LECTURE #22 – SUMMARY

SECTION IV. SIMPLE HARMONIC MOTION

Section IV.1 Definition of Simple Harmonic Motion

Simple harmonic motion is motion in which the position of a point varies with time in a sinusoidal fashion.

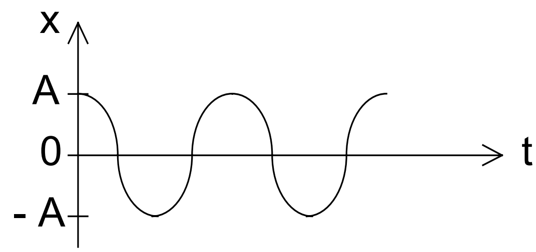
i.e., $\boxed{x(t) = A\cos(\omega t + \delta)}$

A = amplitude

ω = angular frequency (rad/sec)

δ = arbitrary constant = phase shift (rad)

$\omega t + \delta$ = total phase



SHM is important because a large number of physical systems will display SHM when perturbed a small amount from equilbrium. SHM thus describes the response to small departures from equilbrium.

Period of oscillation: $\boxed{T = \frac{2\pi}{\omega}}$ (i.e., $\omega T = 2\pi$ and SHM repeats after 2π rad)

Frequency: $\boxed{f = \frac{\omega}{2\pi}}$ cycles/second $\therefore$ $\boxed{T = \frac{1}{f}}$

Properties of SHM:

Displacement: $x(t) = A\cos(\omega t + \delta)$

Velocity: $v(t) = dx(t)/dt = -A\omega\sin(\omega t + \delta)$

Acceleration: $a(t) = dv(t)/dt = -A\omega^2\cos(\omega t + \delta) = -\omega^2 x(t)$

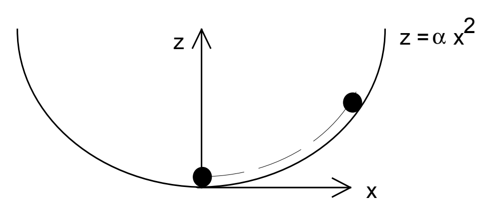
By Newton's Second Law: $F(t) = ma(t) = -m\omega^2 x(t)$

So, in SHM, acceleration $\propto$ displacement and force $\propto$ displacement.

Because the force has opposite sign to the displacement, this is a restoring force.

Example: A bead at the bottom of a bowl.

At equilibrium, the bead sits at the bottom of the bowl. What happens if the bead is perturbed a little bit? SHM in x?



Have $-g\sin\phi = \frac{d^2s}{dt^2}$ and $\sin\phi = \frac{2\alpha x}{\sqrt{1 + (2\alpha x)^2}}$ so $-g\frac{2\alpha x}{\sqrt{1+(2\alpha x)^2}} = \frac{d^2s}{dt^2}$.

If the bowl is small OR the motions are small, then $s \cong x$ and $2\alpha x << 1$.

The equation of motion becomes: $-2g\alpha x \cong \frac{d^2x}{dt^2}$

This is SHM with $\omega = \sqrt{2g\alpha}$ and $x = A\cos(\omega t + \delta)$.