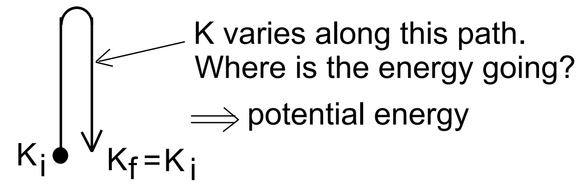
LECTURE #20 – SUMMARY

Conservation of Energy



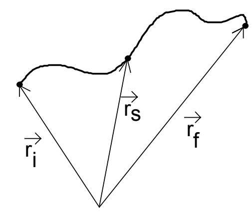
Consider throwing a ball upwards.

$W_{net} = 0 \qquad K_f = K_i \qquad$ in a closed path

Now consider a general path and assume that only conservative forces act:

$$W_{net} = \int_{\vec{r}_i}^{\vec{r}_f} \vec{F}^c_{net} \bullet d\vec{r} = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2$$

In this case, W is independent of the path and ∴ we can make up a path.



$$\int_{\vec{r}_i}^{\vec{r}_f} \vec{F}^c_{net} \bullet d\vec{r} = \int_{\vec{r}_i}^{\vec{r}_s} \vec{F}^c_{net} \bullet d\vec{r} + \int_{\vec{r}_s}^{\vec{r}_f} \vec{F}^c_{net} \bullet d\vec{r} = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2$$

$$\tfrac{1}{2}mv_f^2 - \int_{\vec{r}_s}^{\vec{r}_f} \vec{F}^c_{net} \bullet d\vec{r} = \tfrac{1}{2}mv_i^2 - \int_{\vec{r}_s}^{\vec{r}_i} \vec{F}^c_{net} \bullet d\vec{r} \qquad \text{(a)}$$

There is a TOTAL quantity that doesn't change along the path.

Define potential energy: $\boxed{U(\vec{r}) \equiv U(\vec{r}_s) - \int_{\vec{r}_s}^{\vec{r}} \vec{F}^c_{net} \bullet d\vec{r}}$

where $U(\vec{r}_s)$ is just a reference value, $\vec{F}^c_{net}$ refers only to conservative forces.

Equation (a) becomes: $\tfrac{1}{2}mv_f^2 + U(\vec{r}_f) = \tfrac{1}{2}mv_i^2 + U(\vec{r}_i)$

Define mechanical energy: $\boxed{E_m(\vec{r}) \equiv \tfrac{1}{2}mv^2 + U(\vec{r})}$

Thus, the mechanical energy is constant when only conservative forces are acting. This is the Law of Conservation of Mechanical Energy: $\boxed{E_m(\vec{r}_f) \equiv E_m(\vec{r}_i)}$

It can also be written as: $K + U = \text{constant}$ or $\Delta K + \Delta U = 0$

How do we calculate potential energy? $\Delta U(\vec{r}_s \to \vec{r}) = U(\vec{r}) - U(\vec{r}_s) = -\int_{\vec{r}_s}^{\vec{r}} \vec{F}^c_{net} \bullet d\vec{r}$

= change in potential energy from $\vec{r}_s$ to $\vec{r}$

= - of work done on an object by conservative forces

(1) Gravitational potential energy: $\boxed{\Delta U_{gravitational} = mg\Delta y}$

from $\Delta U(A \to B) = -\int_{\vec{r}_A}^{\vec{r}_B} \vec{F}_g \bullet d\vec{r} = -[(0 \times \Delta x) + (-mg \times \Delta y)] = +mg\Delta y$

(2) Potential energy of springs: $\boxed{\Delta U_{spring} = \tfrac{1}{2}kL^2}$ (or $\Delta U = \tfrac{1}{2}k\Delta x^2$)

from $\Delta U(0 \to L) = -\int_0^L F^s_o dx = -\int_0^L (-kx)dx = \tfrac{1}{2}kL^2$