\vec{r}(t) = R\cos\theta(t)\hat{i} + R\sin\theta(t)\hat{j}$

phase: $\theta(t) = \omega t + \delta$

$\therefore \quad \vec{r}(t) = R\cos(\omega t + \delta)\hat{i} + R\sin(\omega t + \delta)\hat{j}$

$\vec{v}(t) = -\omega R\sin(\omega t + \delta)\hat{i} + \omega R\cos(\omega t + \delta)\hat{j}$

$\vec{a}(t) = -\omega^2 R\cos(\omega t + \delta)\hat{i} - \omega^2 R\sin(\omega t + \delta)\hat{j}$

$\therefore \quad a_x(t) = -\omega^2 x(t)$ and $a_y(t) = -\omega^2 y(t)$

So, UCM is the superposition of SHM along the x and y axes with

- equal amplitudes (R) and equal angular velocities (ω)
- $\pi/2$ phase difference since $R\sin(\omega t + \delta + \frac{\pi}{2}) = R\cos(\omega t + \delta)$

Uniform Circular Motion: $\boxed{\vec{r}(t) = R\cos(\omega t + \delta)\hat{i} + R\cos(\omega t + \delta - \frac{\pi}{2})\hat{j}}$

Any 2-D Periodic Motion: $\boxed{\vec{r}(t) = R_1\cos(\omega t + \delta_1)\hat{i} + R_2\cos(\omega t + \delta_2 - \frac{\pi}{2})\hat{j}}$

Section IV.2 Springs and Pendulums – Examples of SHM

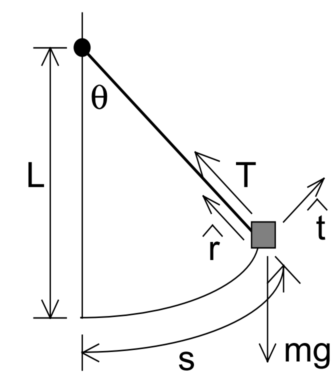
(1) Springs

F_o^s = force of spring on block (restoring force): $F_o^s = -kx$, $ma = -kx$, $a = -\frac{k}{m}x$

This is SHM because $a \propto x$ and a is oppositely directed to x.

$\therefore \; x(t) = A\cos(\omega t + \delta)$ and $\omega^2 = -\frac{a}{x} = \frac{k}{m}$ so $\boxed{\omega = \sqrt{\frac{k}{m}}}$ $\boxed{T = 2\pi\sqrt{\frac{m}{k}}}$

(2) Pendulums



Force tangent to the trajectory: $F_{\hat{t}} = -mg\sin\theta$

Also have: $F_{\hat{t}} = ma_{\hat{t}} = m\frac{d^2 s}{dt^2} = mL\frac{d^2\theta}{dt^2}$ using $s = L\theta$

$\therefore \quad \frac{d^2\theta}{dt^2} = -\frac{g}{L}\sin\theta \cong -\frac{g}{L}\theta$ using $\sin\theta \cong \theta$ for small θ

This is again SHM, with acceleration $\propto$ displacement, and in the opposing direction. Here, gravity acts as the restoring force.

In this case: $\theta(t) = \theta_o\cos(\omega t + \delta)$ with $\boxed{\omega = \sqrt{\frac{g}{L}}}$ and $\boxed{T = 2\pi\sqrt{\frac{L}{g}}}$

So a simple pendulum perturbed slightly from equilibrium ($\theta \approx 0$) exhibits SHM. If θ is large, then the situation becomes non-linear and is one of the simplest systems which exhibits chaotic behaviour.